SIMULATION AND INTERPRETATION OF BOREHOLE GEOPHYSICAL MEASUREMENTS USING *hp* FINTE ELEMENTS

***hp*-FEM team:** **D. Pardo, M. J. Nam , L. Demkowicz,**

C. Torres-Verdín, V. M. Calo,

M. Paszynski, and P. J. Matuszyk

8th Annual Formation Evaluation
Research Consortium Meeting
August 14-15, 2008

THE UNIVERSITY OF TEXAS AT AUSTIN

Overview

1. Main Lines of Research and Applications (D. Pardo)
   - Previous work
   - Main features of our technology
2. Application 1: Tri-Axial Induction Instruments (M. J. Nam)
3. Application 2: Dual-Laterolog Instruments (M. J. Nam)
4. Multi-Physics Inversion (D. Pardo)
5. Sonic Instruments (L. Demkowicz)

Previous Work

Type of Problems We Can Solve with our *hp*-FEM software

Applications	Borehole Measurements		Marine Controlled Source EM
Spatial Dimensions	2D		3D
Well Type	Vertical Well	Deviated Well	Eccentered Tool
Logging Instruments	LWD/MWD	Normal/Laterolog	Dual-Laterolog
	Induction	Dielectric Instruments	Cross-Well
Frequency	0 ~ 10 GHz		
Materials	Isotropic	Anisotropic	
Physical Devices	Magnetic Buffers	Insulators	Casing
	Casing Imperfections	Displacement Currents	Combination of All
Sources	Finite Size Antennas	Dipoles in Any Direction	Solenoidal Antennas
	Toroidal Antennas	Electrodes	Combination of All
Invasion	Water	Oil	etc.

MOST (OIL-INDUSTRY) GEOPHYSICAL PROBLEMS

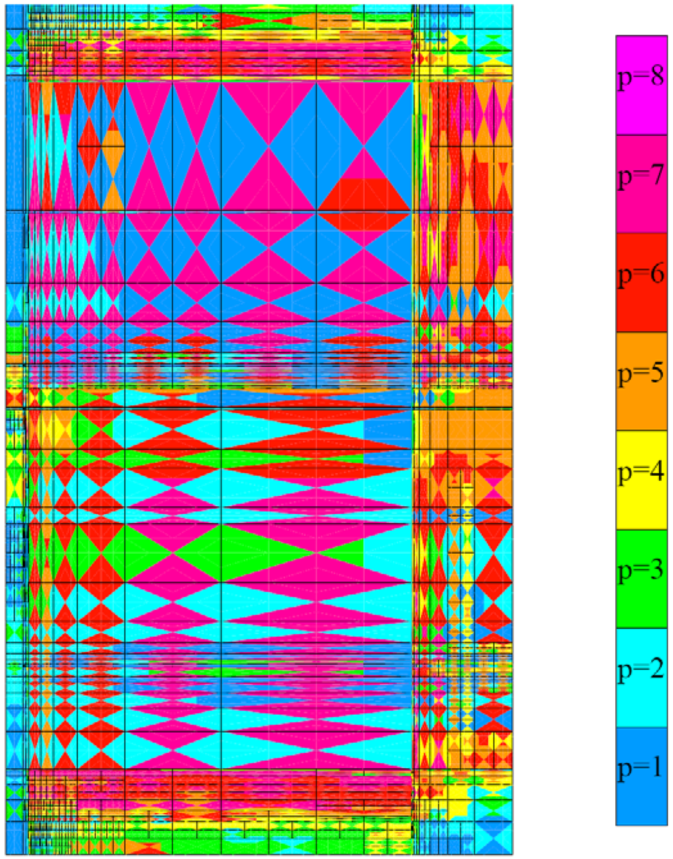
Main Features of Our Technology

1. Self-Adaptive Goal-Oriented *hp*-Refinements

2. Fourier Finite-Element Method

3. Parallel Implementation

Self-Adaptive Goal-Oriented *hp*-FEM



We vary locally the element size *h* and the polynomial order of approximation *p* throughout the grid.

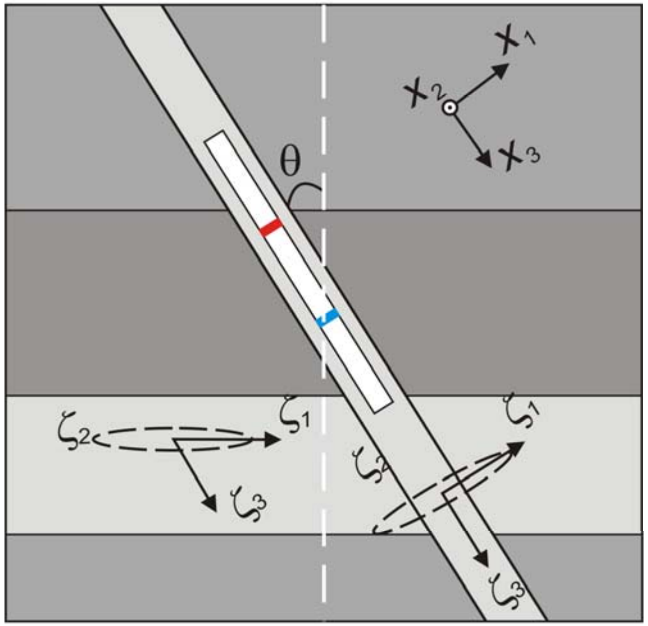
Optimal grids are automatically generated by the *hp*-algorithm.

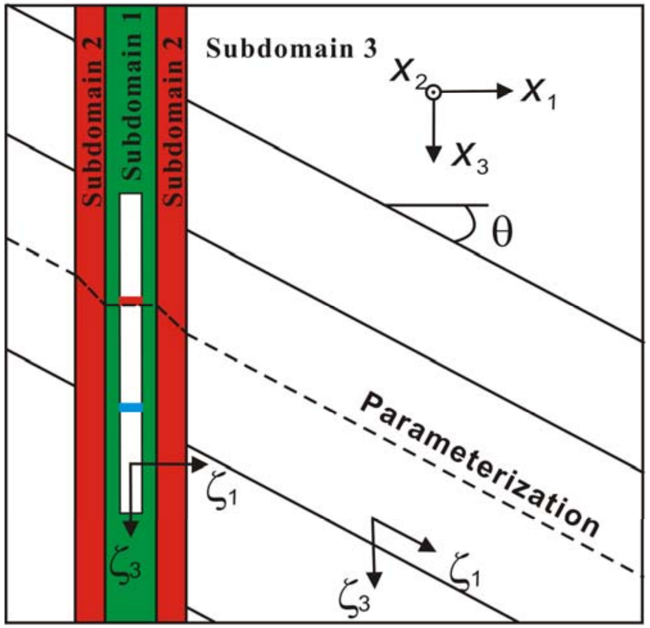
The self-adaptive goal-oriented *hp*-FEM provides exponential convergence rates in terms of the CPU time vs. the error in a user prescribed quantity of Interest.

3D Deviated Well

Cartesian system of coordinates: (x_1, x_2, x_3)

New non-orthogonal system of coordinates: $(\zeta_1, \zeta_2, \zeta_3)$





Subdomain 1

$$\begin{cases} x_1 = \zeta_1 \cos \zeta_2 \\ x_2 = \zeta_1 \sin \zeta_2 \\ x_3 = \zeta_3 \end{cases}$$

Subdomain 2

$$\begin{cases} x_1 = \zeta_1 \cos \zeta_2 \\ x_2 = \zeta_1 \sin \zeta_2 \\ x_3 = \zeta_3 + \tan\theta \dfrac{\zeta_1 - \rho_1}{\rho_2 - \rho_1} \rho_2 \cos \zeta_2 \end{cases}$$

Subdomain 3

$$\begin{cases} x_1 = \zeta_1 \cos \zeta_2 \\ x_2 = \zeta_1 \sin \zeta_2 \\ x_3 = \zeta_3 + \zeta_1 \tan\theta \cos \zeta_2 \end{cases}$$

3D Deviated Well

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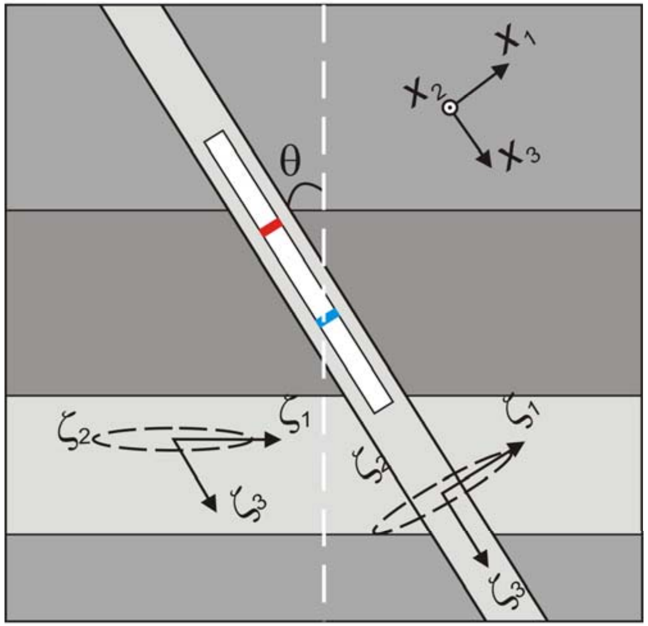
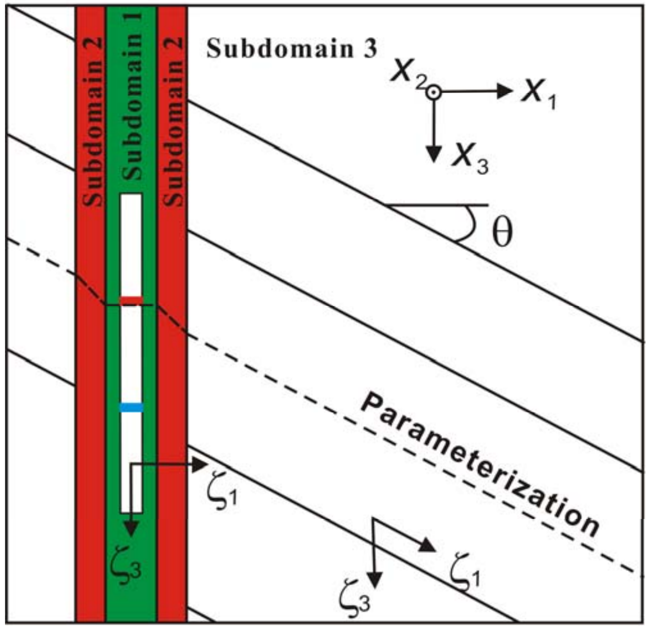
New non-orthogonal system of coordinates: $(\zeta_1, \zeta_2, \zeta_3)$





Constant material coefficients in the quasi-azimuthal direction ζ_2 in the new non-orthogonal system of coordinates!!!!

3D Deviated Well

For each Fourier mode, we obtain a 2D problem. Each 2D problem couples with up to five different 2D problems corresponding to different Fourier modes, therefore, constituting the resulting 3D problem.

When we use 9 Fourier Modes for the Solution:

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{2,1} & A_{2,2} & A_{2,3} & A_{2,4} & 0 & 0 & 0 & 0 & 0 \\ A_{3,1} & A_{3,2} & A_{3,3} & A_{3,4} & A_{3,5} & 0 & 0 & 0 & 0 \\ 0 & A_{4,2} & A_{4,3} & A_{4,4} & A_{4,5} & A_{4,6} & 0 & 0 & 0 \\ 0 & 0 & A_{5,3} & A_{5,4} & A_{5,5} & A_{5,6} & A_{5,7} & 0 & 0 \\ 0 & 0 & 0 & A_{6,4} & A_{6,5} & A_{6,6} & A_{6,7} & A_{6,8} & 0 \\ 0 & 0 & 0 & 0 & A_{7,5} & A_{7,6} & A_{7,7} & A_{7,8} & A_{7,9} \\ 0 & 0 & 0 & 0 & 0 & A_{8,6} & A_{8,7} & A_{8,8} & A_{8,9} \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{9,7} & A_{9,8} & A_{9,9} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \\ b_8 \\ b_9 \end{bmatrix}$$

$A_{i,j}$: represents a full 2D problem for each Fourier basis function

3D Deviated Well

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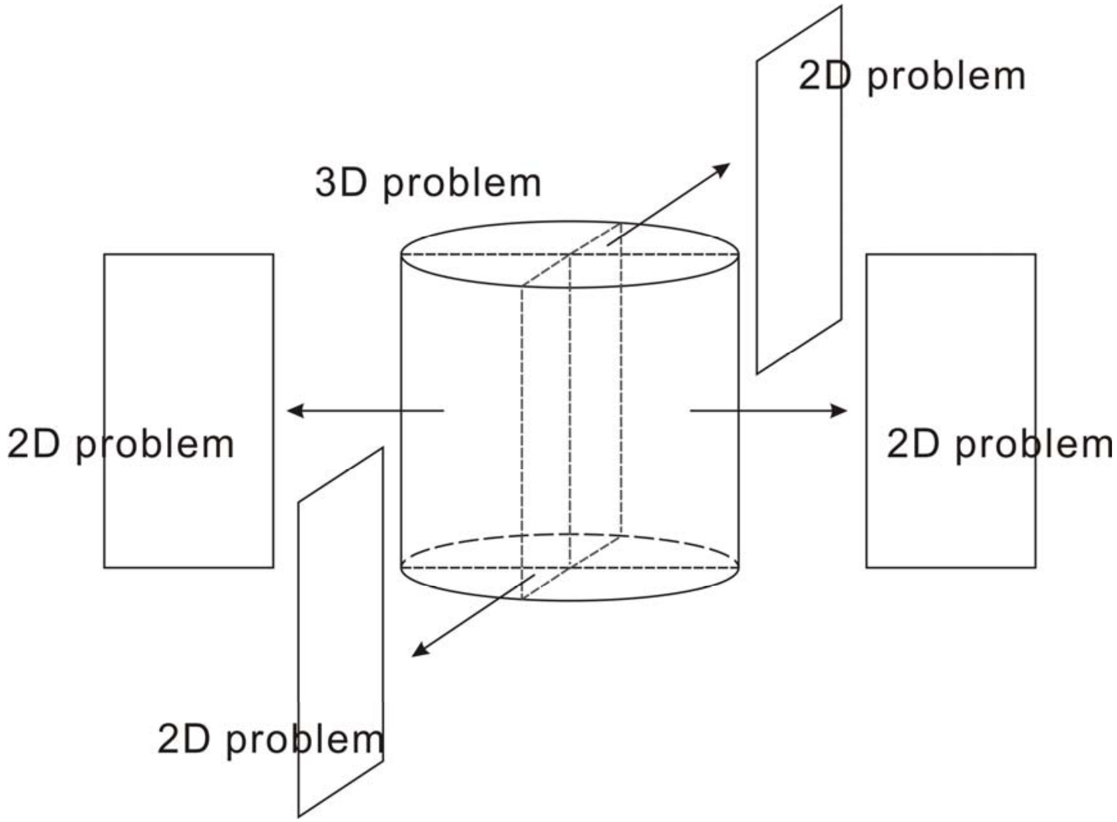
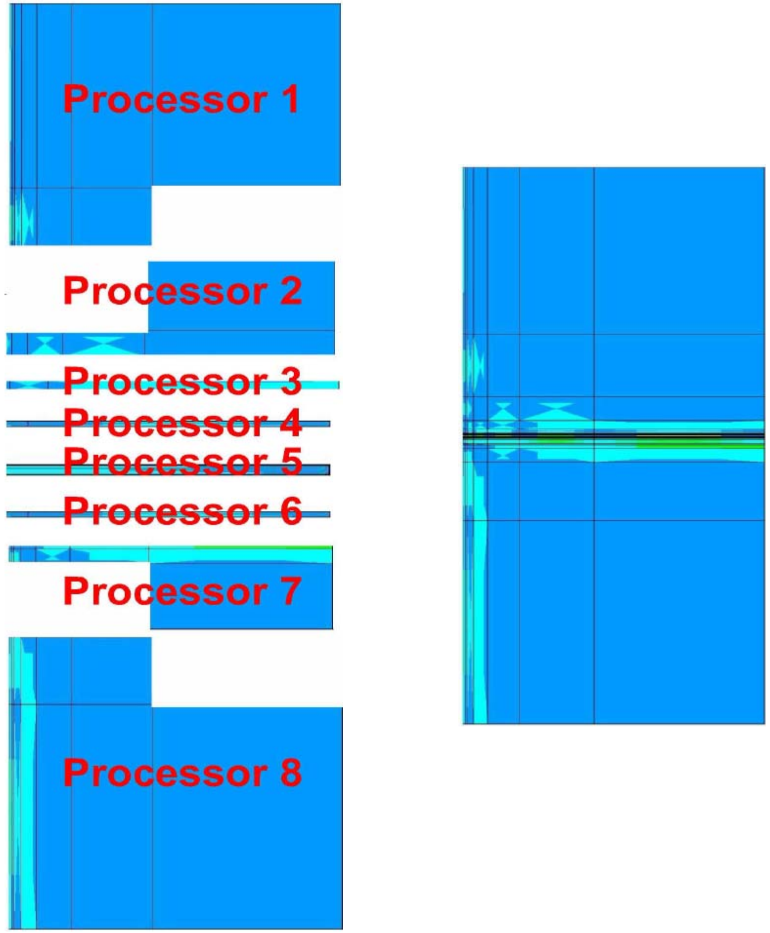
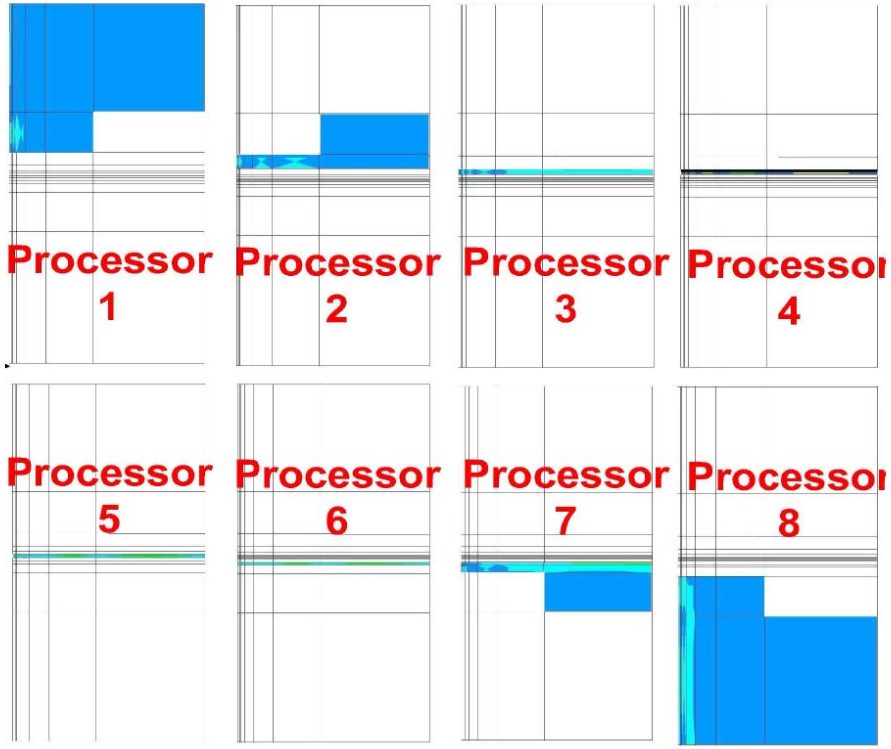


3D Parallelization Implementation

Distributed Domain Decomposition



Shared Domain Decomposition!!



SELF-ADAPTIVE *hp* FINITE-ELEMENT SIMULATION OF MULTI-COMPONENT INDUCTION MEASUREMENTS ACUIRED IN DIPPING, INVADED, AND ANISOTROPIC FORMATIONS

M. J. Nam, D. Pardo, and C. Torres-Verdín,
The University of Texas at Austin

***hp*-FEM team: D. Pardo, M. J. Nam, L. Demkowicz, C. Torres-Verdín,**
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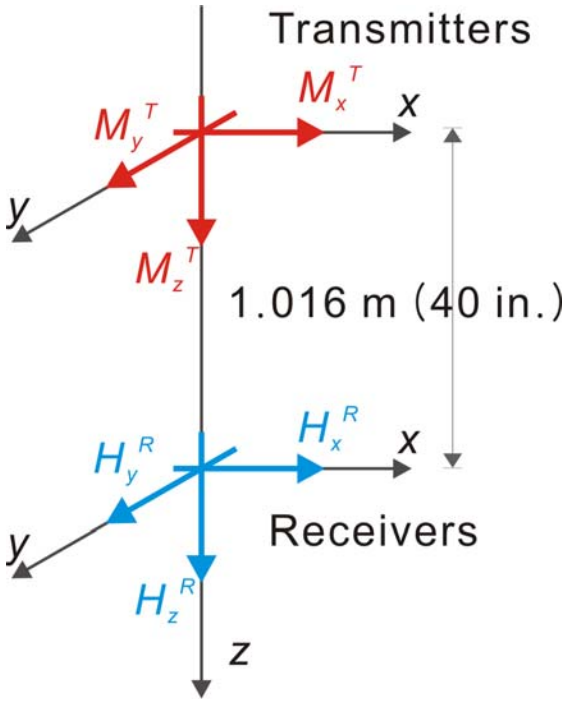
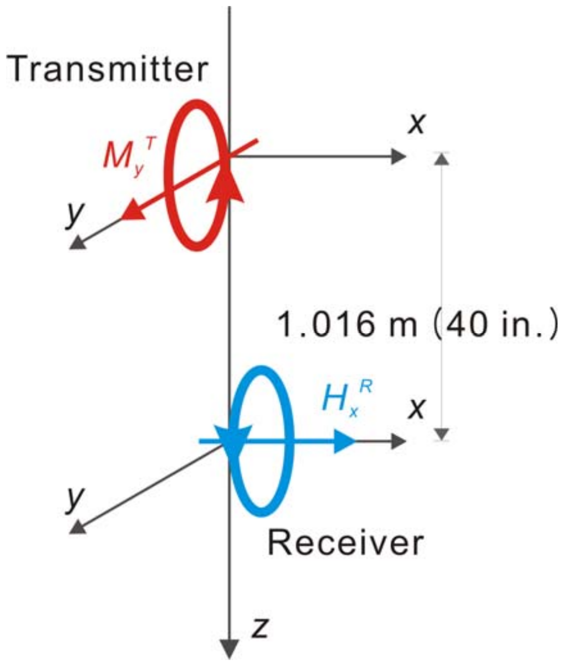
4. **Multi-Physics Inversion (D. Pardo)**
5. **Sonic Instruments (L. Demkowicz)**

Outline

- Introduction to Tri-Axial Induction
- Method
- Numerical Results:
  - Verification of 3D Method for Tri-Axial Induction Tool
  - Dipping, Invaded, Anisotropic Formations
- Conclusions

Tri-Axial Induction Tool

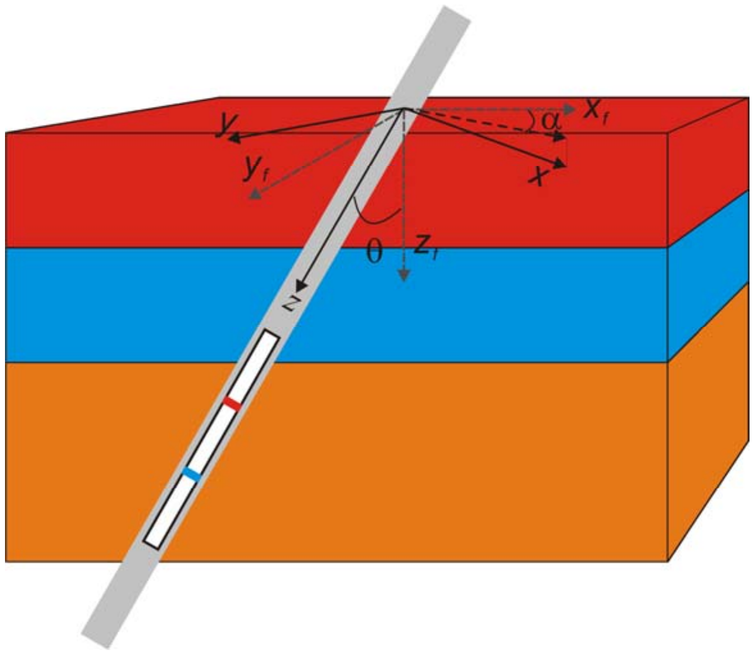




H_{xy}

L = 1.016 m (40 In.)

Operating frequency: 20 kHz



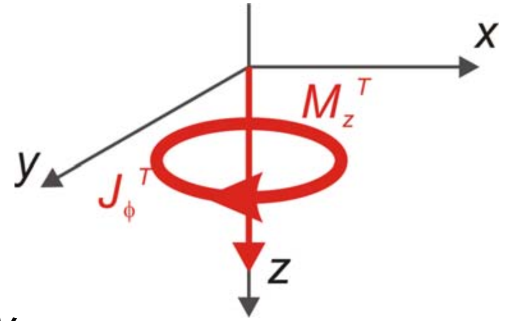
θ: dip angle

α: tool orientation angle

3D Source Implementation

1. Solenoidal Coil (J_ϕ) for M_z

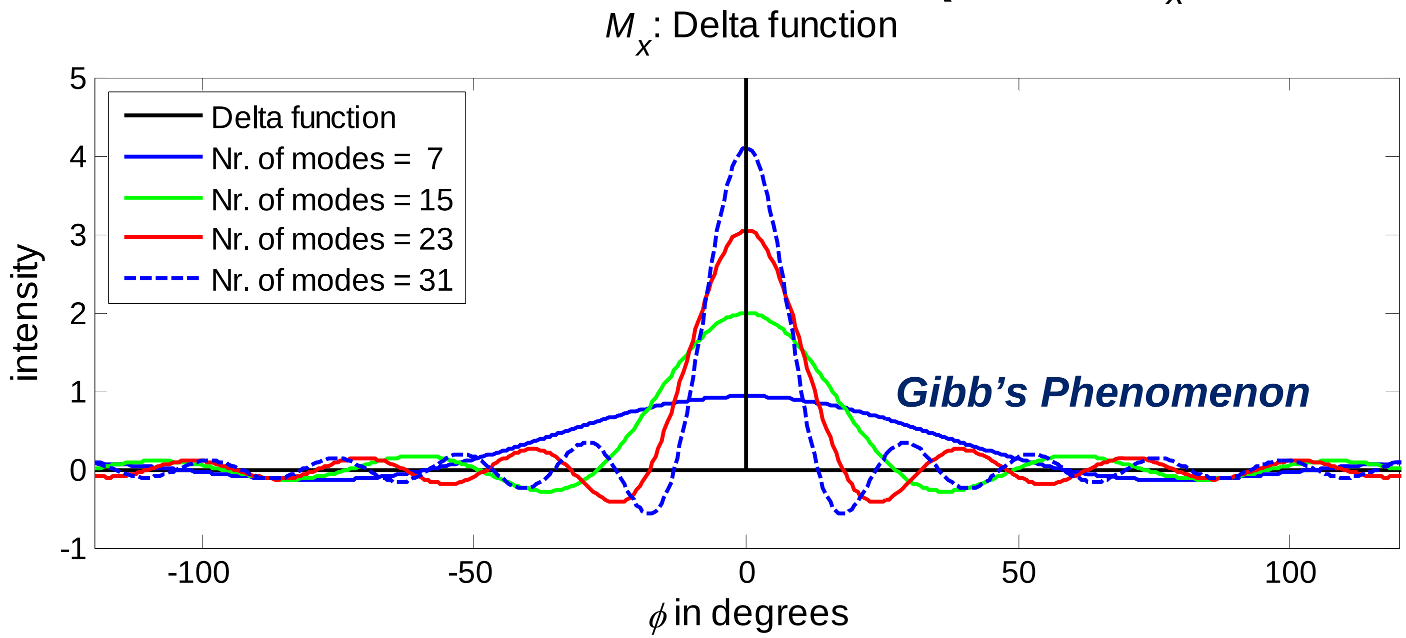
→ ***becoming a 2D source in (ρ, ϕ, z)***

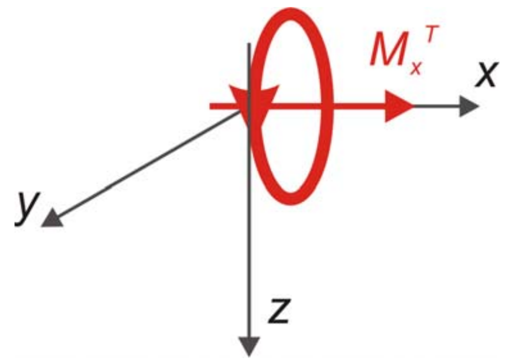


2. Delta Function for 3D source M_x or M_y

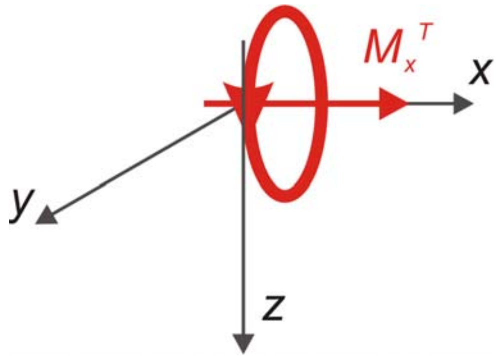
$$f(\phi) = \delta(\phi - \phi_0)$$

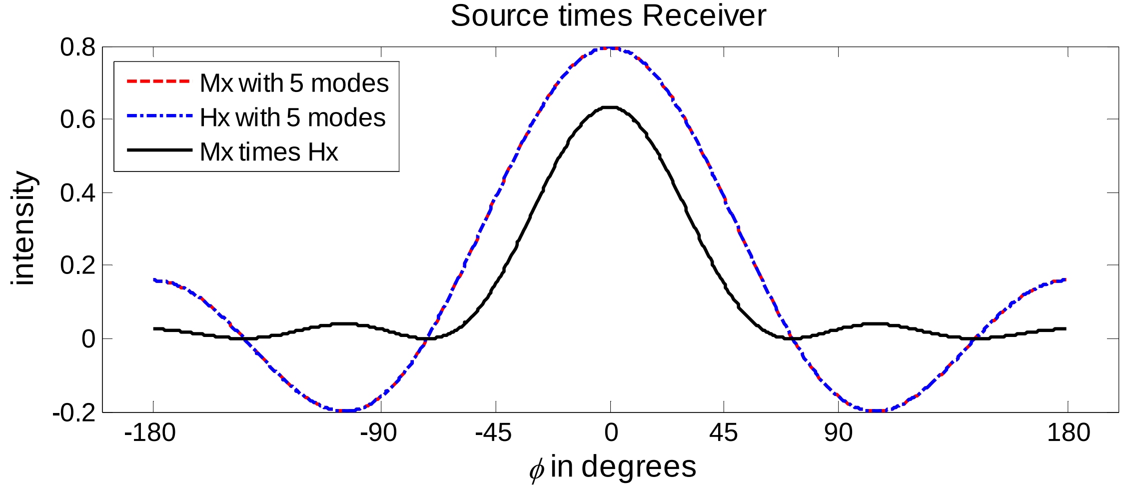
ϕ_0: the position of the center of the peak (0° for M_x; 90° for M_y)

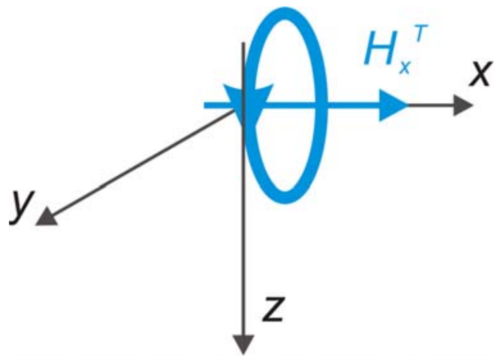


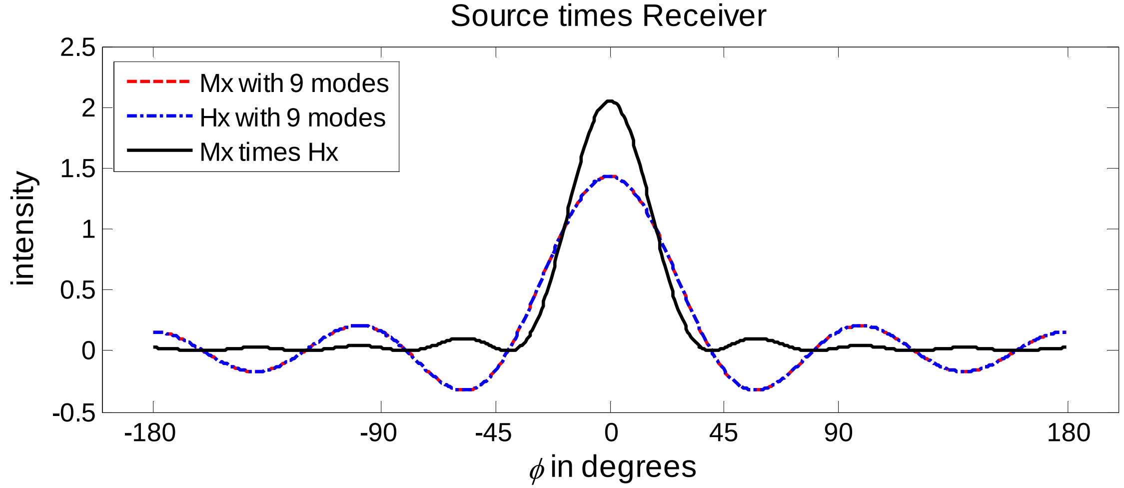


3D Source and Receiver (Delta Functions)









Coupling between source and receiver: less Gibb's phenomenon

Method

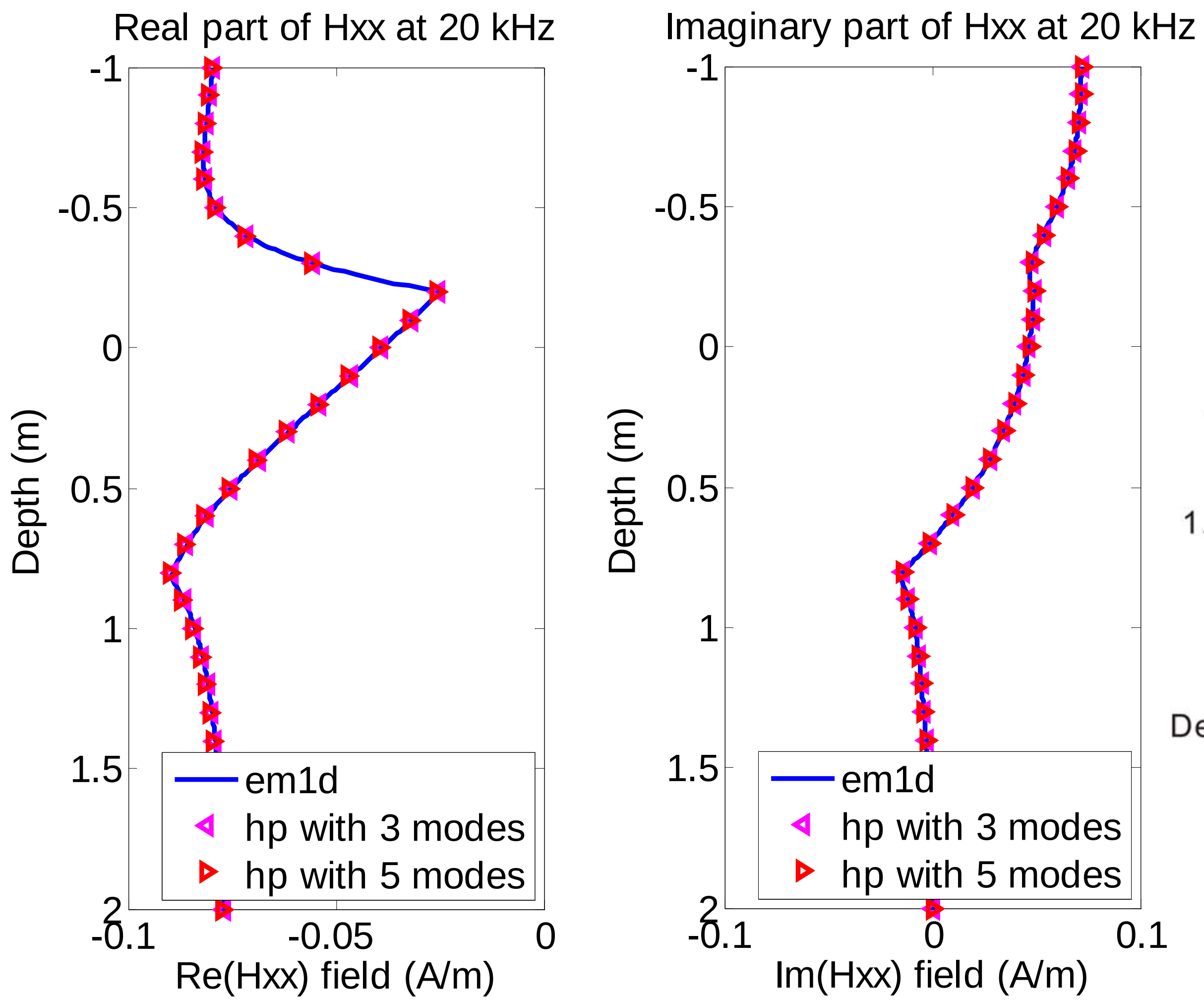
Combination of:

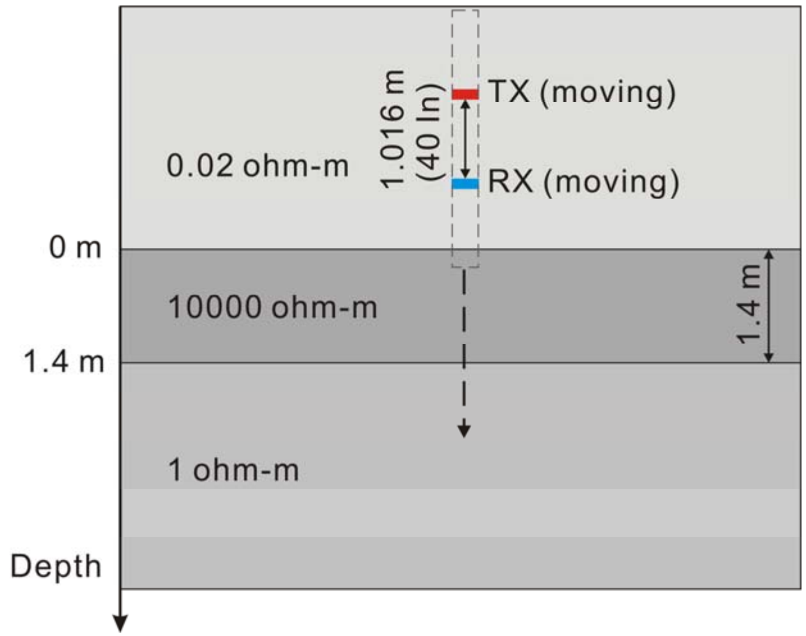
1. **A Self-Adaptive Goal-Oriented *hp*-FEM for AC problems**

2. **A Fourier Series Expansion in a Non-Orthogonal System of Coordinates**

3. **Parallel Implementation**

Verification of 2.5D Simulation ($H_{xx} = H_{yy}$)

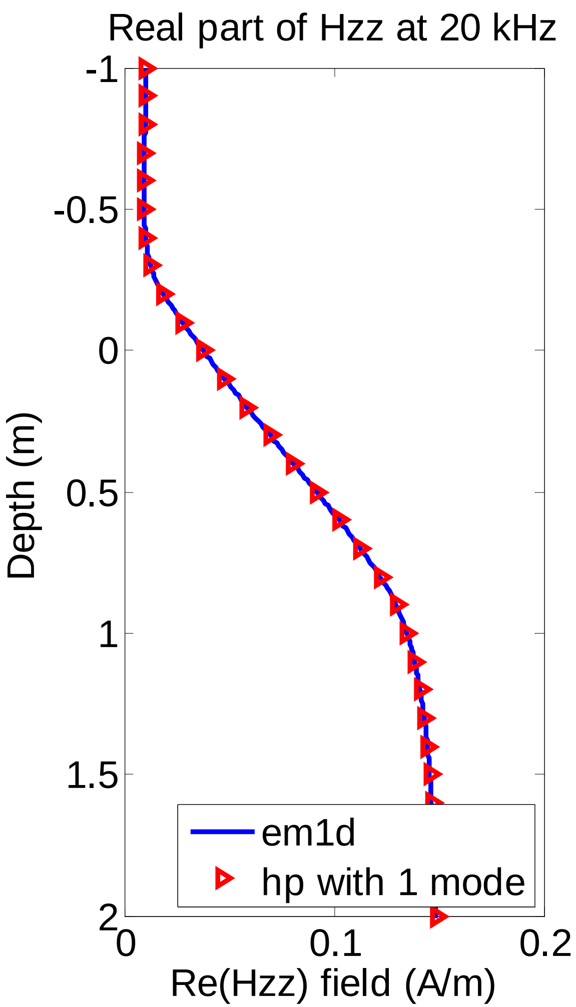


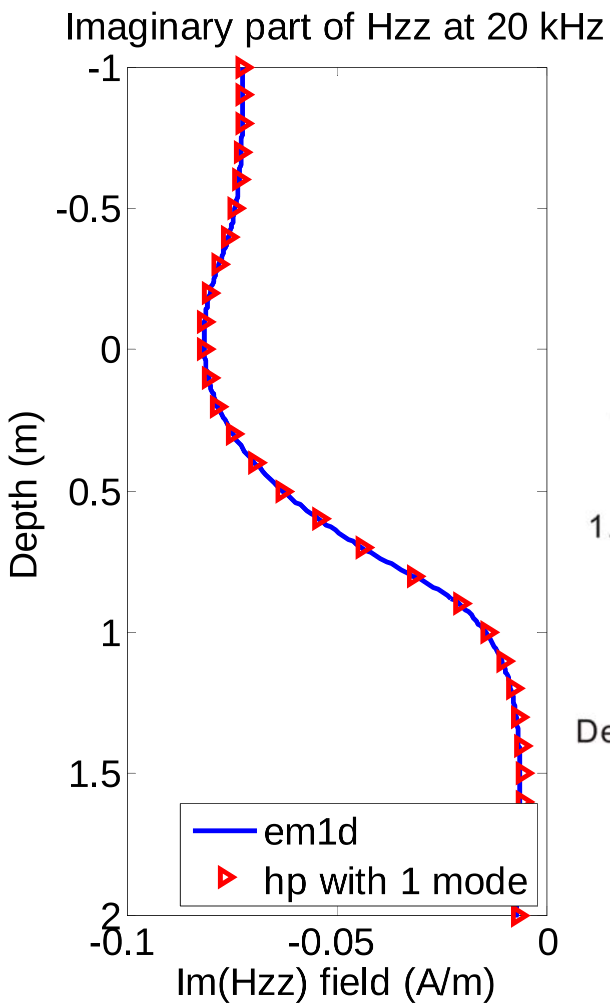


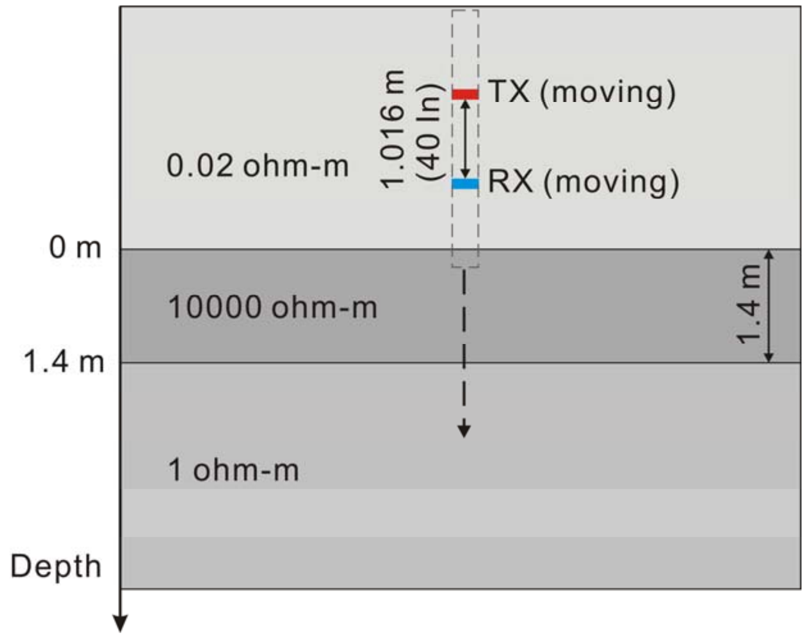
Converged solutions with 3 Fourier modes

em1D: K. H. Lee 1984, pers. comm.

Verification of 2.5D Simulation (H_{zz})

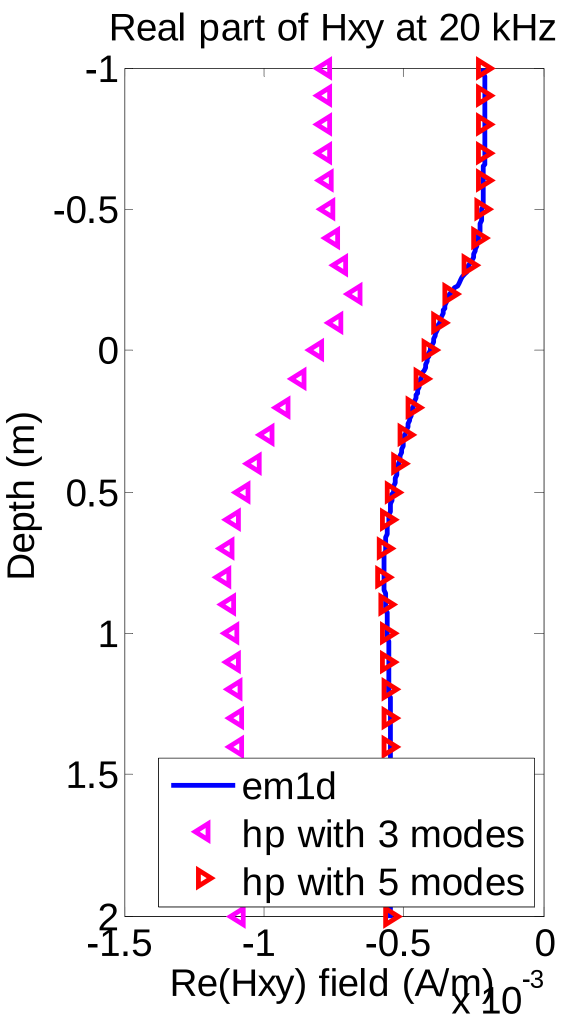


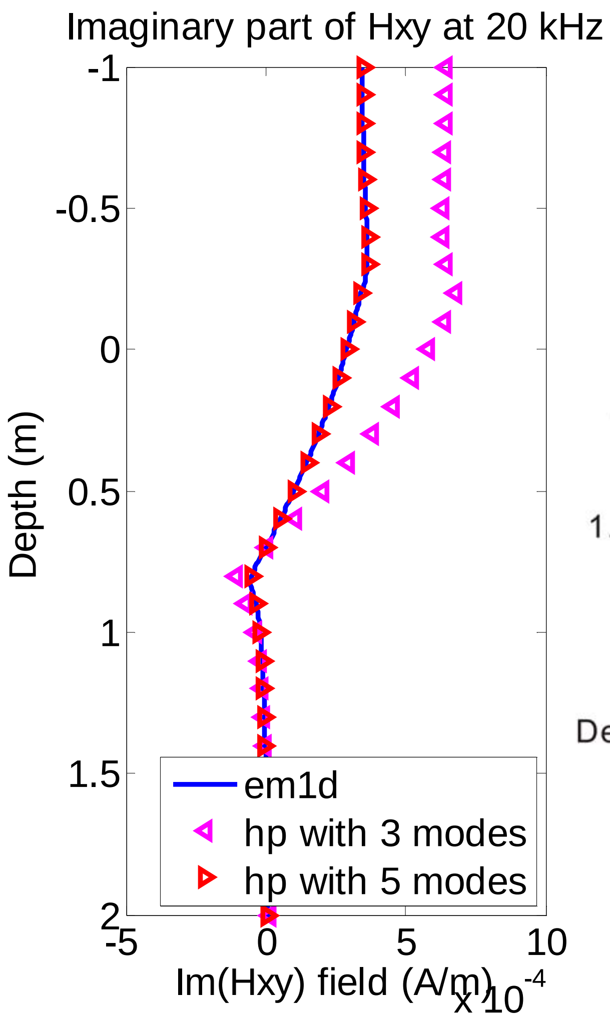


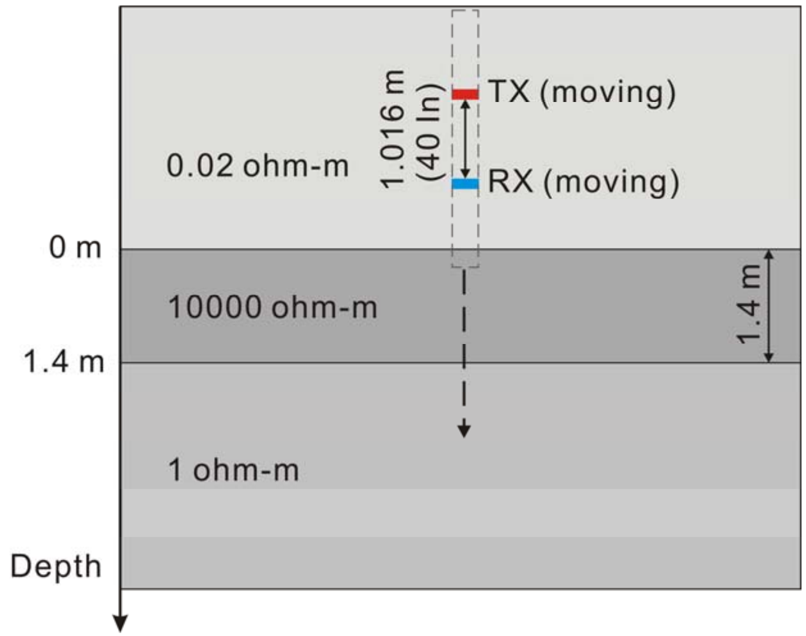


The same solutions with 1 Fourier mode

Verification of 2.5D Simulation ($H_{xy} = H_{yx}$)

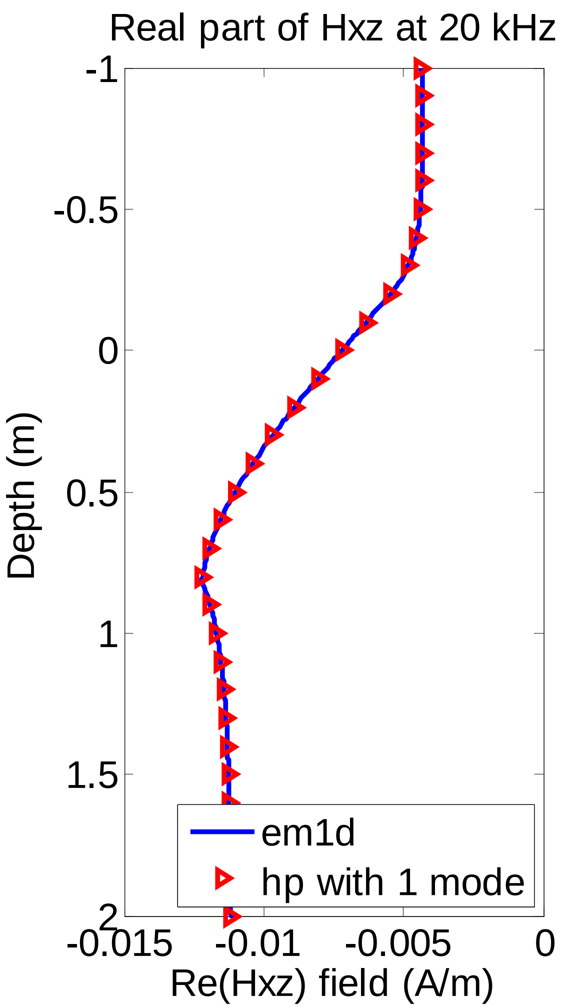


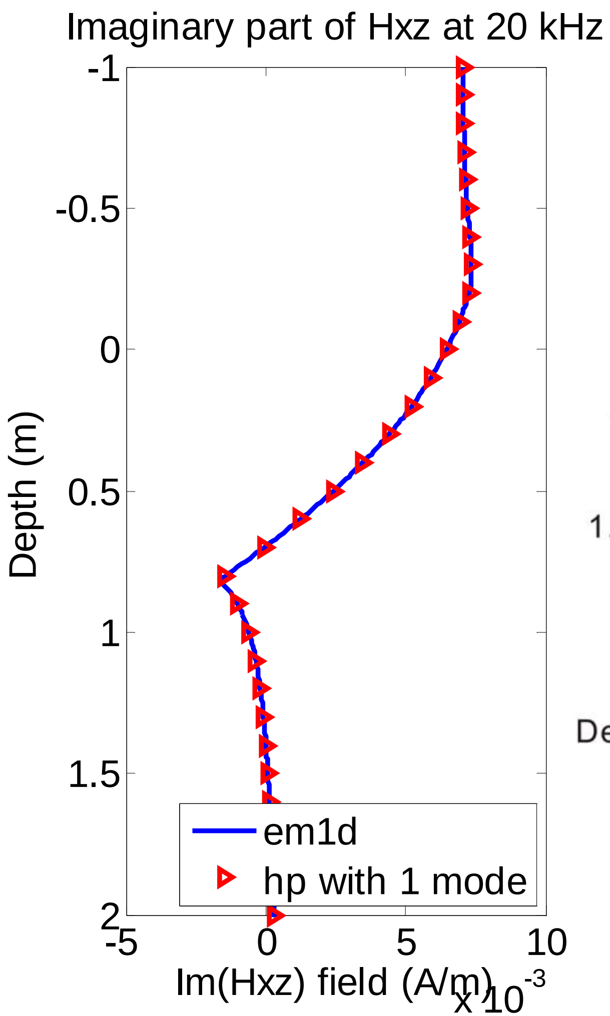


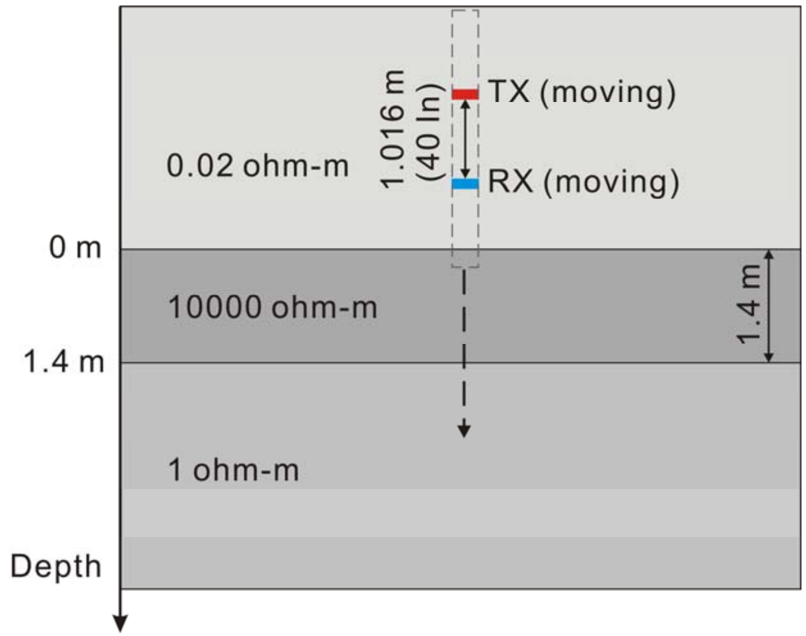


Converged solutions with 5 Fourier modes

Verification of 2.5D Simulation ($H_{xz} = H_{zx}$)

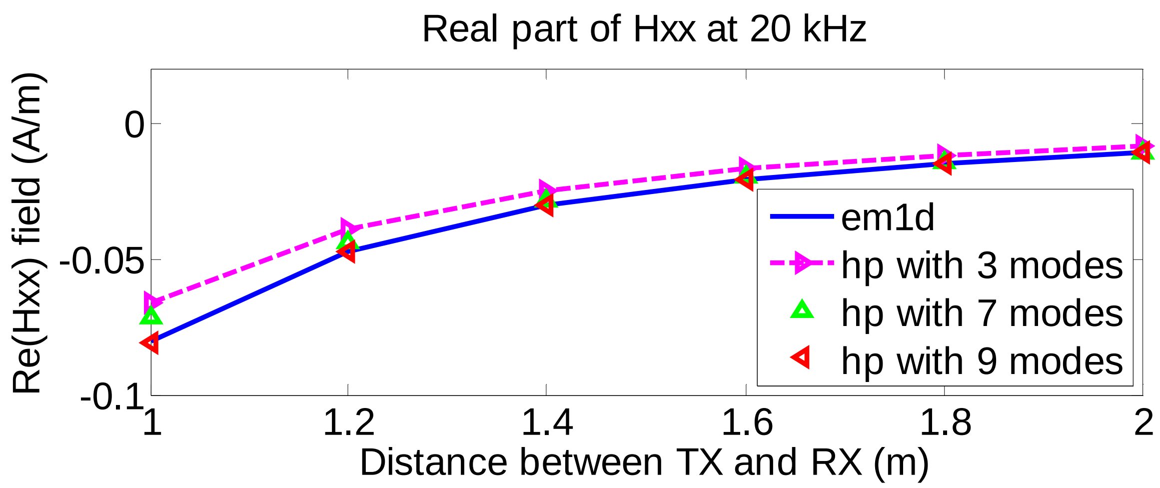


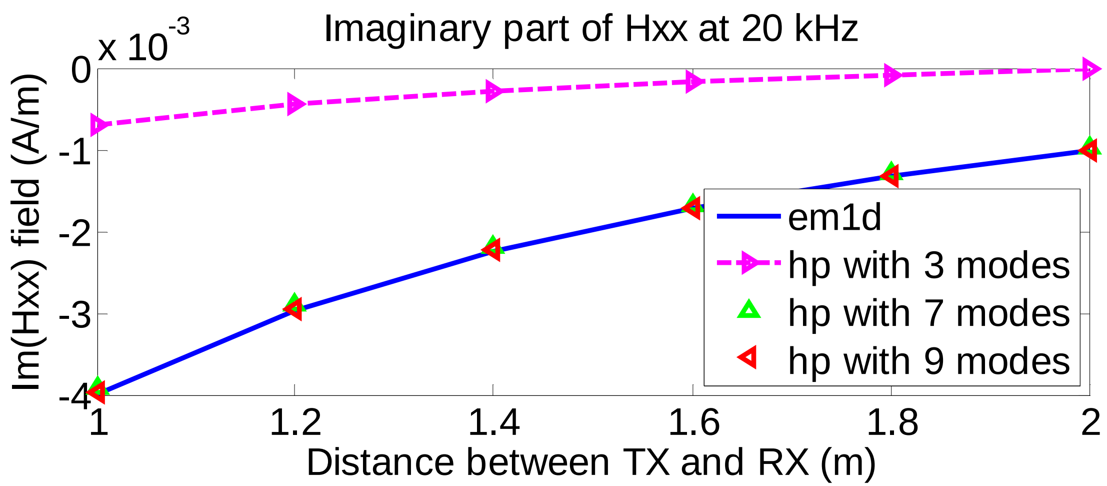




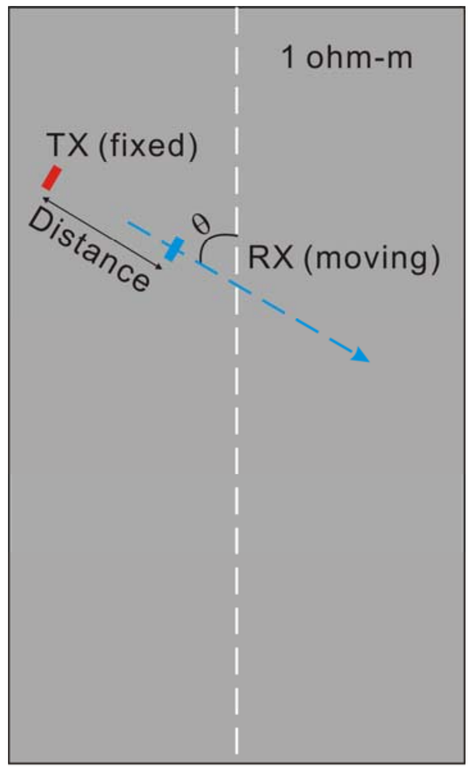
The same solutions with 1 Fourier mode

Verification of 3D Simulation ($H_{xx} = H_{yy}$)



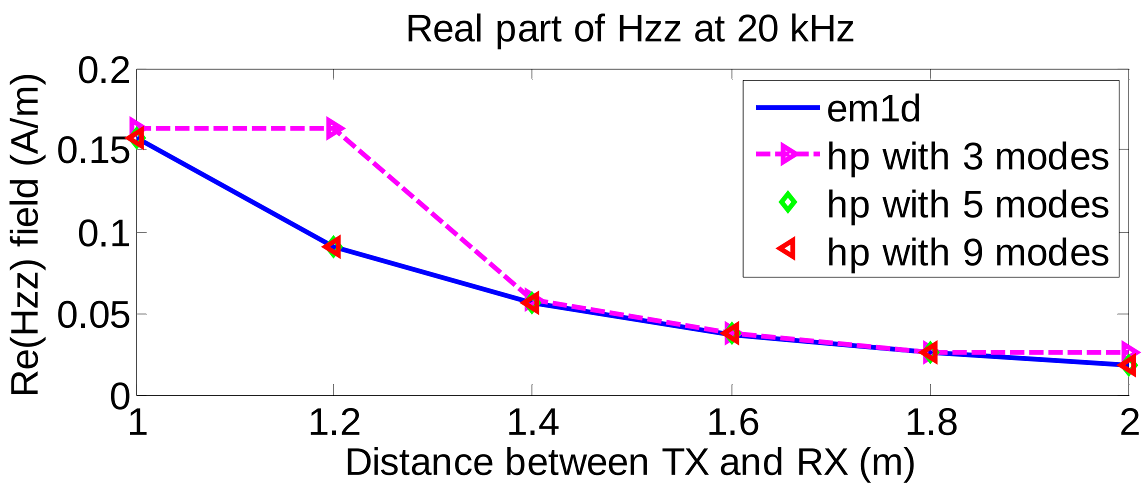


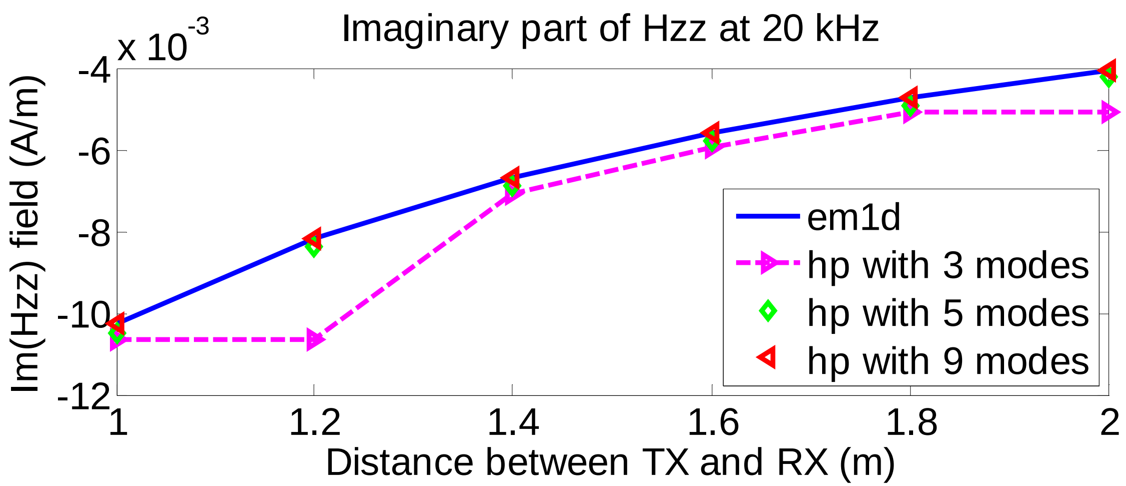
Dip angle: 60 degrees



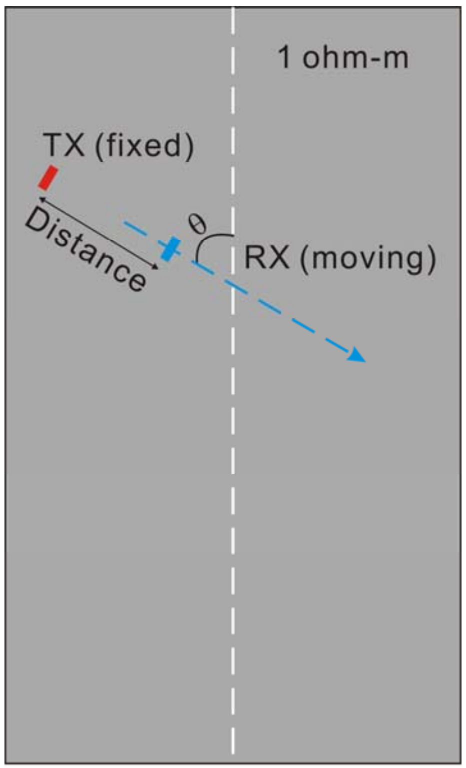
Converged solutions with 9 Fourier mode

Verification of 3D Simulation (H_{zz})



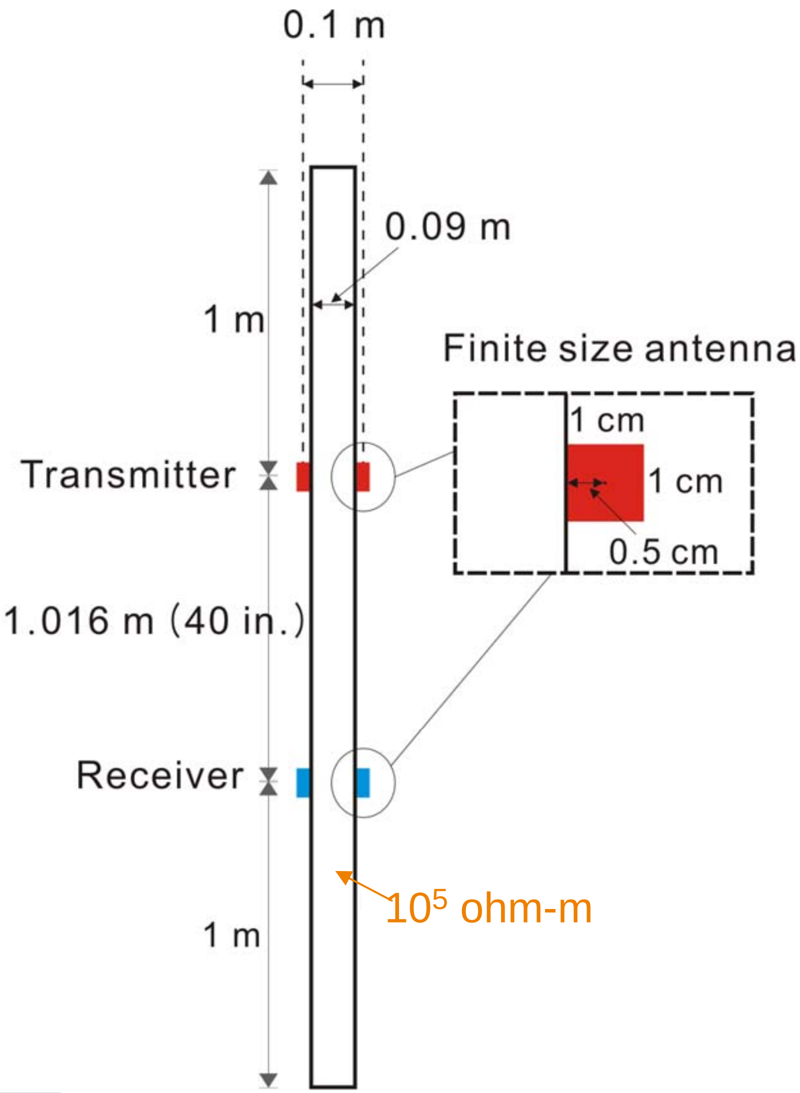


Dip angle: 60 degrees

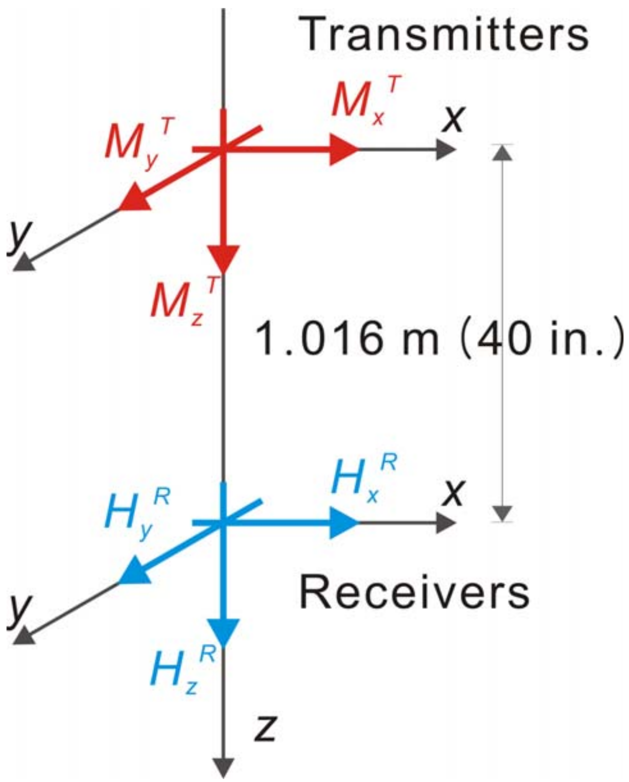


Converged solutions with 5 Fourier mode

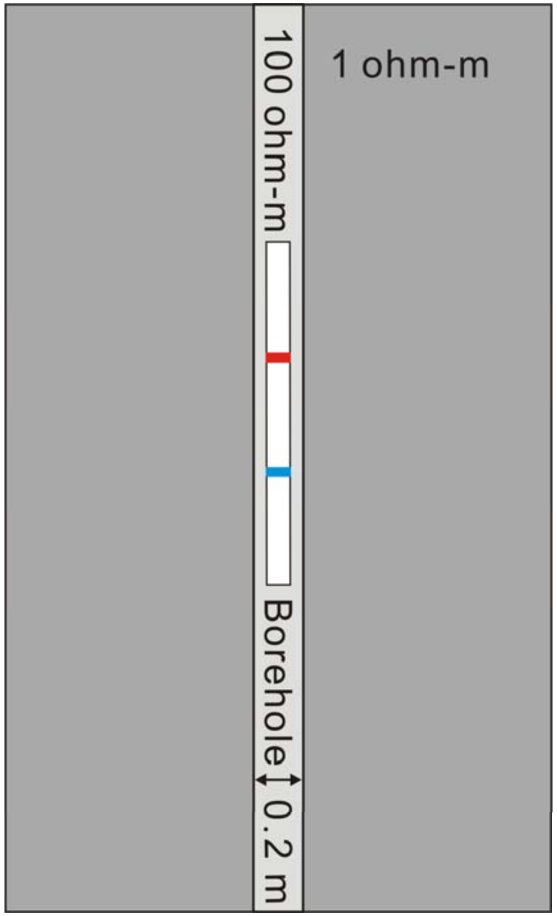
Description of the Tri-Axial Tool



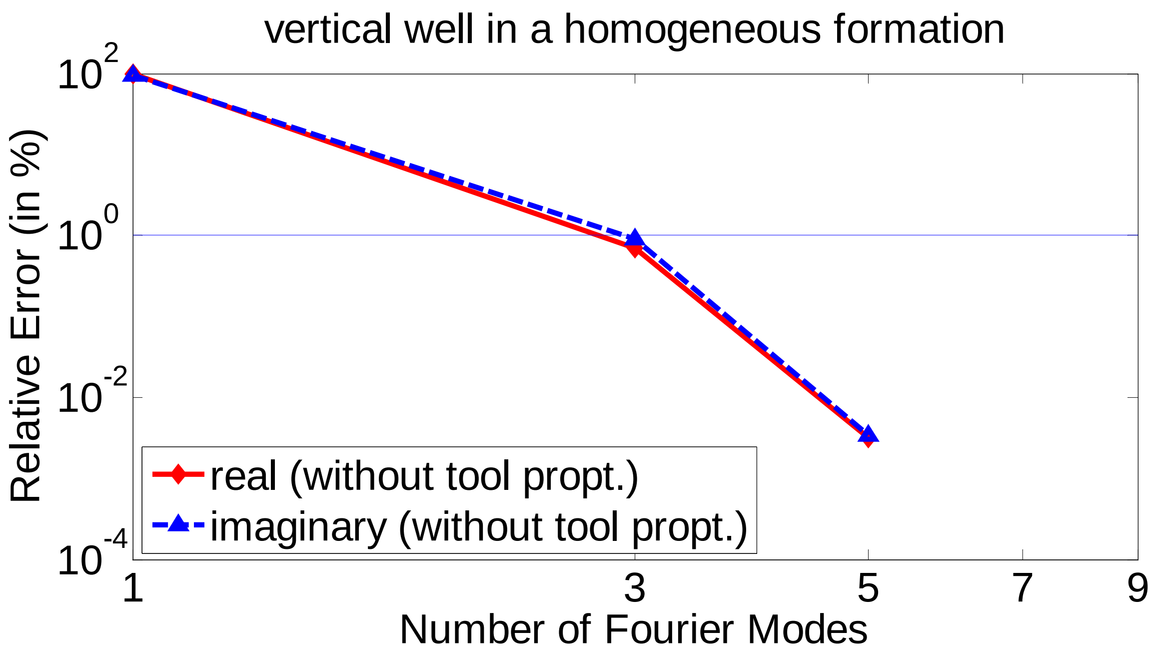
Operating frequency: 20 kHz



Verification of 2.5D Simulation (H_{xx})

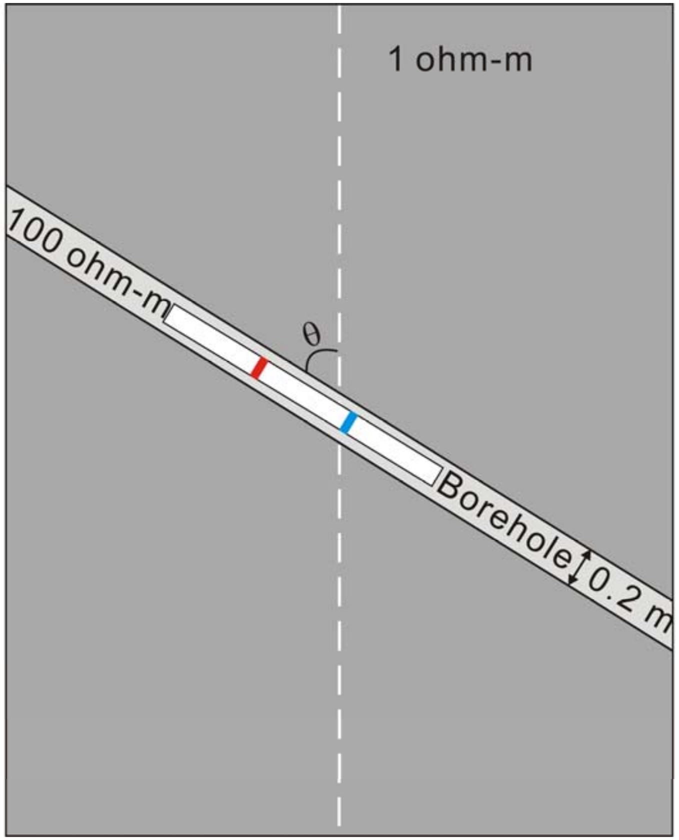


Relative errors of tri-axial induction solutions with respect to the solution with 9 Fourier modes

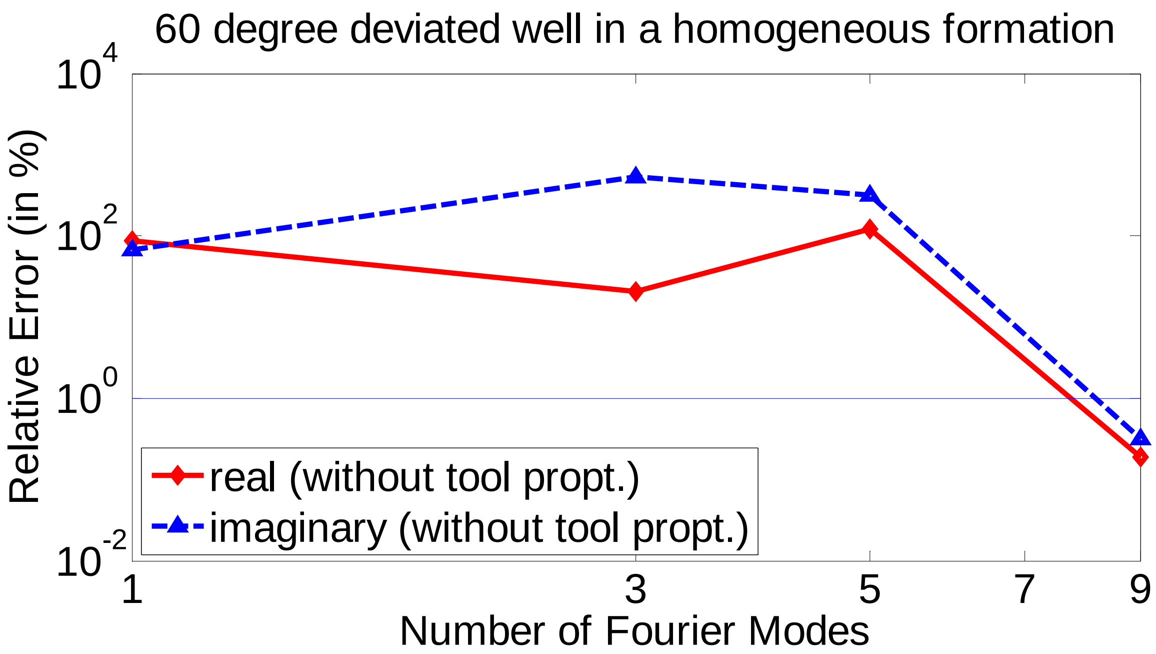


Verification of 3D Simulation (H_{xx})

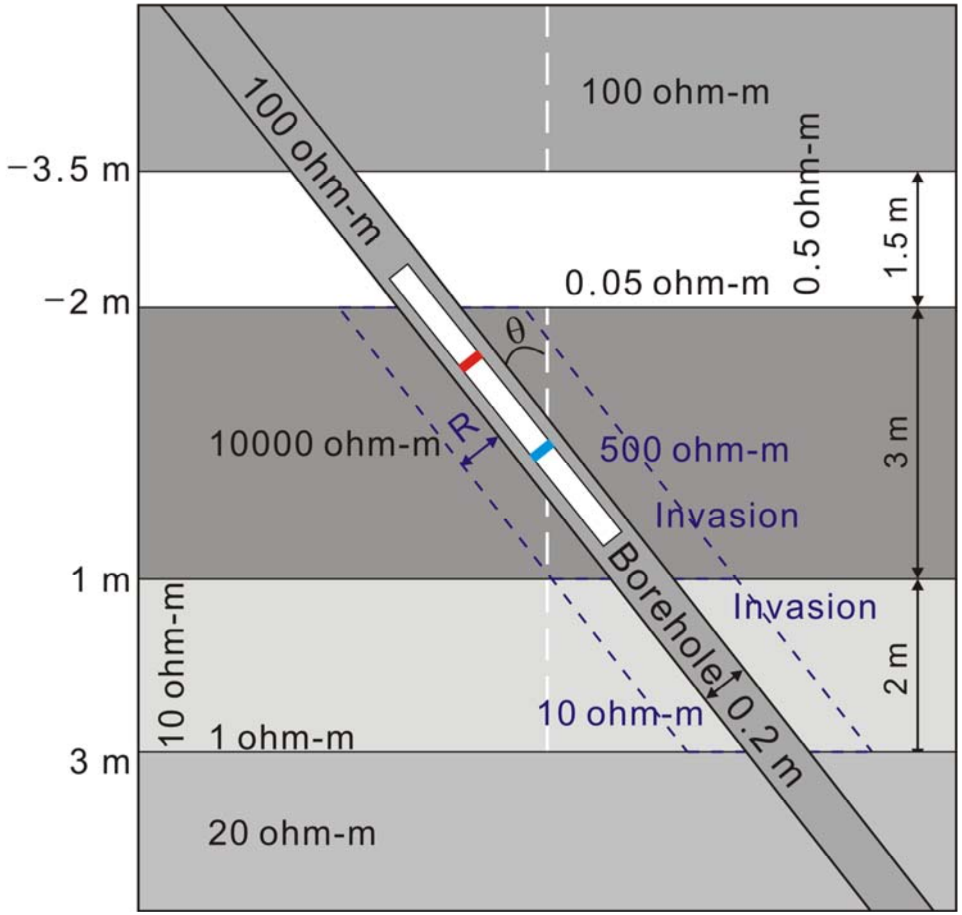
θ = 60 degrees



Relative errors of tri-axial Induction solutions with respect to the solution for the vertical well



Model for Numerical Experiments



Five layers: 100, 0.05, 10000, 1 and 20 ohm-m from top to bottom

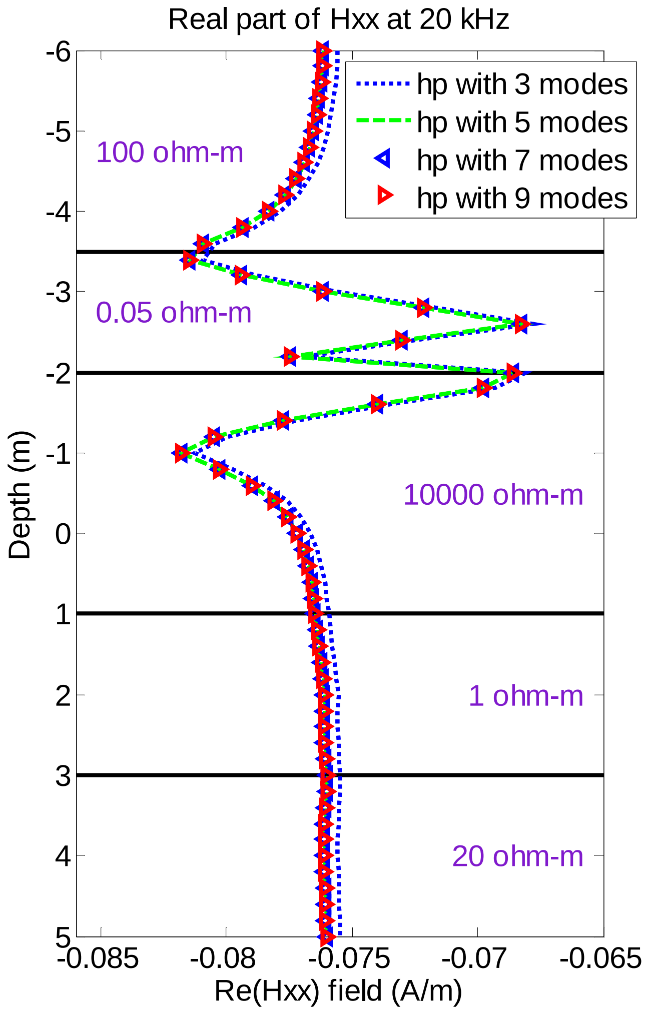
Borehole: 0.1 m in radius
100 ohm-m in resistivity

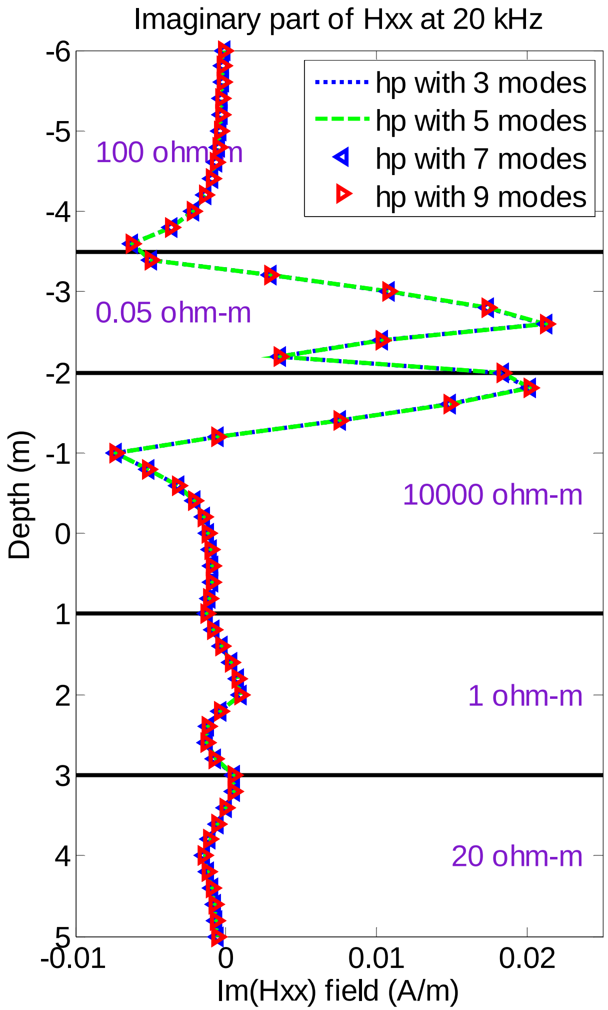
Invasion in the third and fourth layers

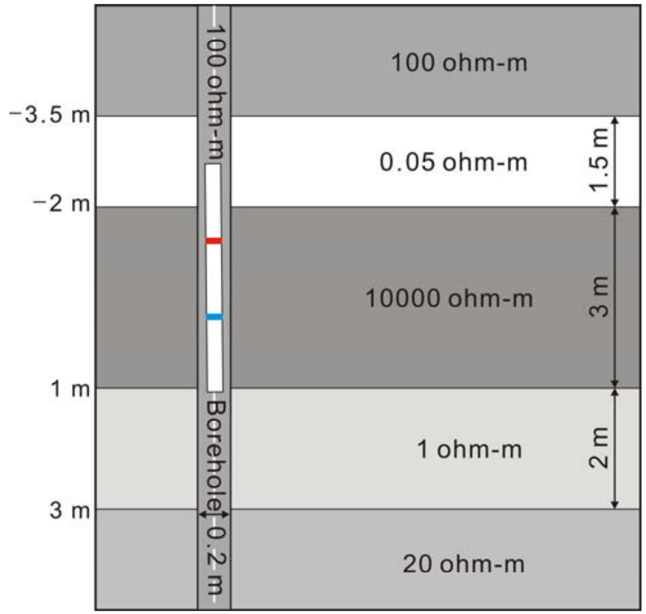
Anisotropy in the second and fourth layers

θ **= 0, 30 and 60 degrees**

Convergence History of H_{xx} in Vertical Well

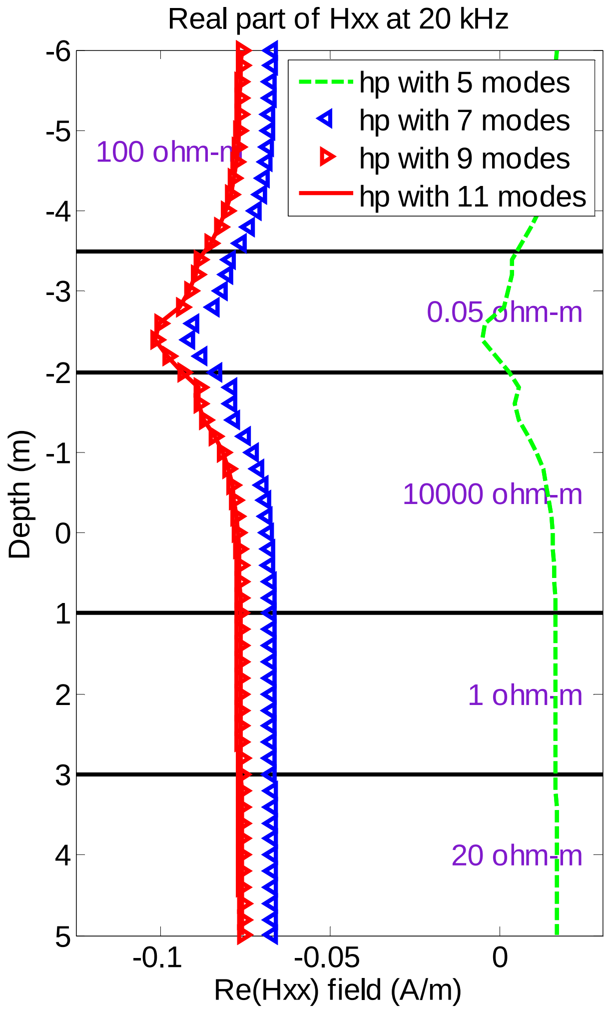


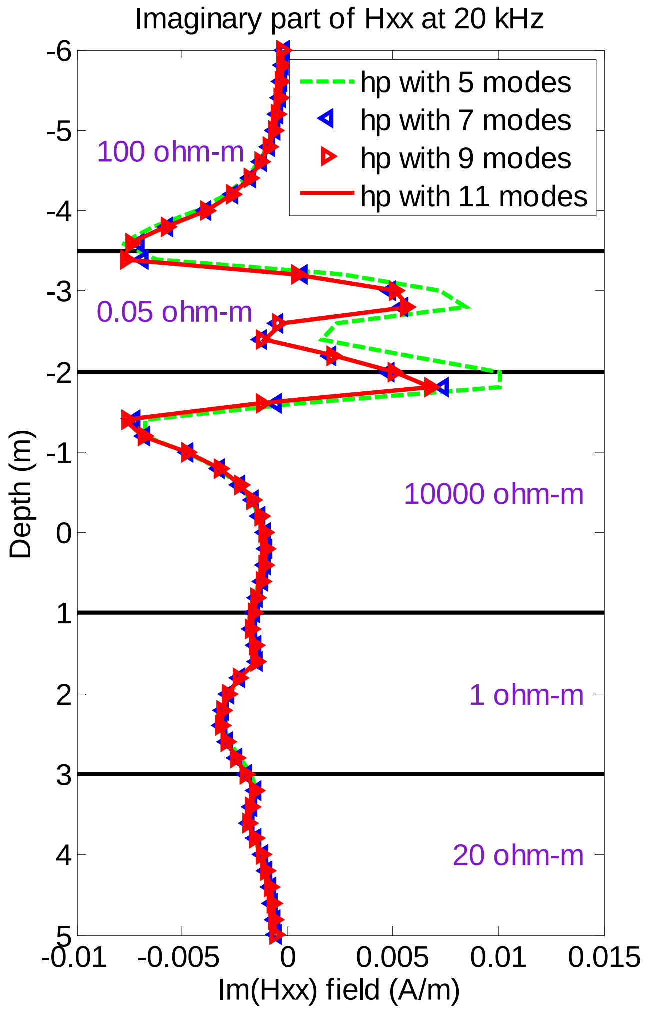




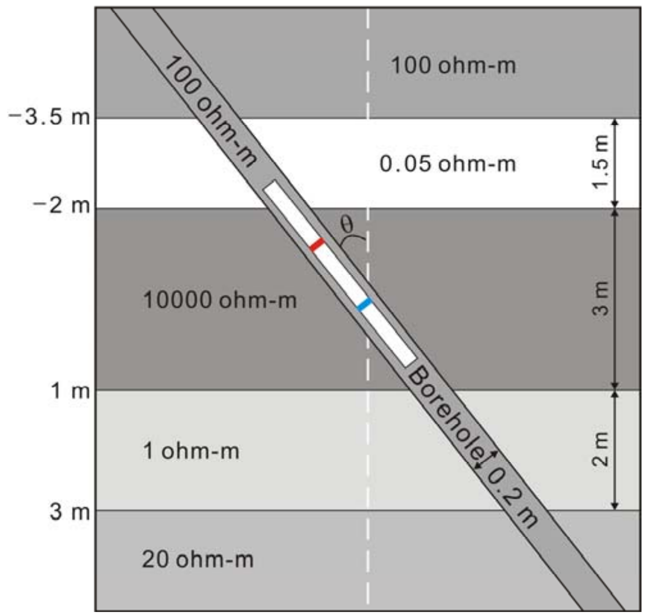
Converged solutions with 5 Fourier modes

Convergence History of H_{xx} in Deviated Well



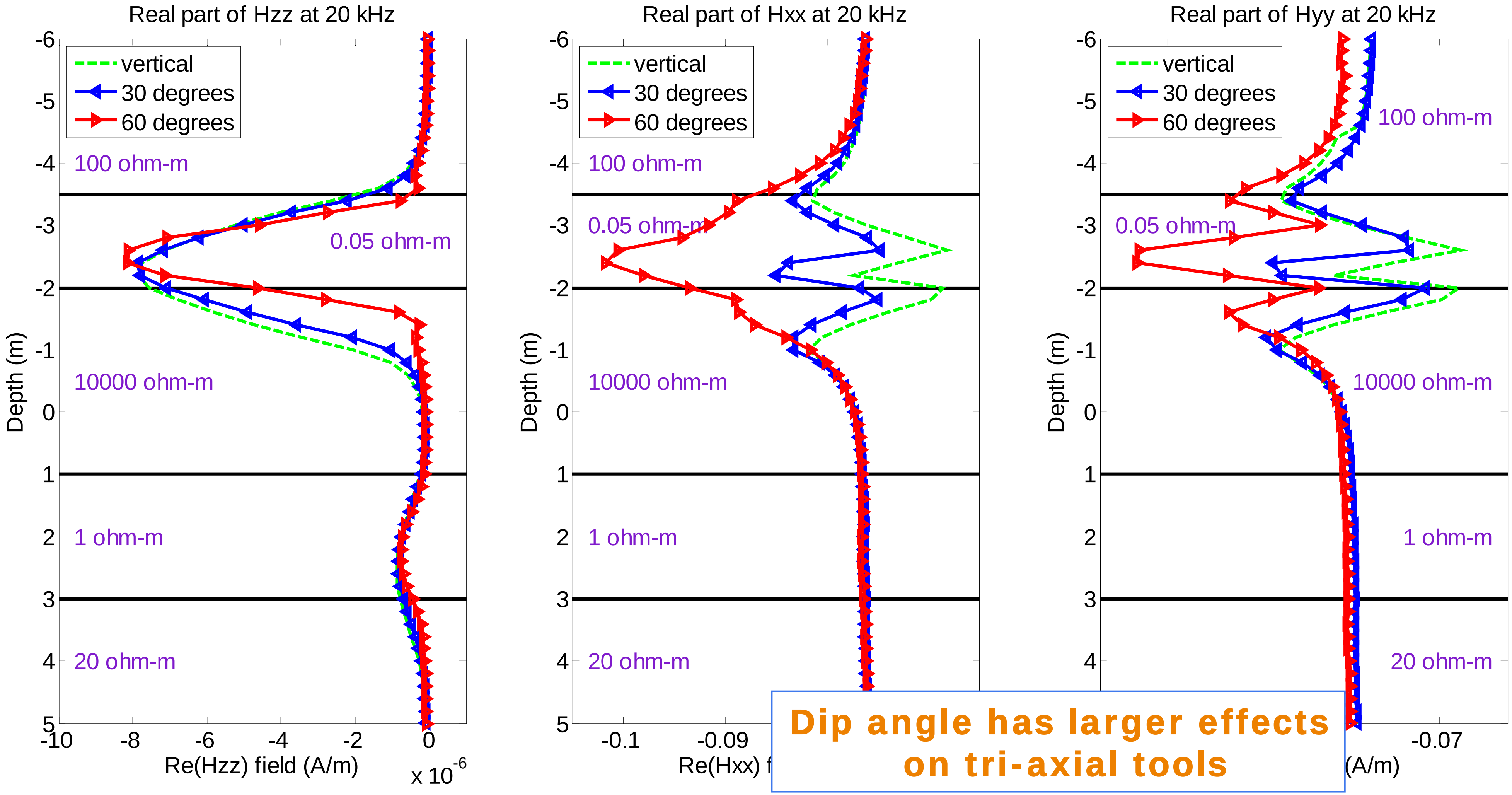


θ = 60 degrees



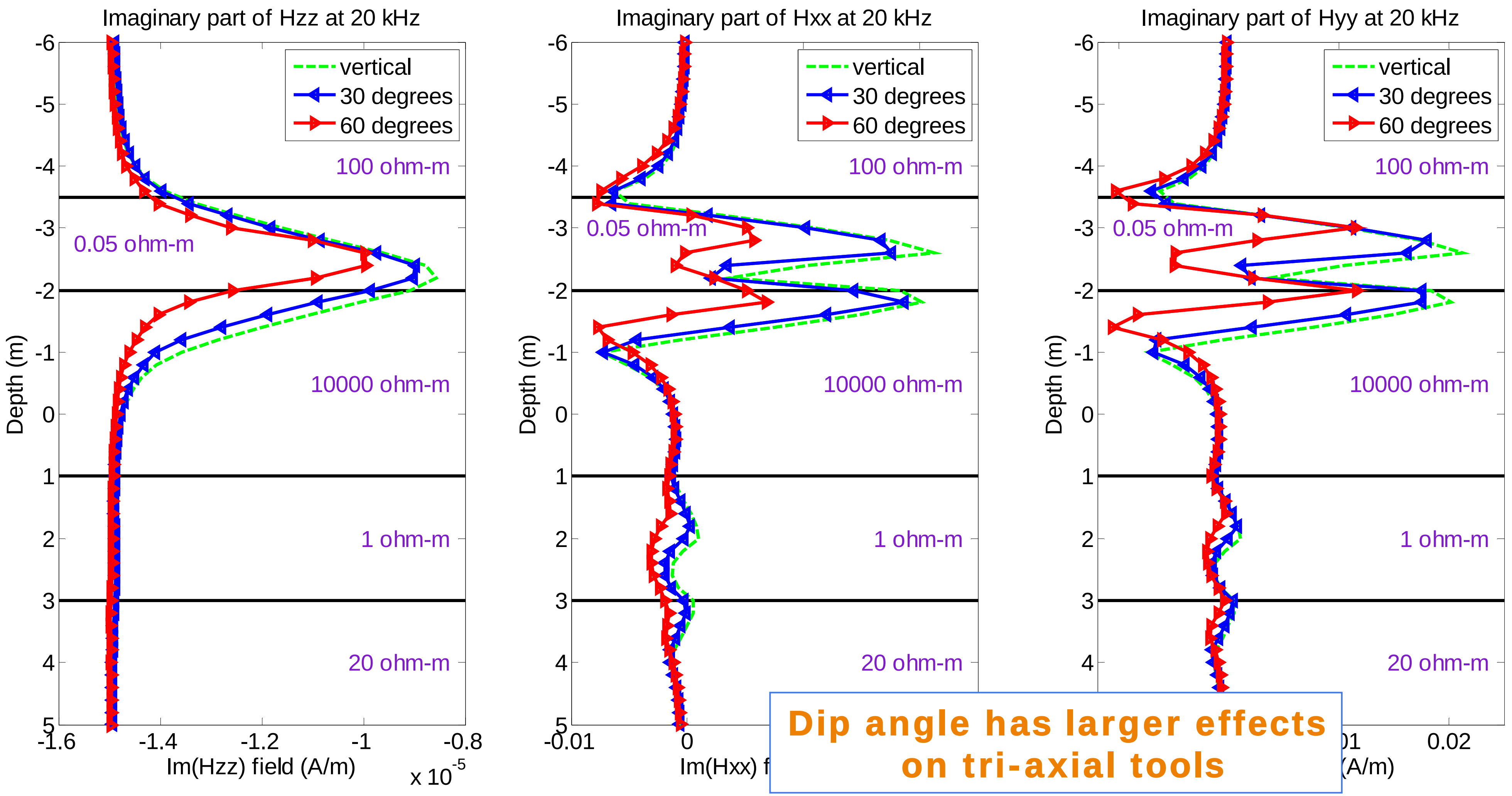
Converged solutions with 9 Fourier modes

Deviated Wells (0, 30 & 60 degrees)



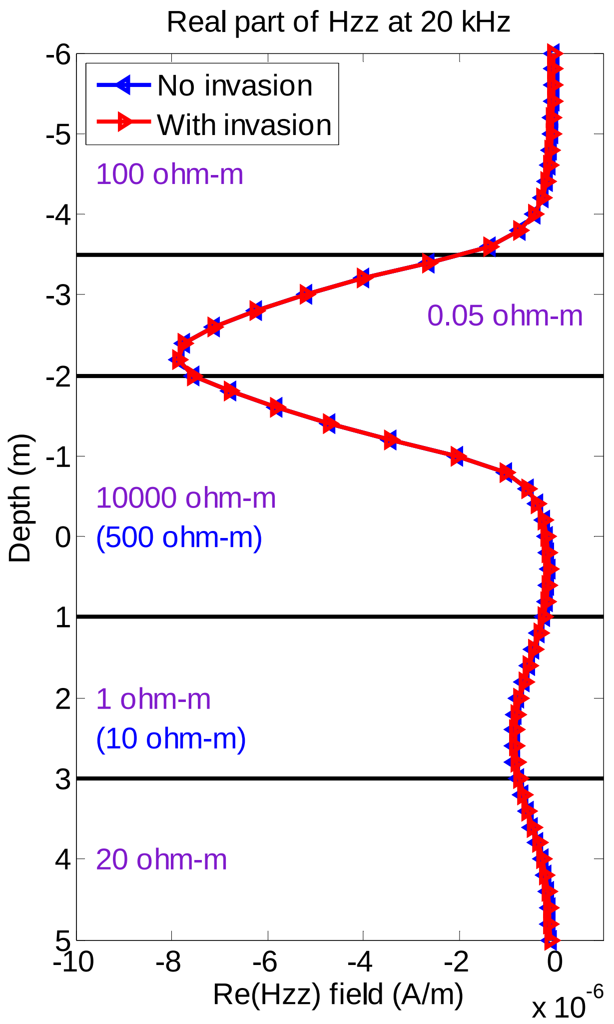
Dip angle has larger effects on tri-axial tools

Deviated Wells (0, 30 & 60 degrees)

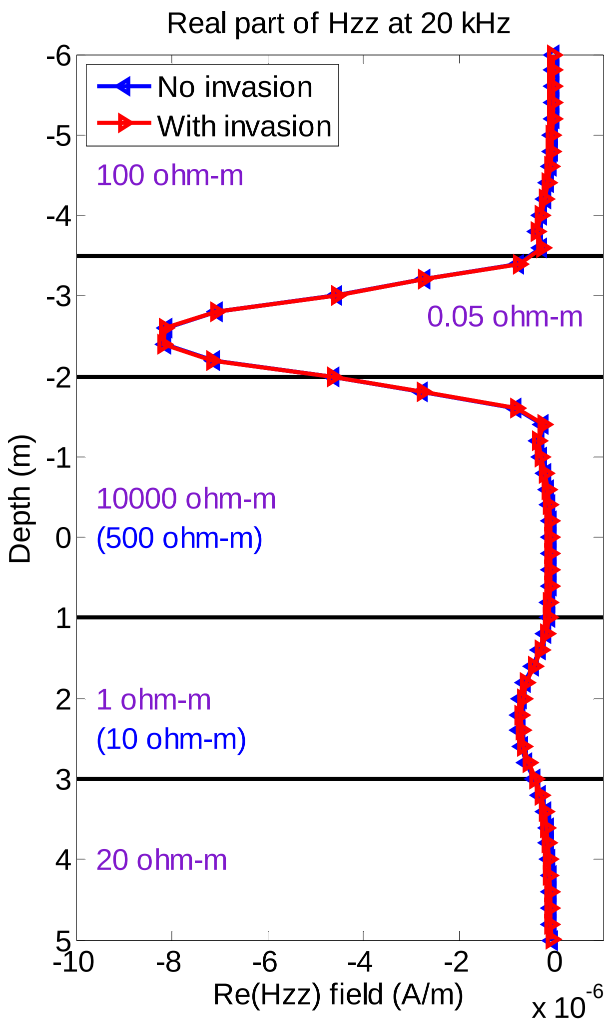


Dip angle has larger effects on tri-axial tools

H_{zz} in Deviated Wells with Invasion (Re.)

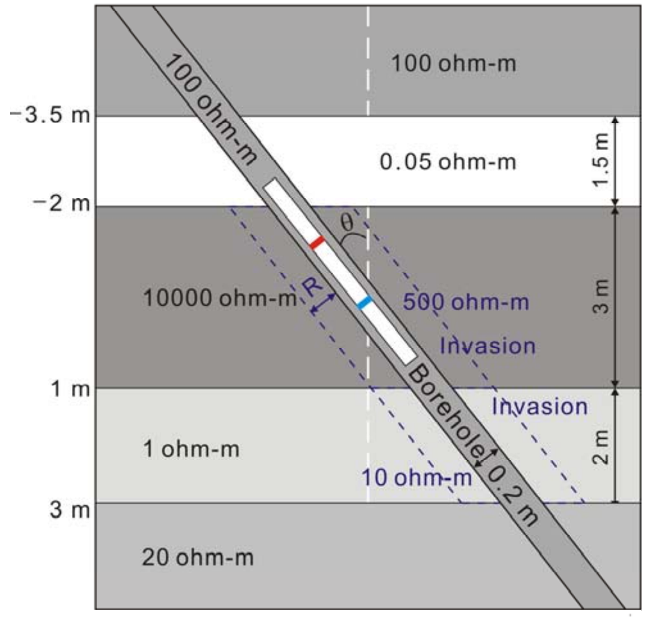


vertical

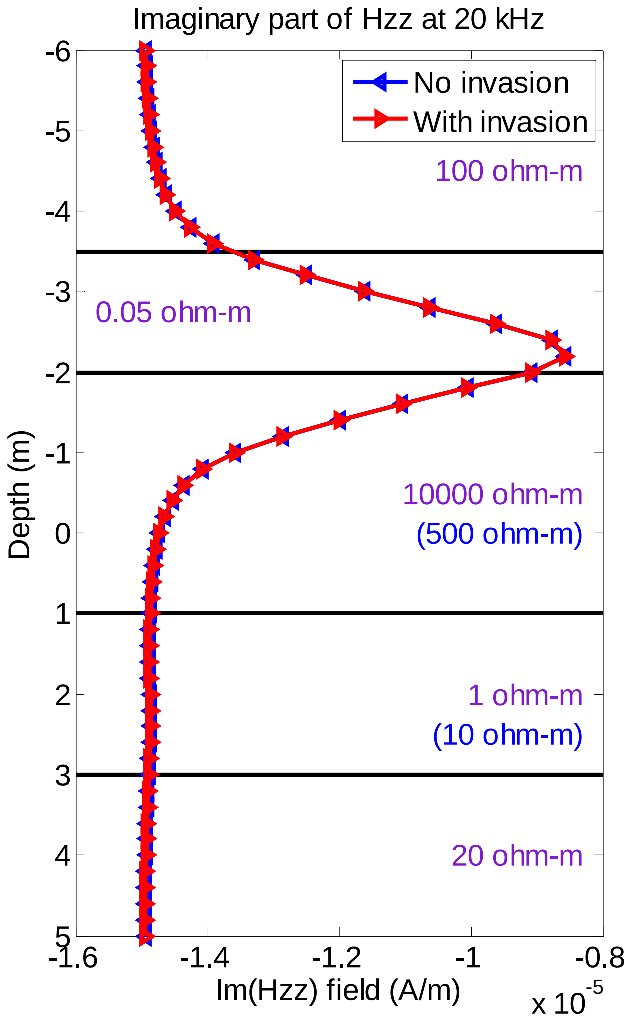


60 degrees

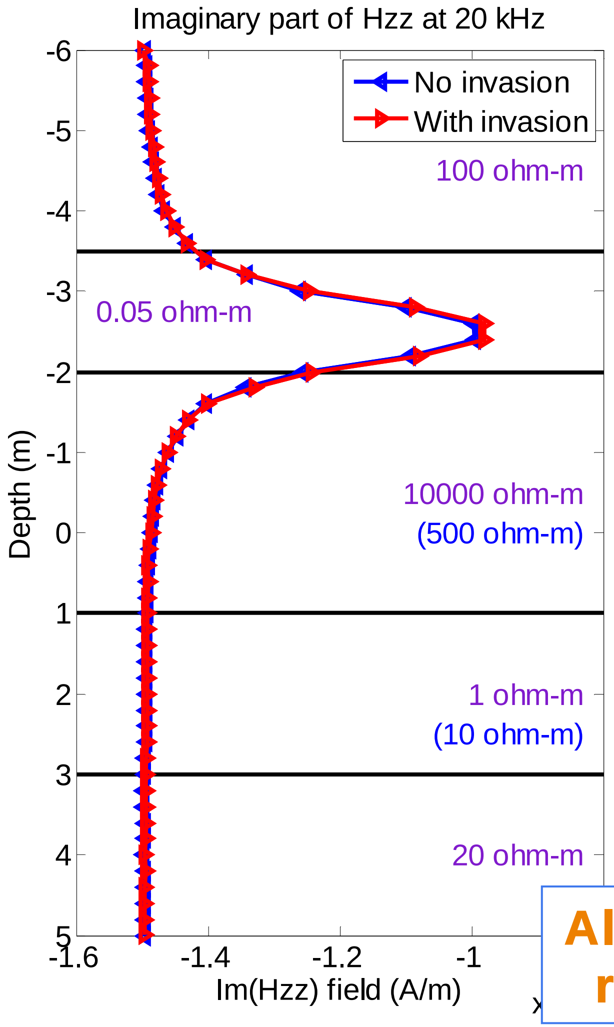
Shallow invasion with R = 0.1 m



H_{zz} in Deviated Wells with Invasion (Im.)

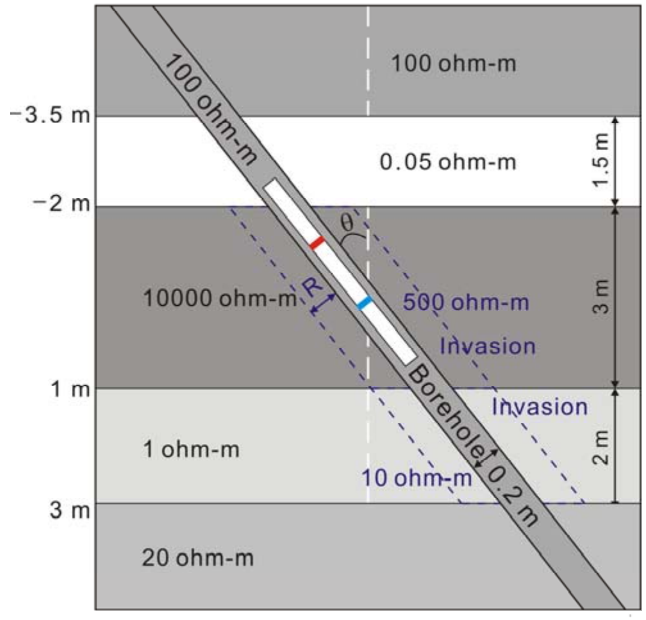


vertical



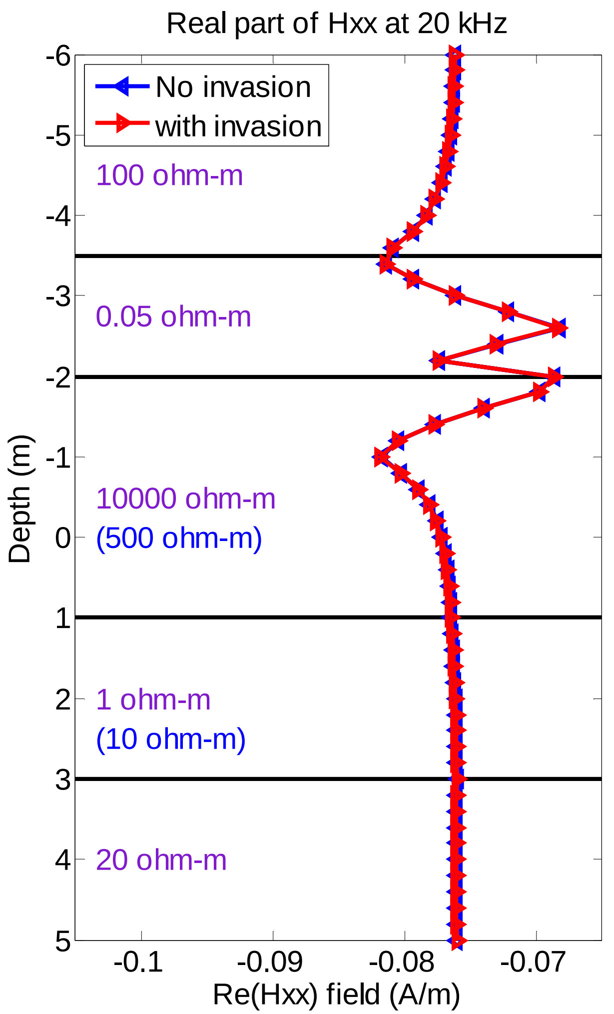
60 degrees

Shallow invasion with R = 0.1 m

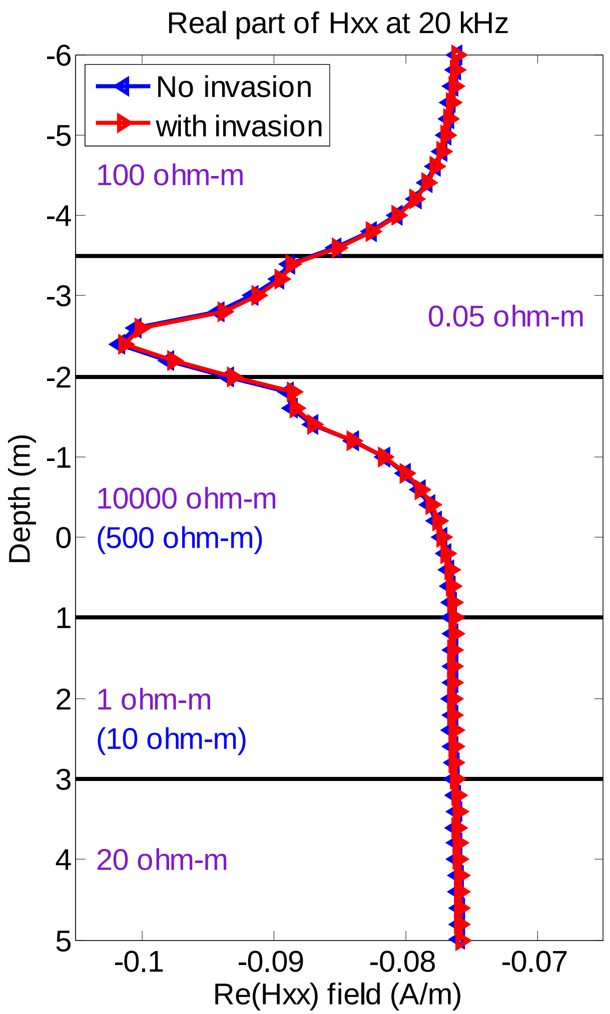


Almost no effects of invasion regardless of the dip angle

H_{xx} in Deviated Wells with Invasion (Re.)

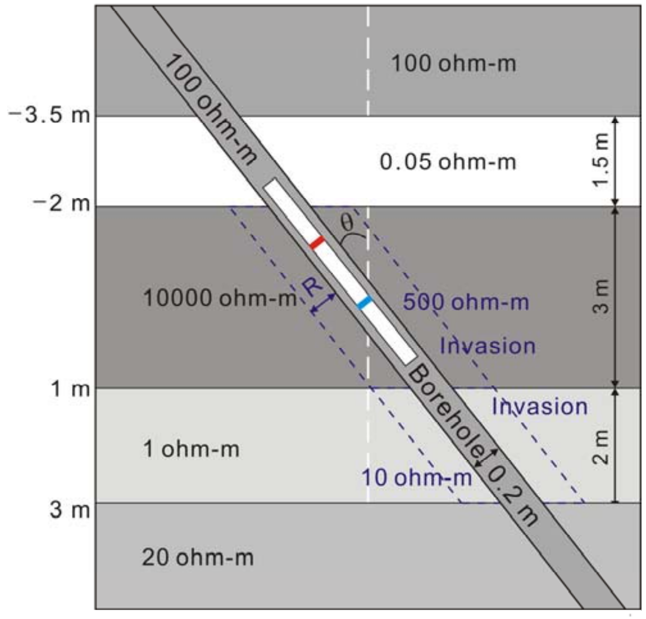


vertical

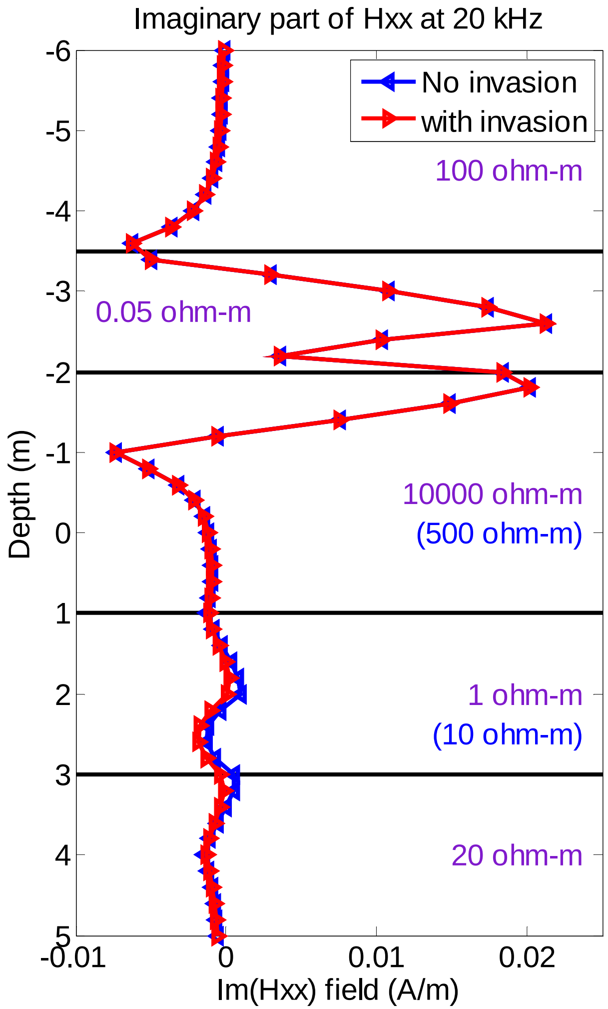


60 degrees

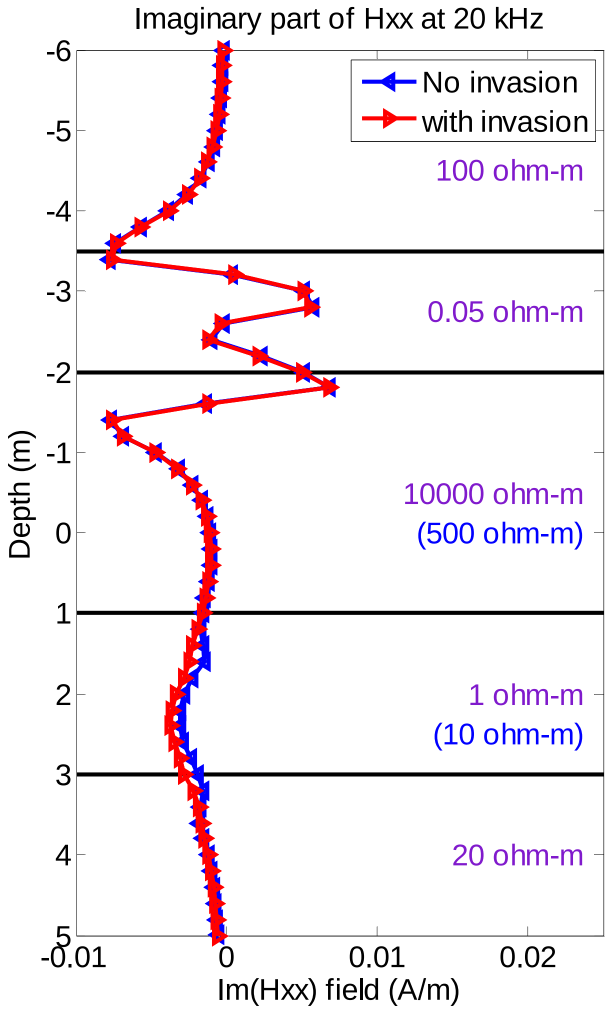
Shallow invasion with R = 0.1 m



H_{xx} in Deviated Wells with Invasion (Im.)

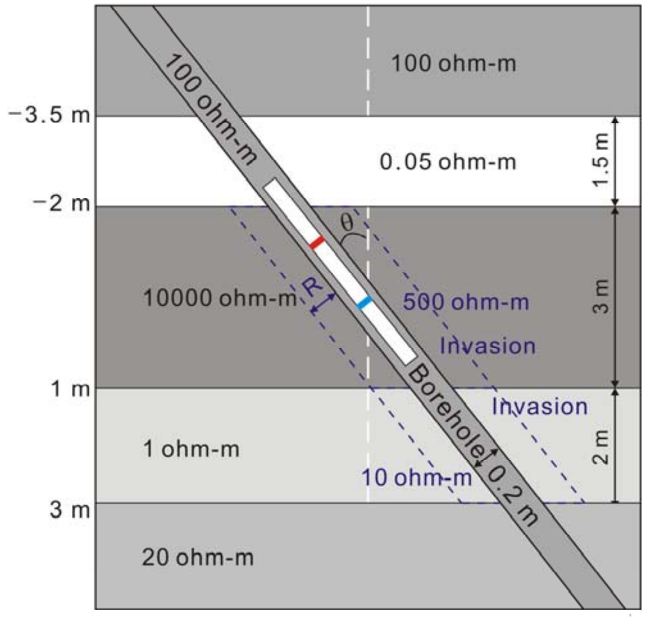


vertical



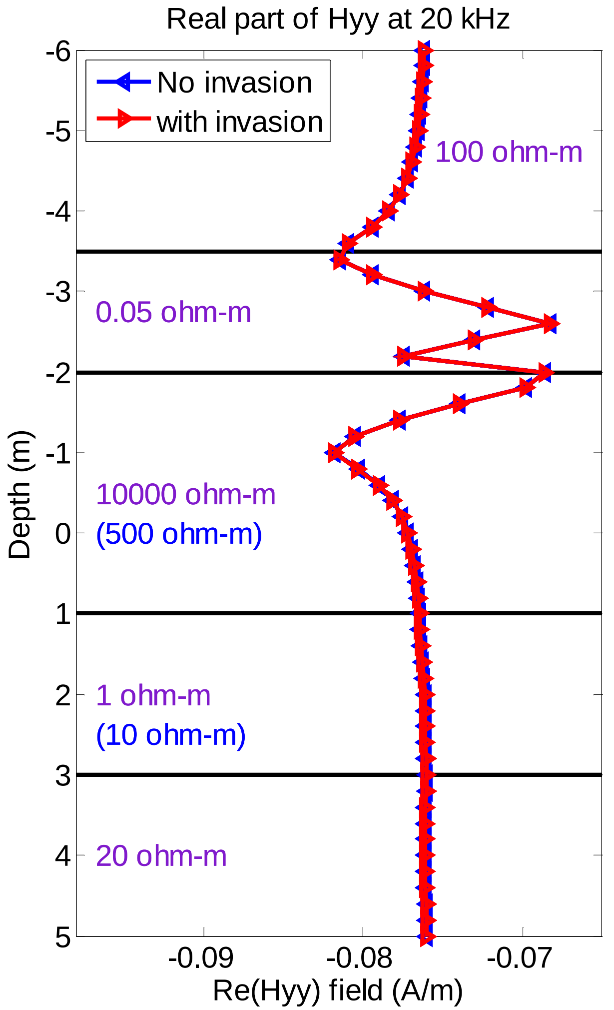
60 degrees

Shallow invasion with R = 0.1 m

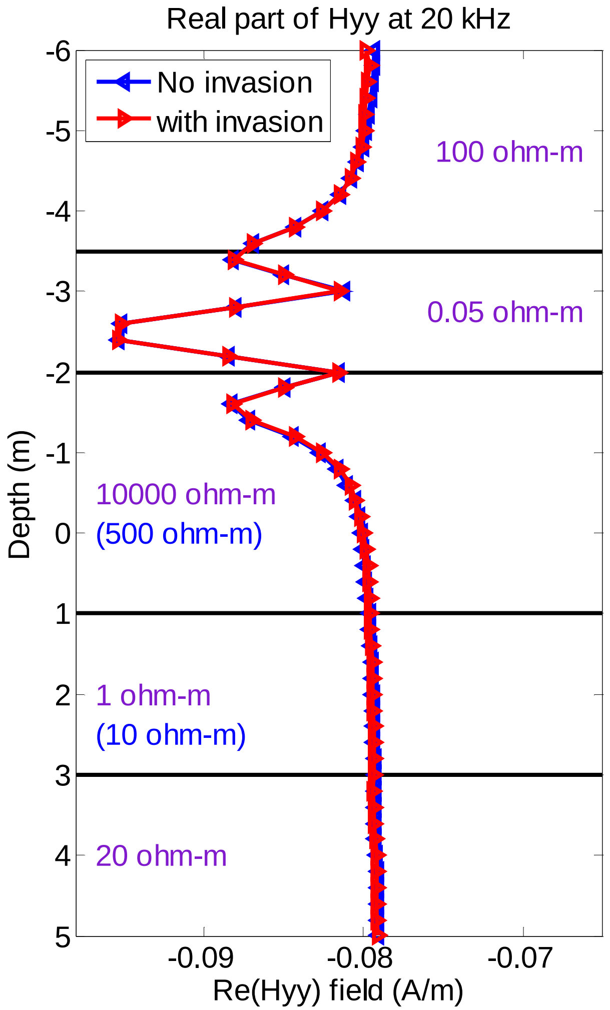


Small effects of invasion

H_{yy} in Deviated Wells with Invasion (Re.)

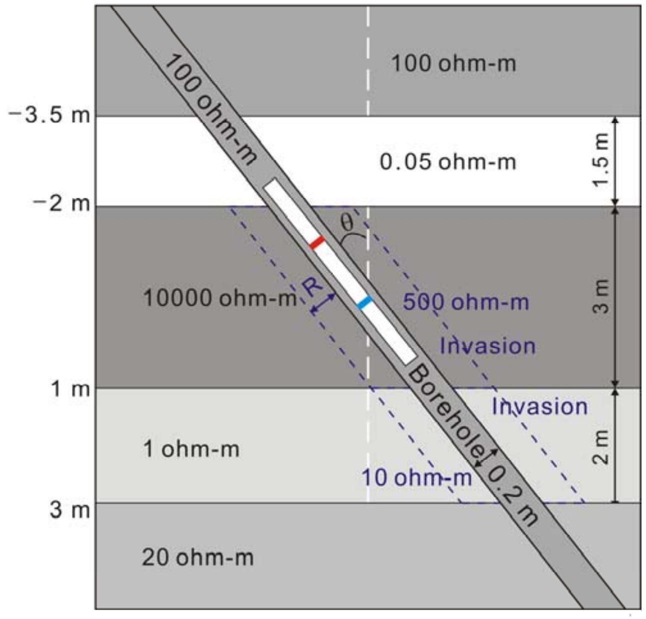


vertical

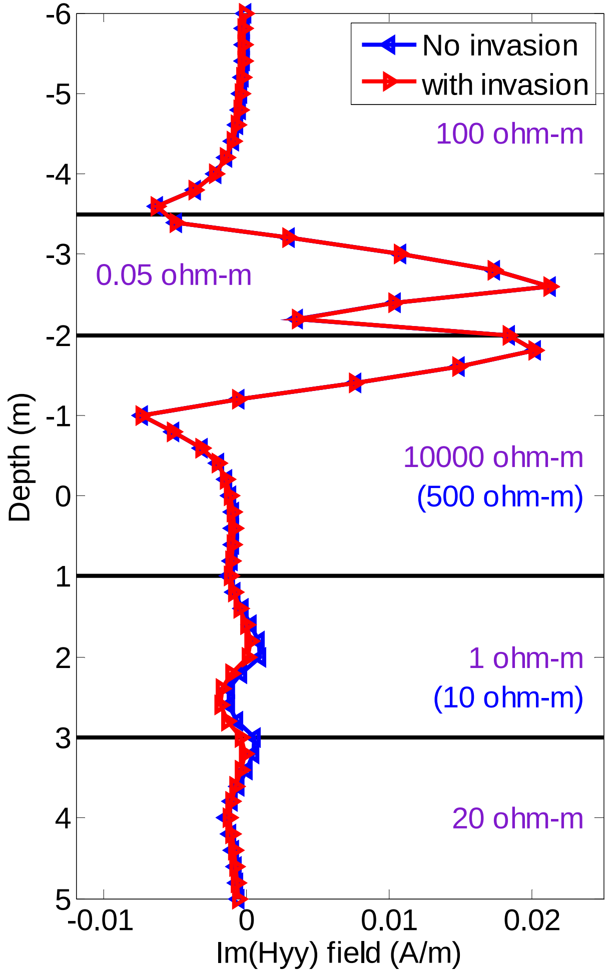


60 degrees

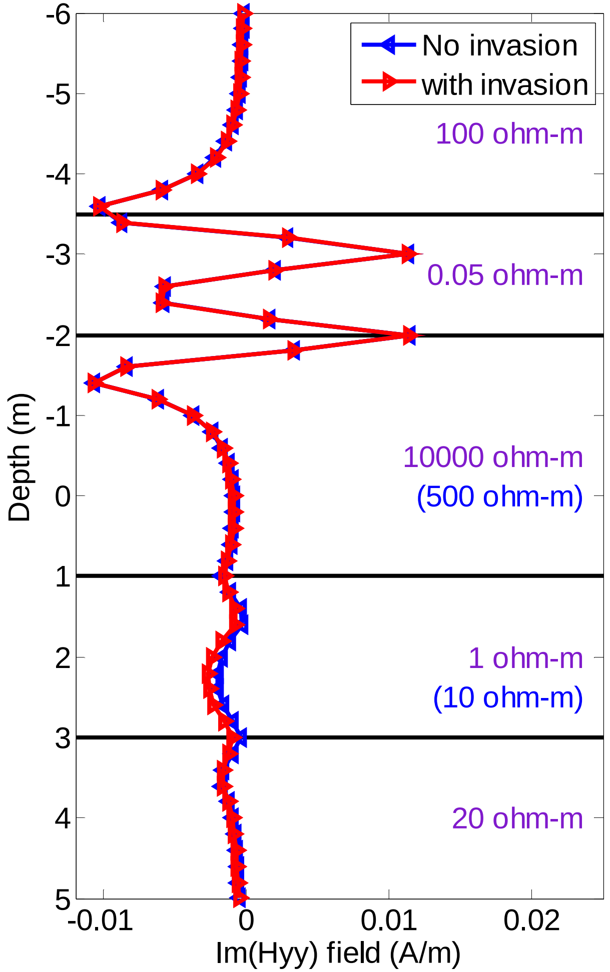
Shallow invasion with R = 0.1 m



H_{yy} in Deviated Wells with Invasion (Im.)

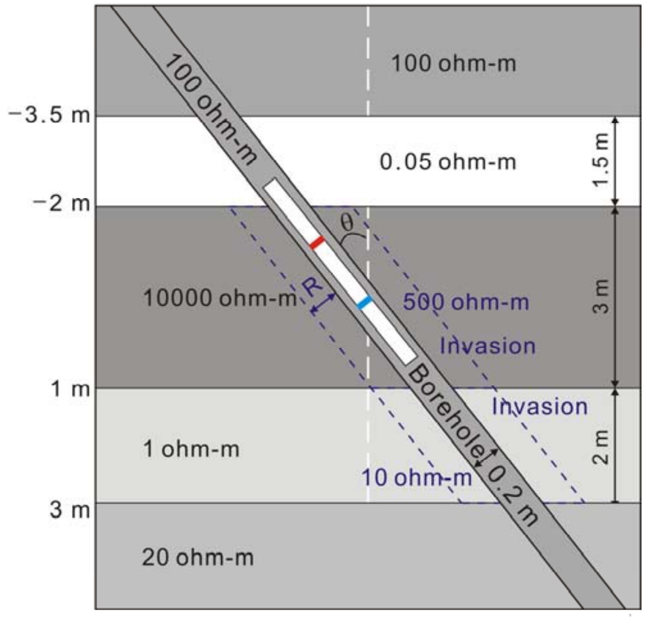


vertical



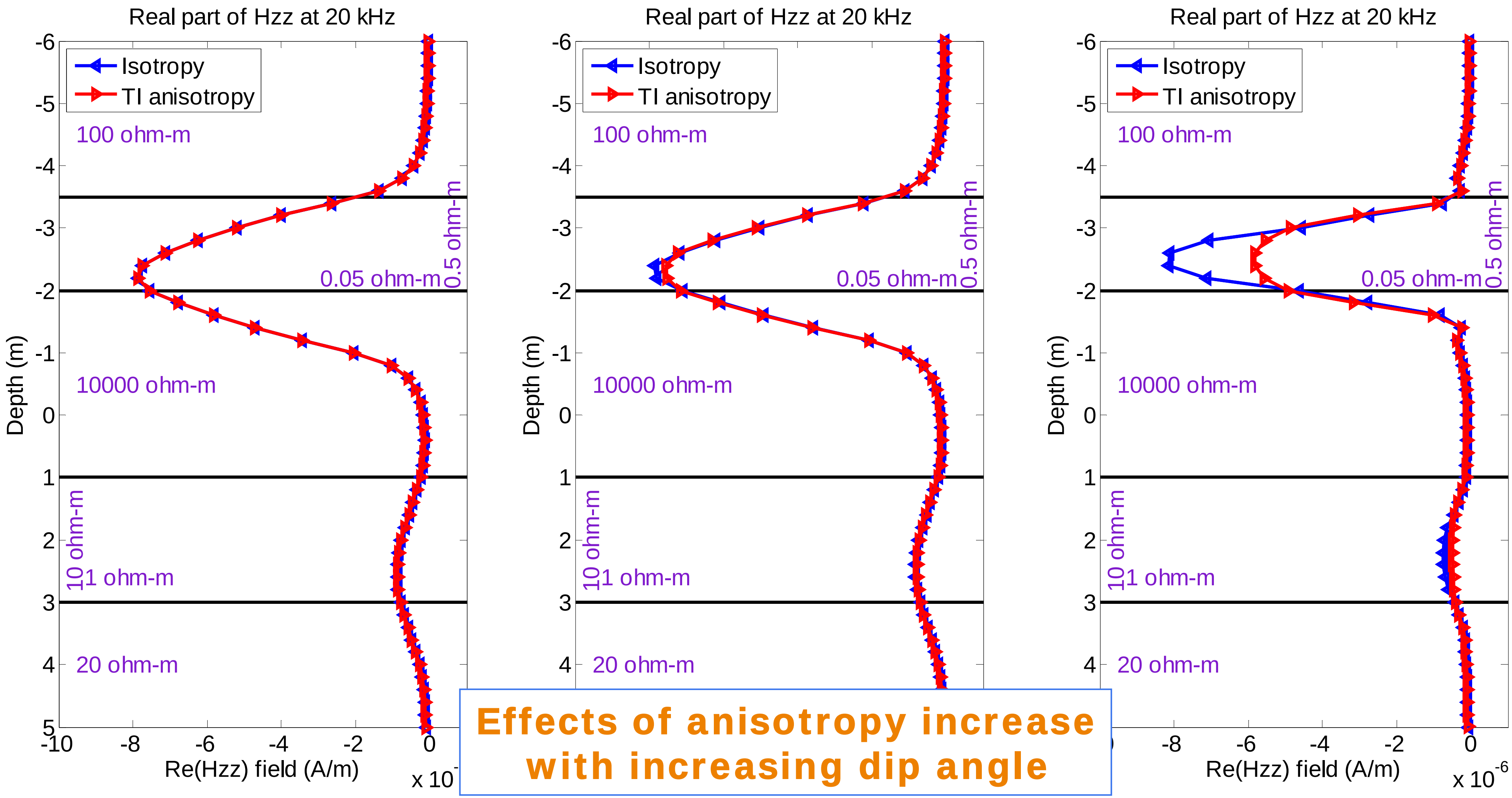
60 degrees

Shallow invasion with R = 0.1 m



Small effects of invasion

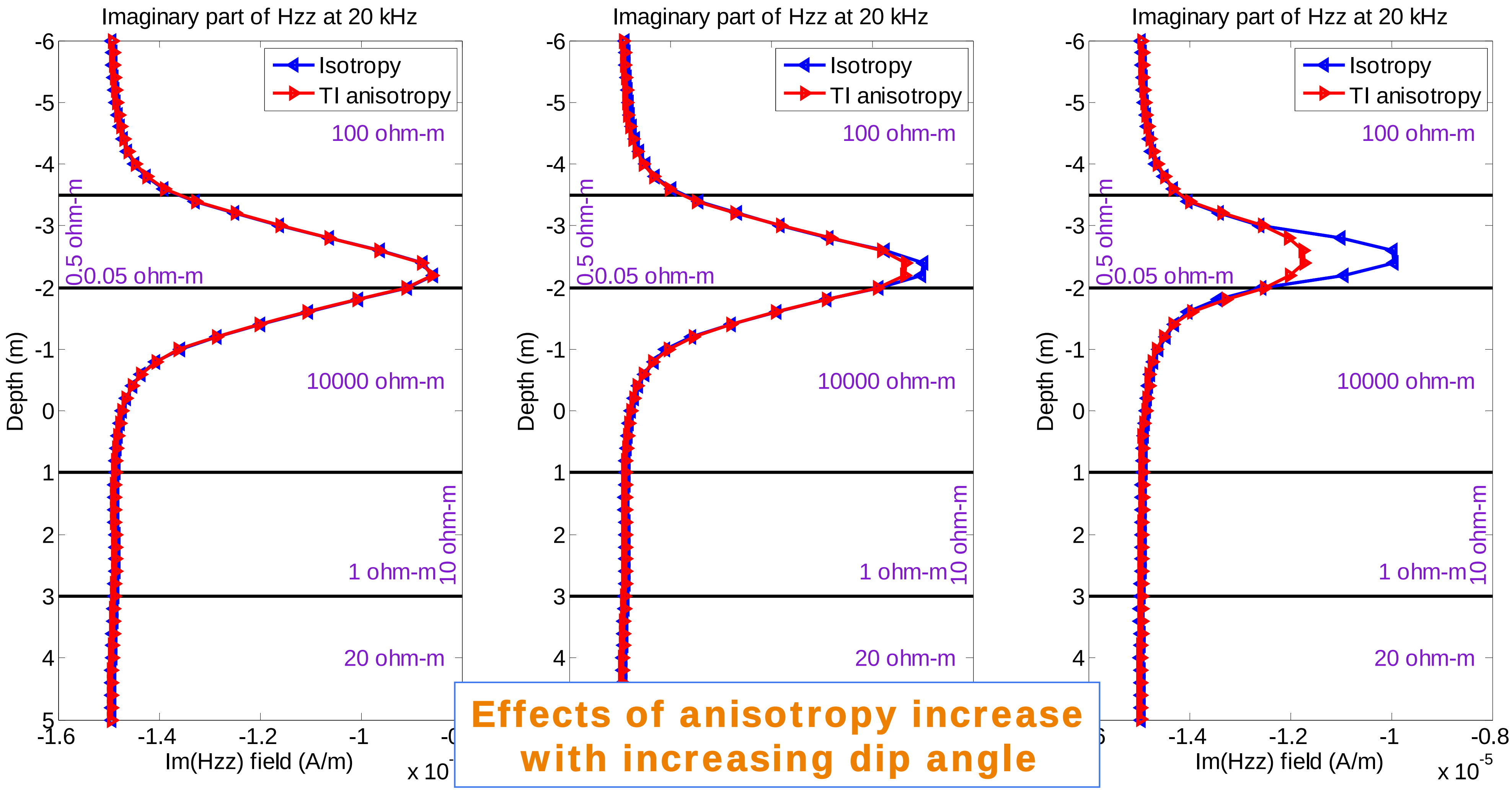
H_{zz} in Deviated Wells with Anisotropy (Re.)



Effects of anisotropy increase with increasing dip angle

vertical | **30 degrees** | **60 degrees**

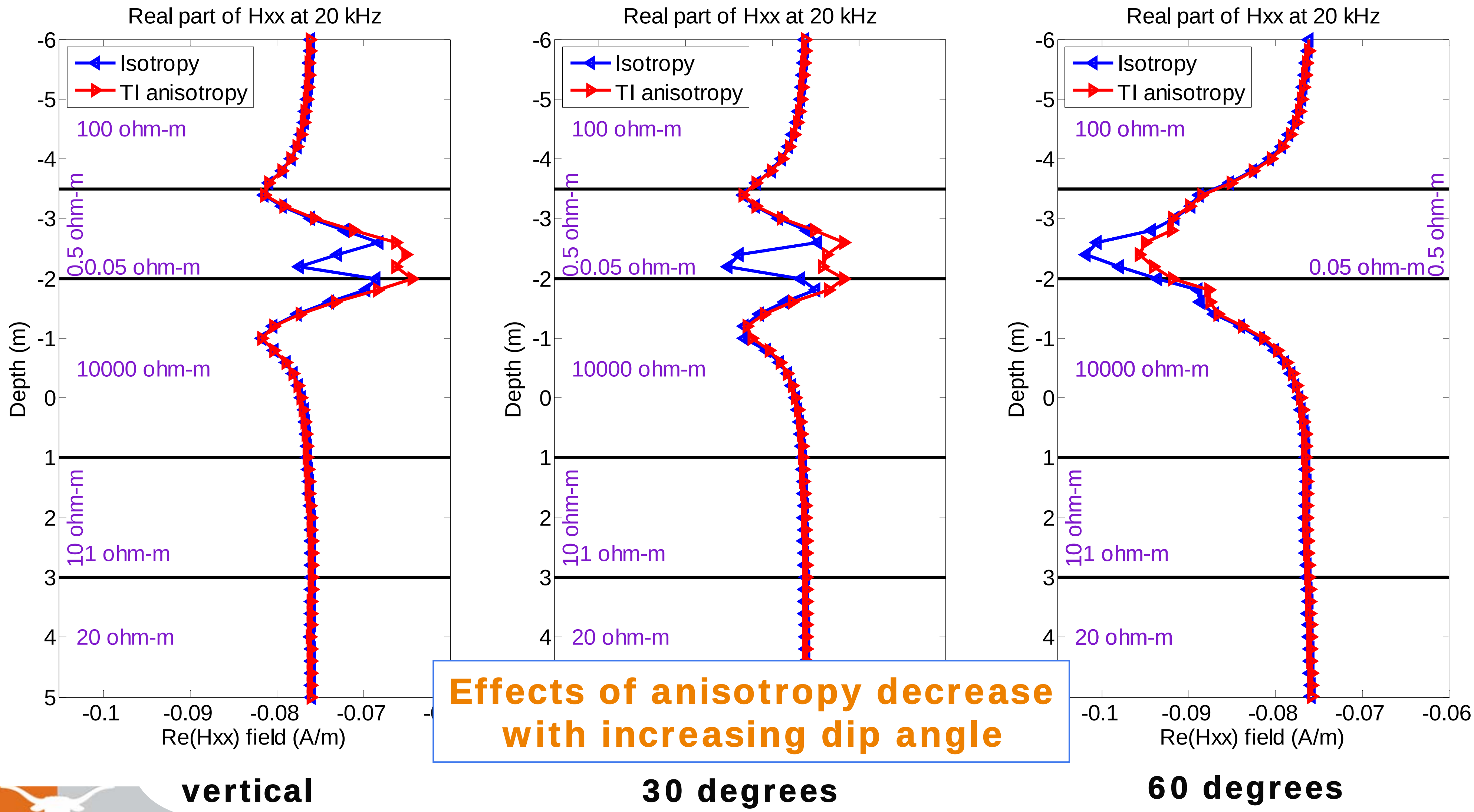
H_{zz} in Deviated Wells with Anisotropy (Im.)



Effects of anisotropy increase with increasing dip angle

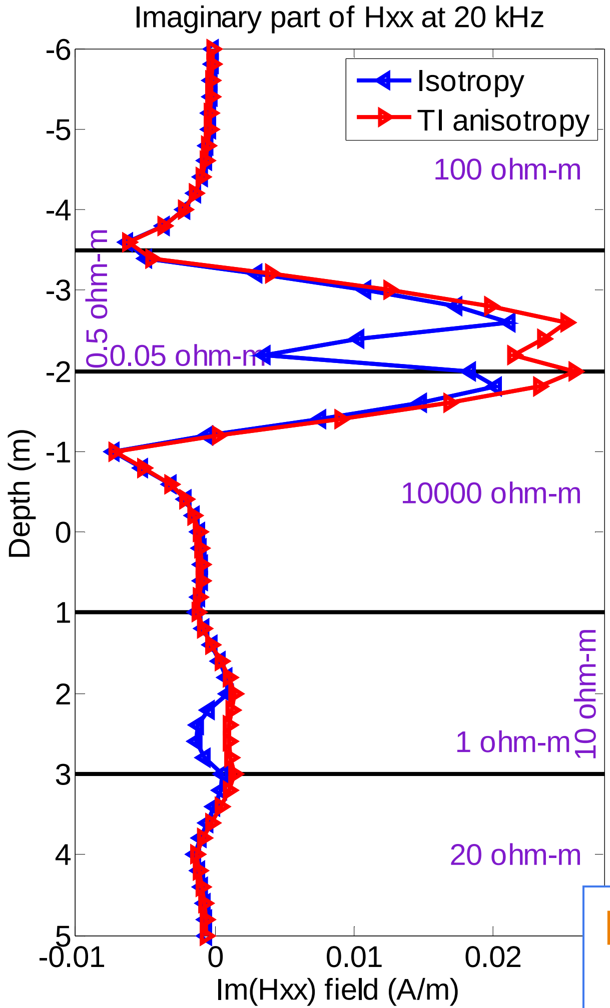
vertical **30 degrees** **60 degrees**

H_{xx} in Deviated Wells with Anisotropy (Re.)

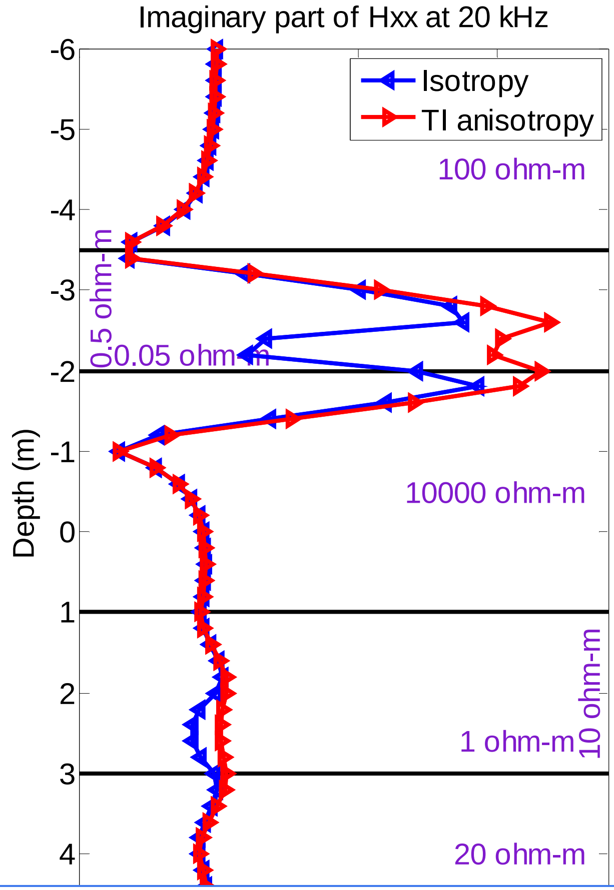


Effects of anisotropy decrease with increasing dip angle

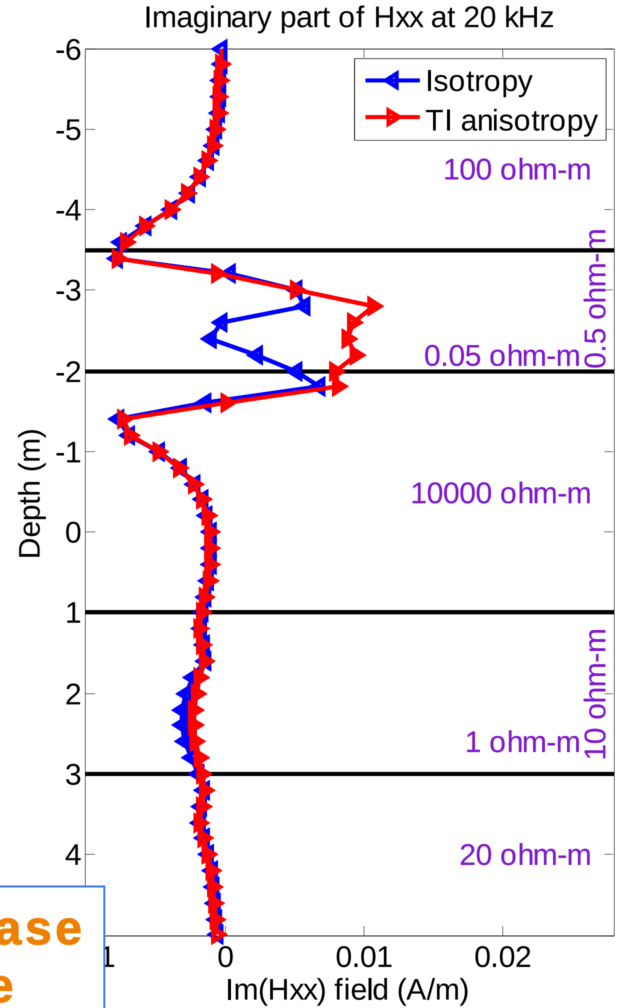
H_{xx} in Deviated Wells with Anisotropy (Im.)



vertical



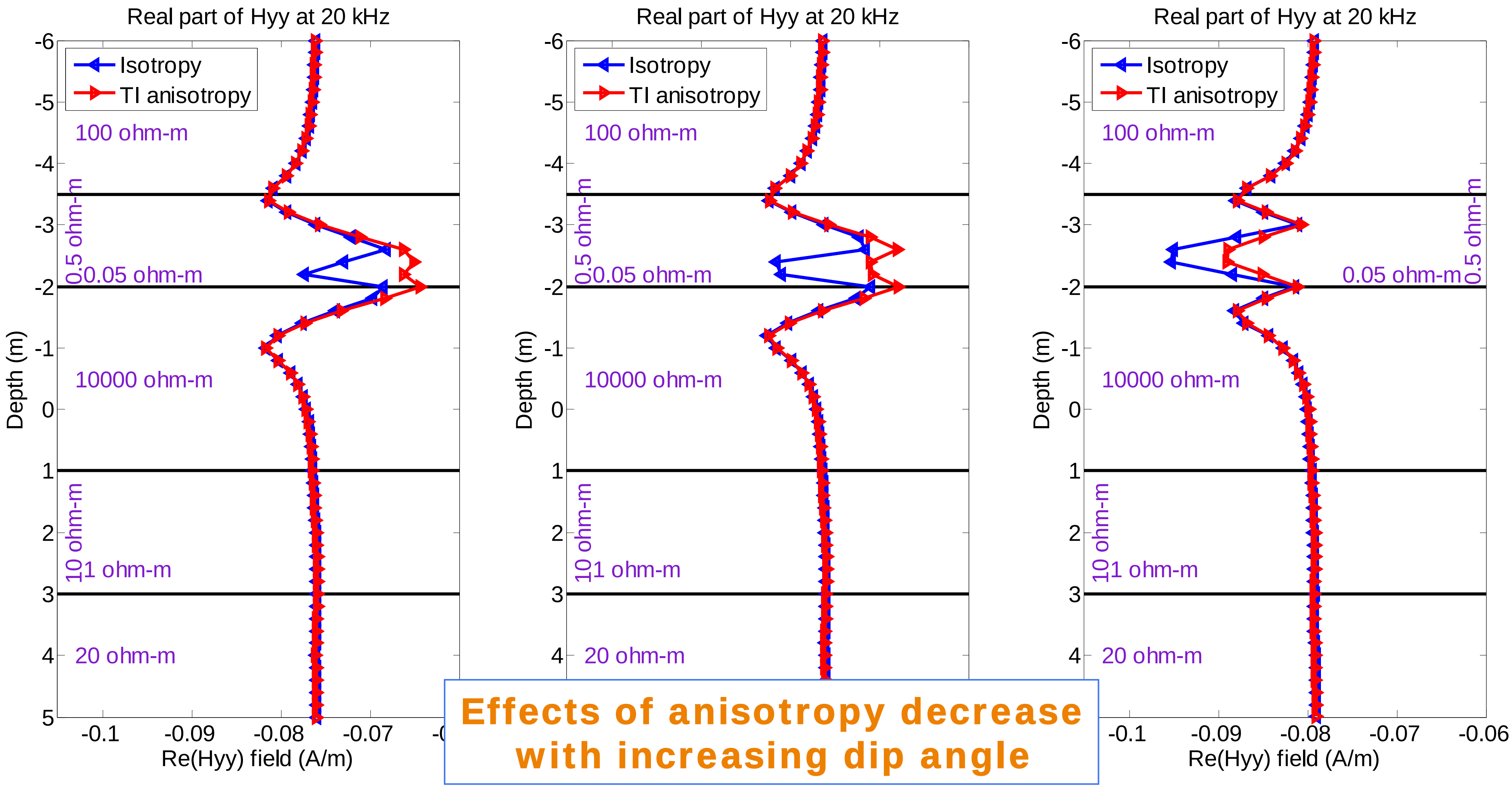
30 degrees



60 degrees

Effects of anisotropy decrease with increasing dip angle

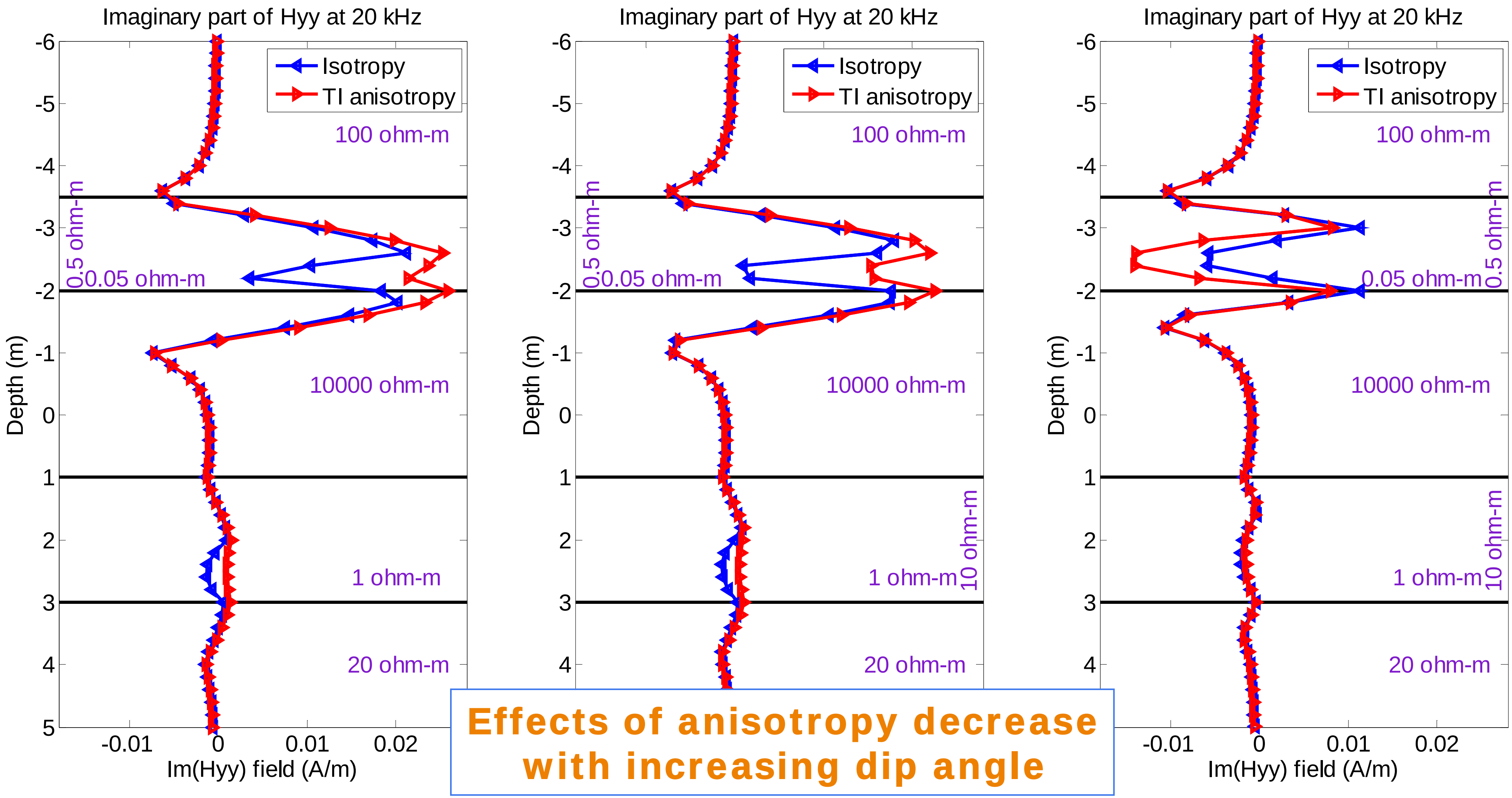
H_{yy} in Deviated Wells with Anisotropy (Re.)



Effects of anisotropy decrease with increasing dip angle

vertical **30 degrees** **60 degrees**

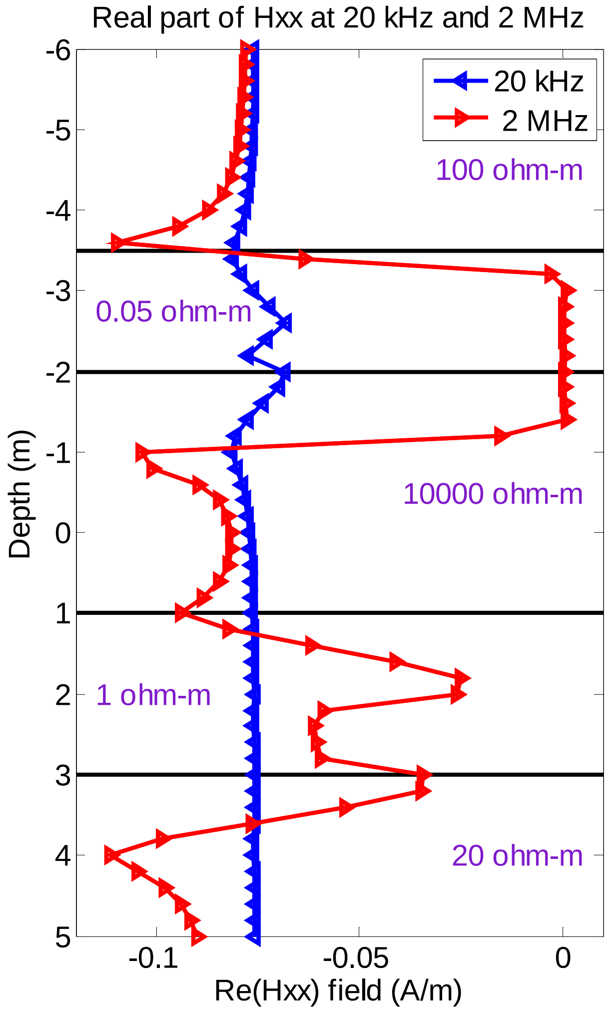
H_{yy} in Deviated Wells with Anisotropy (Im.)

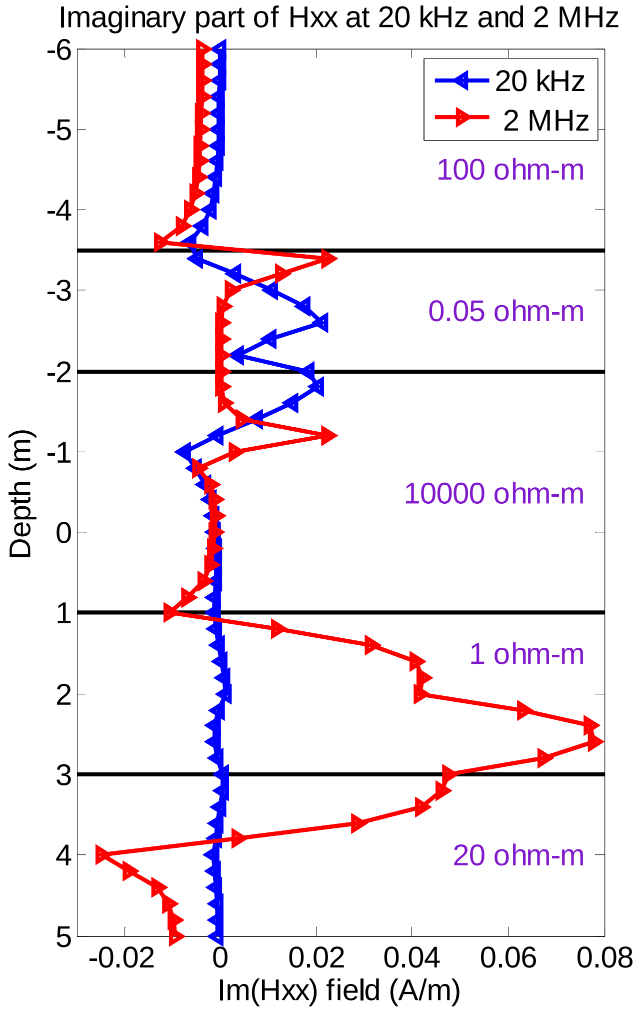


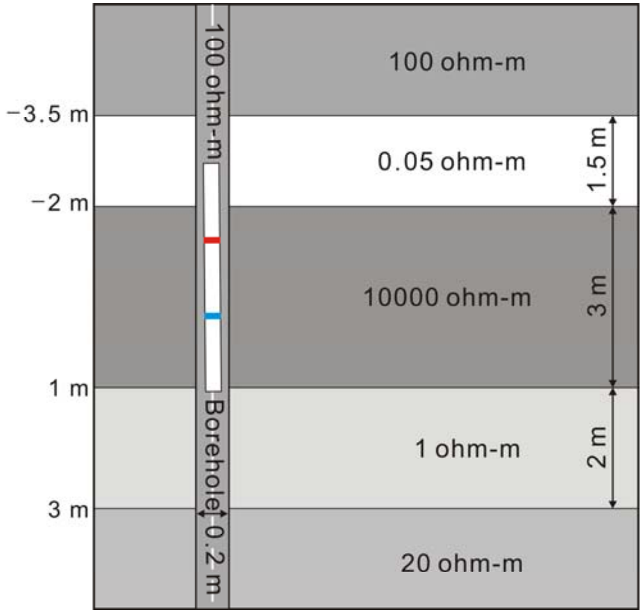
Effects of anisotropy decrease with increasing dip angle

vertical **30 degrees** **60 degrees**

H_{xx} at 20 KHz and 2 MHz in Vertical Well







Larger variations at 2 MHz than at 20 kHz

Conclusions

- We successfully simulated 3D tri-axial induction measurements by combining the use of a Fourier series expansion in a non-orthogonal system of coordinates with a 2D high-order, self-adaptive *hp* finite-element method.

- Dip angle effects on tri-axial tools are larger than on more traditional induction logging instruments.

- Anisotropy effects on H_{xx} and H_{yy} decrease with increasing dip angle, while those on H_{zz} increase.

- H_{xx} at 20 kHz exhibits smaller variations than at 2 MHz.

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