MAFELAP 2009

Parallel Goal-Oriented Adaptivity for a hp Fourier-Finite-Element Method. Applications to the Oil Industry

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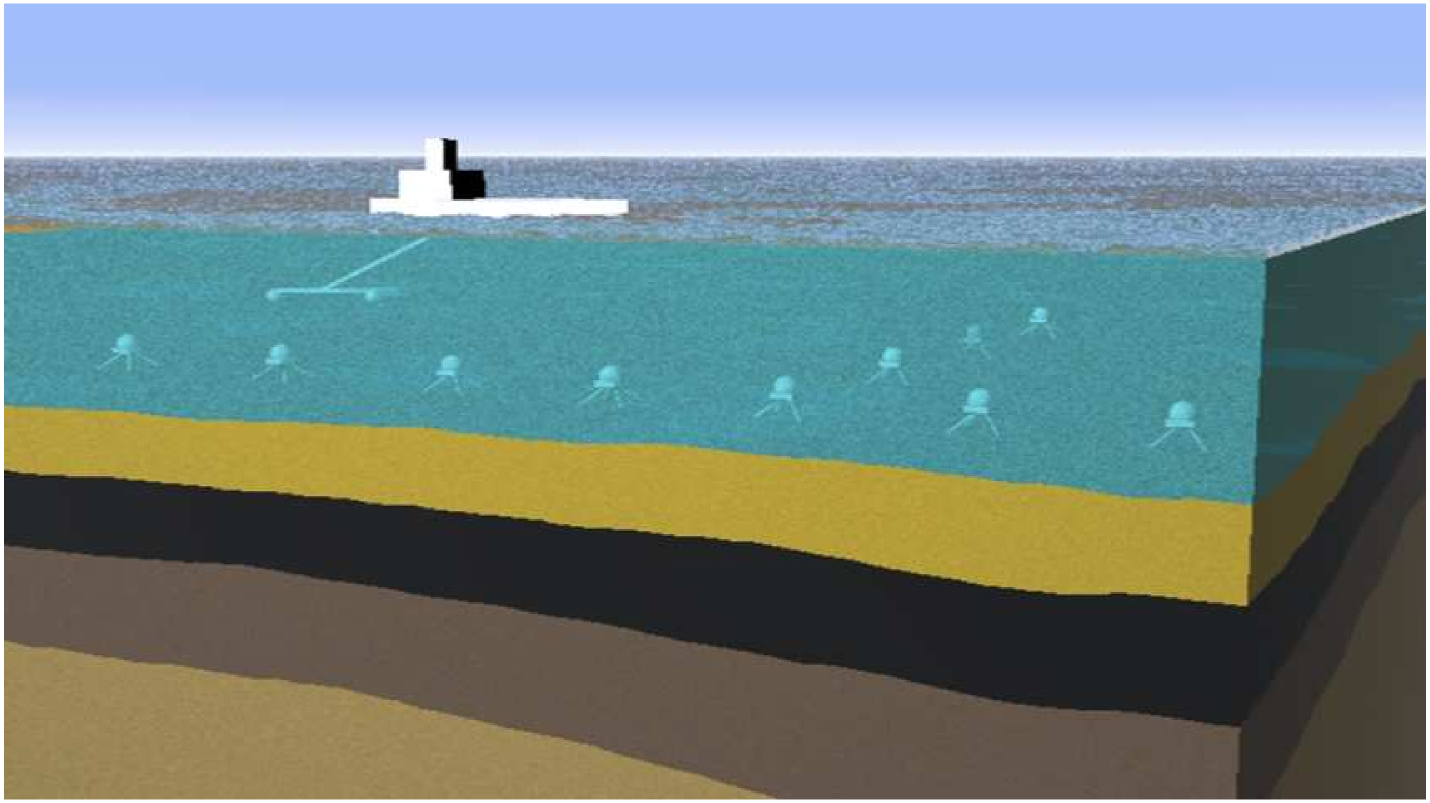
overview

1. Motivation and Objectives: Joint Multiphysics Inversion.
2. Method:
   - Fourier hp-Finite Element Method
   - Self-Adaptive Goal-Oriented hp Adaptivity.
   - Multiphysics Implementation.
   - Parallel Implementation
3. Numerical Results.
4. Conclusions.
5. Future Work.

motivation and objectives

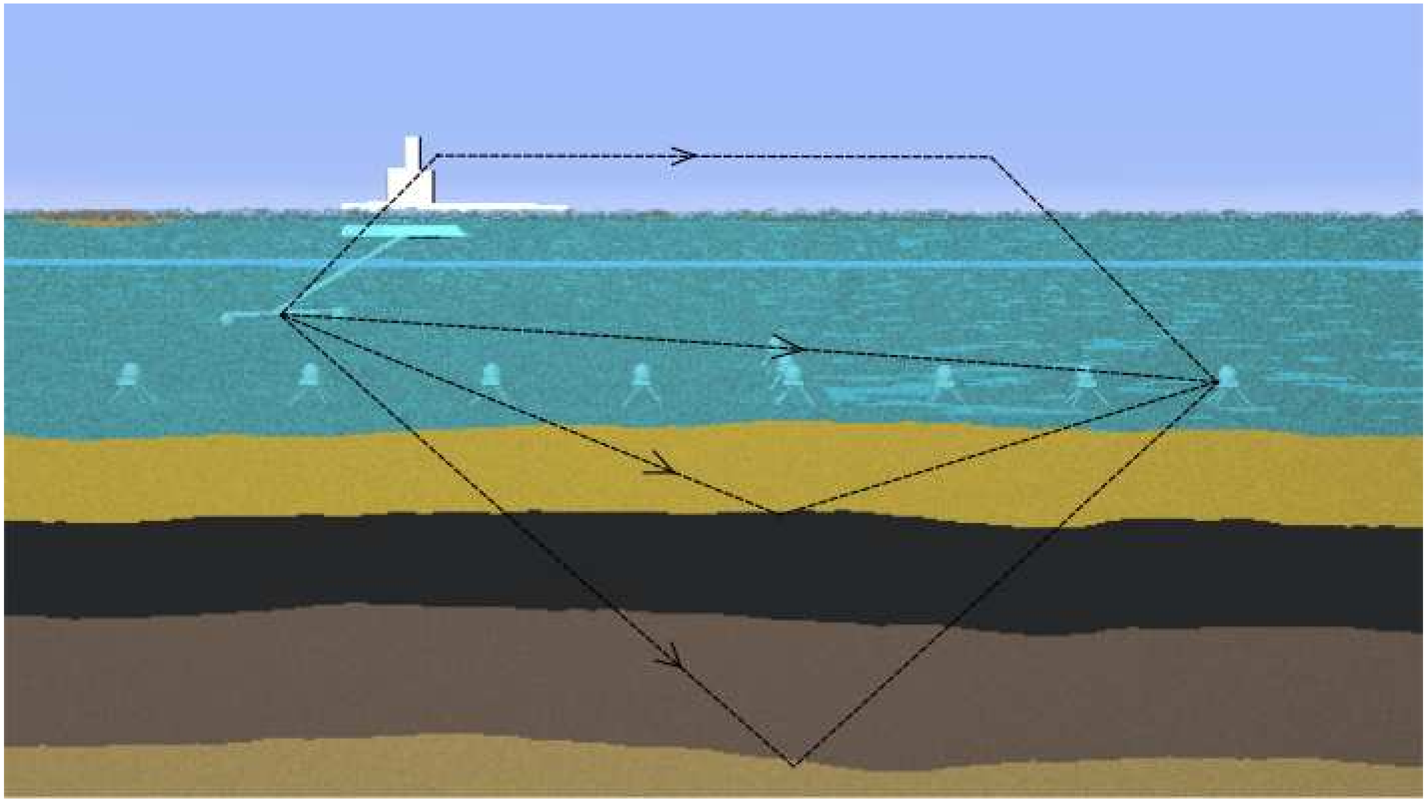
Marine Controlled Source Electromagnetics (CSEM)



CSEM Scenario

motivation and objectives

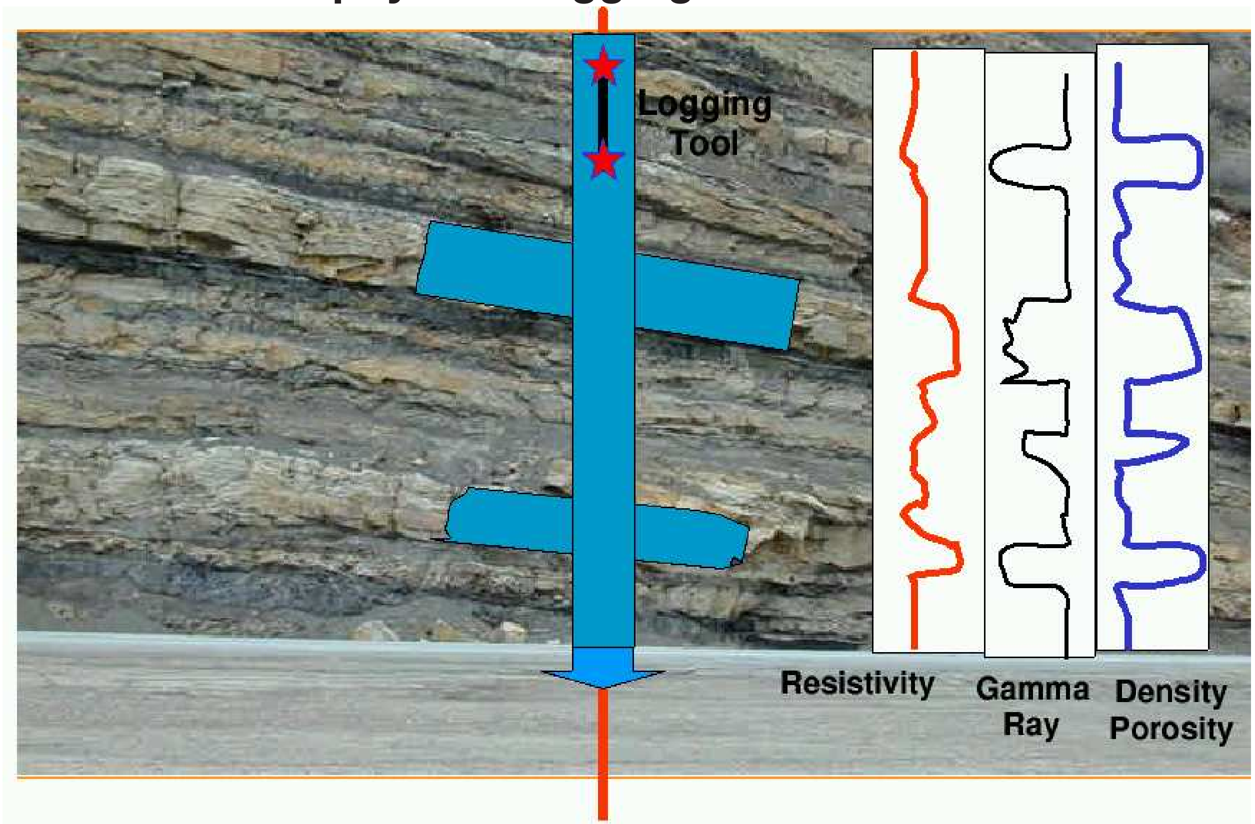
Marine Controlled Source Electromagnetics (CSEM)



EM waves travelling through the air, sea, and sub-surface.

motivation and objectives

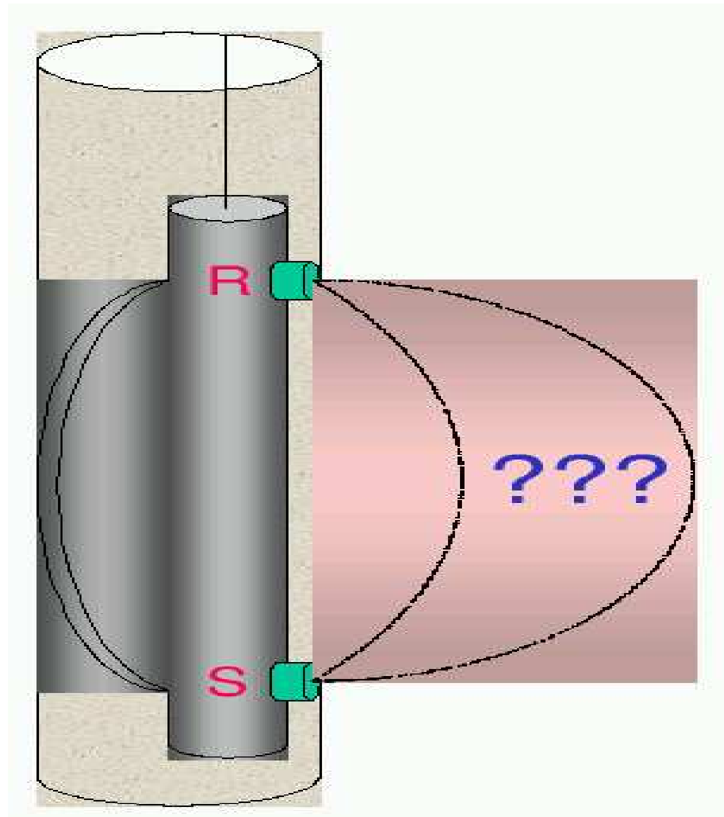
Multiphysics Logging Measurements



OBJECTIVES: To determine payzones (porosity), amount of oil/gas (saturation), and ability to extract oil/gas (permeability).

motivation and objectives

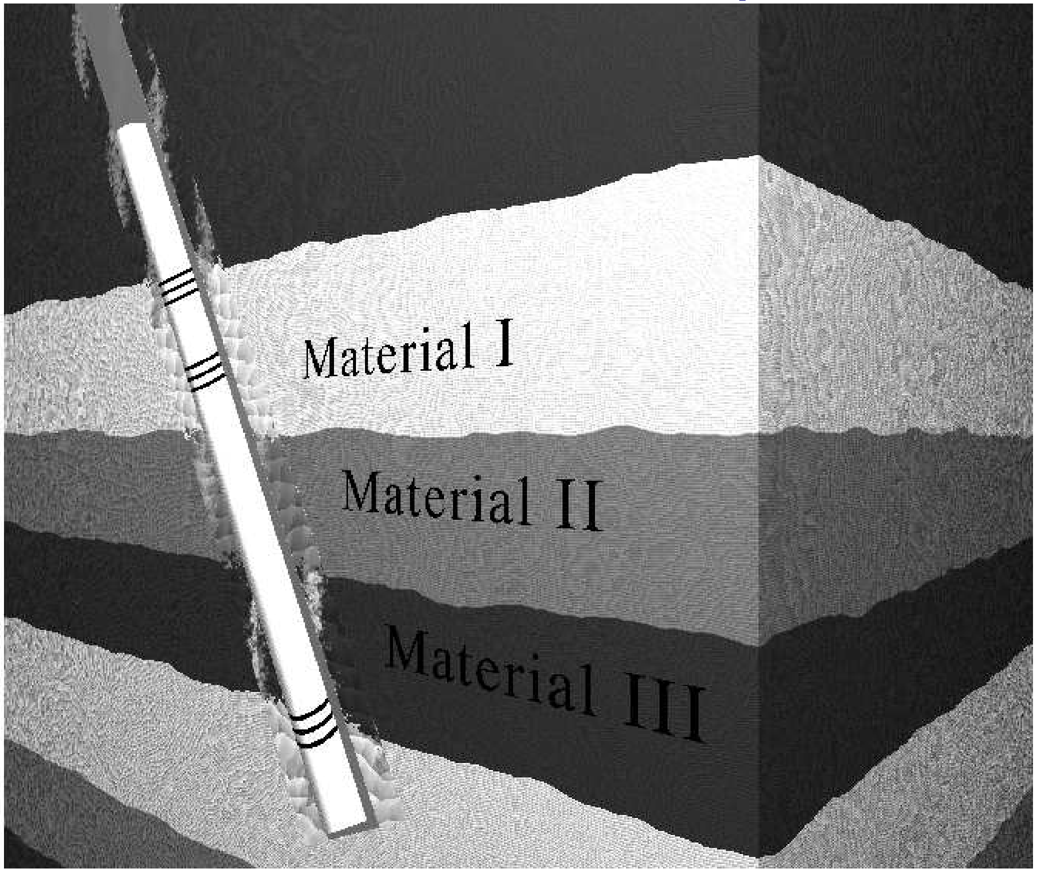
Main Objective: To Solve a Multiphysics Inverse Problem



Software to solve the DIRECT problem is essential in order to solve the INVERSE problem.

motivation and objectives

Deviated Wells (Forward Problem)

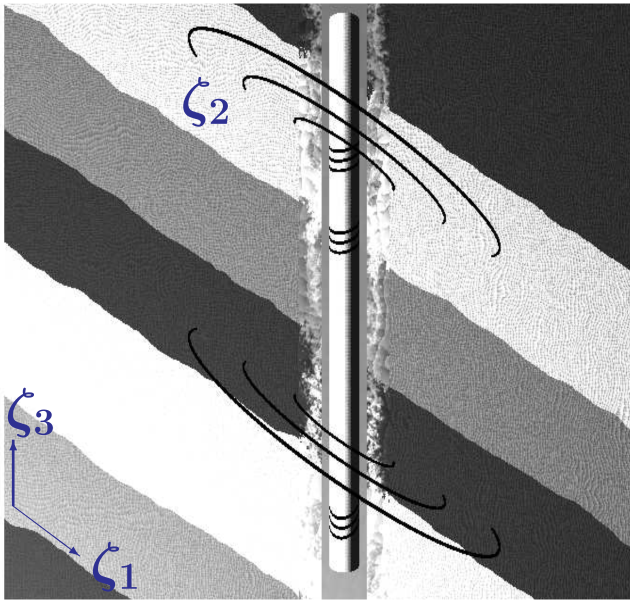


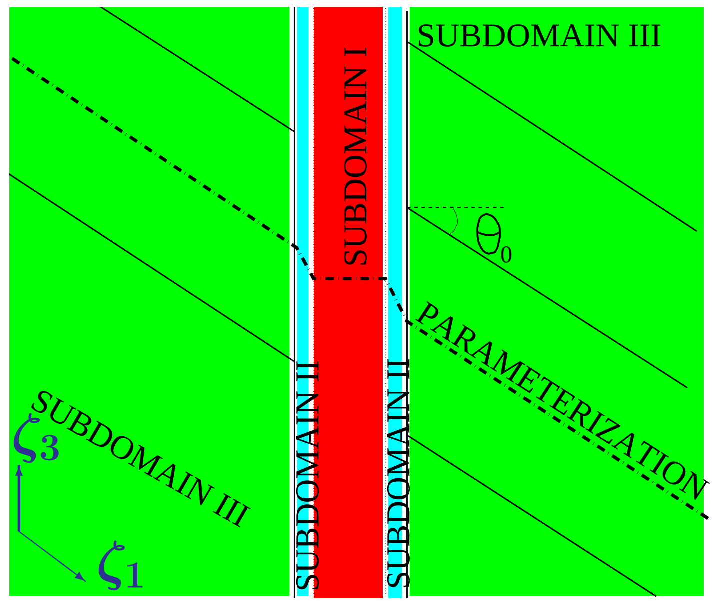
- Dip Angle
- Invasion
- Anisotropy
- Different Sources (Triaxial Induction)
- Eccentric Logging Instruments
- Laterolog
- Through-Casing
- Induction-LWD
- Induction-Wireline

Goal: To find the EM fields at the receiver antennas.

Fourier series expansion

Cartesian system of coordinates: $\mathrm{x} = (x, y, z)$.
New non-orthogonal system of coordinates: $\zeta = (\zeta_1, \zeta_2, \zeta_3)$.



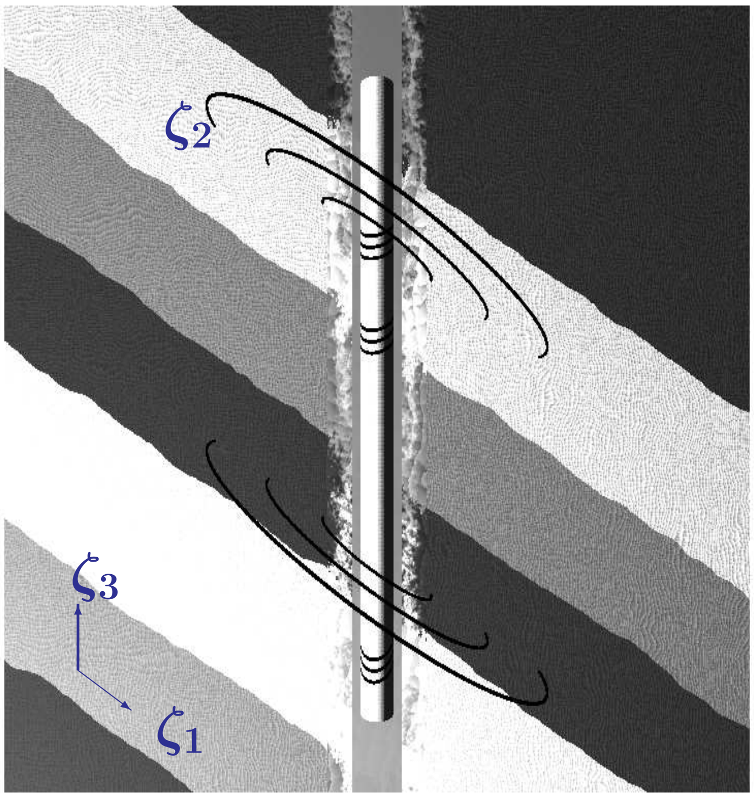


Subdomain I ; **Subdomain II** ; **Subdomain III**

$$\left\{\begin{array}{l} x = \zeta_1 \cos\zeta_2 \\ y = \zeta_1 \sin\zeta_2 \\ z = \zeta_3 \end{array}\right. ; \quad \left\{\begin{array}{l} x = \zeta_1 \cos\zeta_2 \\ y = \zeta_1 \sin\zeta_2 \\ z = \zeta_3 + \tan\theta_0 \dfrac{\zeta_1 - \rho_1}{\rho_2 - \rho_1}\rho_2 \end{array}\right. ; \quad \left\{\begin{array}{l} x = \zeta_1 \cos\zeta_2 \\ y = \zeta_1 \sin\zeta_2 \\ z = \zeta_3 + \tan\theta_0 \zeta_1 \end{array}\right.$$

Fourier series expansion

Non-Orthogonal System of Coordinates



Fourier Series Expansion in ζ_2

DC Problems: $-\nabla\sigma\nabla u = f$

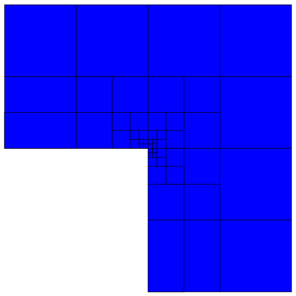
$$u(\zeta_1,\zeta_2,\zeta_3) = \sum_{l=-\infty}^{l=\infty} u_l(\zeta_1,\zeta_3)e^{jl\zeta_2}$$

$$\sigma(\zeta_1,\zeta_2,\zeta_3) = \sum_{m=-\infty}^{m=\infty} \sigma_m(\zeta_1,\zeta_3)e^{jm\zeta_2}$$

$$f(\zeta_1,\zeta_2,\zeta_3) = \sum_{n=-\infty}^{n=\infty} f_n(\zeta_1,\zeta_3)e^{jn\zeta_2}$$

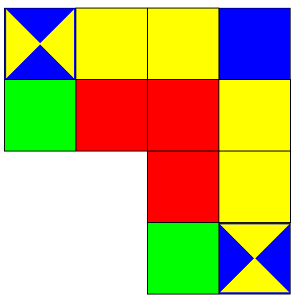
Fourier modes $e^{jl\zeta_2}$ are orthogonal high-order basis functions that are (almost) invariant with respect to the gradient operator.

hp finite element method



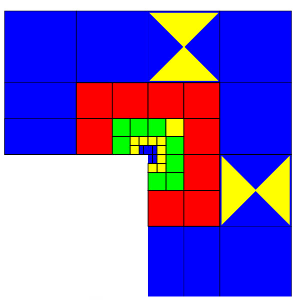
The h-Finite Element Method

1. Convergence limited by the polynomial degree, and large material contrasts.
2. Optimal h-grids do NOT converge exponentially in real applications.
3. They may "lock" (100$\%$ error).



The p-Finite Element Method

1. Exponential convergence feasible for analytical ("nice") solutions.
2. Optimal *p*-grids do NOT converge exponentially in real applications.
3. If initial *h*-grid is not adequate, the *p*-method will fail miserably.



The hp-Finite Element Method

1. Exponential convergence feasible for ALL solutions.
2. Optimal *hp*-grids DO converge exponentially in real applications.
3. If initial *hp*-grid is not adequate, results will still be great.

Fourier finite element method

2D Finite Elements + 1D Fourier

3D Problem (using a Fourier Finite Element Method):

- $H(\mathrm{curl})$ **(Nedelec elements) for the meridian components ($\mathrm{E}_{\rho,z}$), and**
- H^1 **(Lagrange elements) for the azimuthal component (E_ϕ).**

2.5D Problem (using a Fourier Finite Element Method):

- $H(\mathrm{curl})$ **(Nedelec elements) for the meridian components ($\mathrm{E}_{\rho,z}$), and**
- H^1 **(Lagrange elements) for the azimuthal component (E_ϕ).**

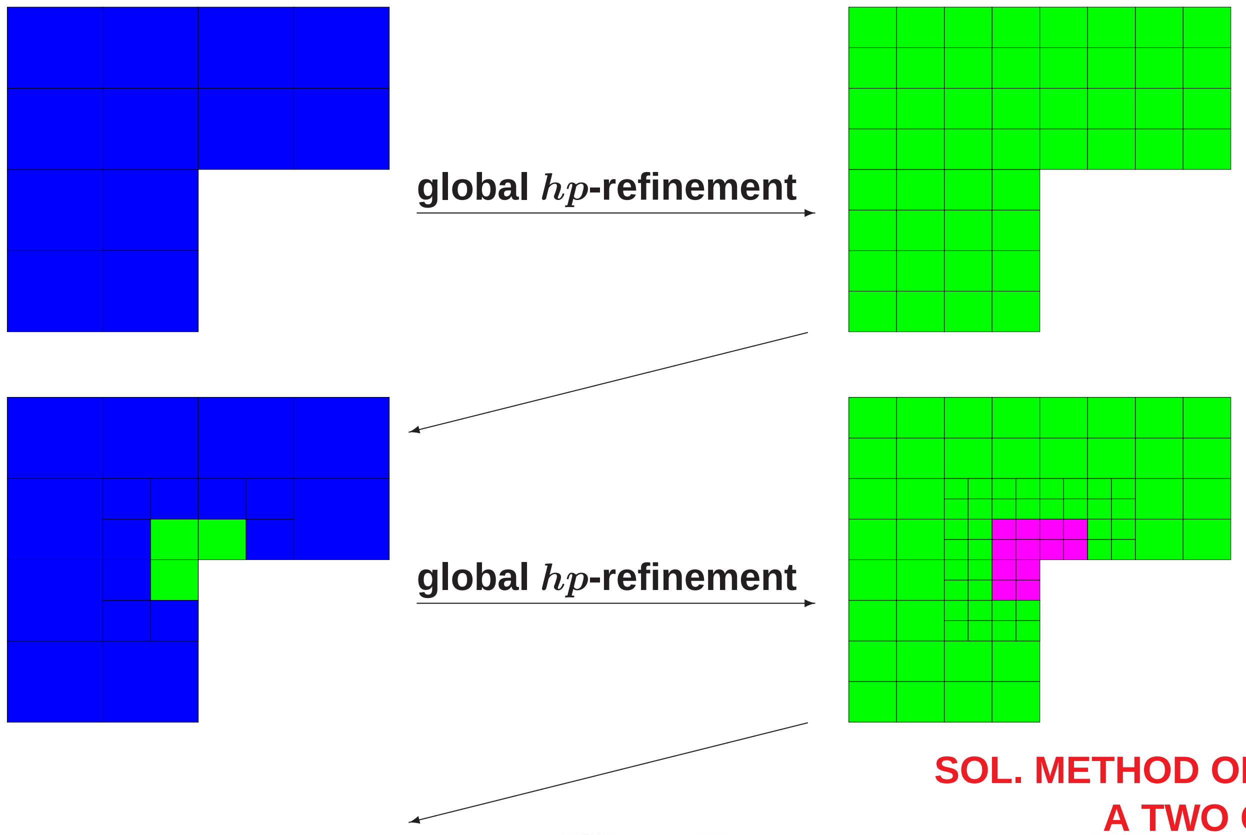
2D Problem:

- $H(\mathrm{curl})$ **(Nedelec elements) in terms of the meridian components ($\mathrm{E}_{\rho,z}$), or**
- H^1 **(Lagrange elements) in terms of the azimuthal component (E_ϕ).**

hp-adaptivity

Energy norm based fully automatic *hp*-adaptive strategy

Coarse grids (hp) **Fine grids** $(h/2, p+1)$



SOL. METHOD ON FINE GRIDS:
A TWO GRID SOLVER

hp-adaptivity

Refinement strategy

Notation:

- K is an element of the hp-grid.
- $E_C = E_{hp}$ (coarse grid) $\prec \quad E_{\widehat{hp}} \quad \prec \quad E_F = E_{h/2,p+1}$ (fine grid).

The adaptive strategy maximizes the following quantity:

$$\widehat{hp} = arg \max_{\widetilde{hp}} \sum_K \frac{|E_F - \Pi^K_{hp} E_F|^2_{?,K} - |E_F - \Pi^K_{\widetilde{hp}} E_F|^2_{?,K}}{(N_{\widetilde{hp}} - N_{hp})^2},$$

where $\Pi^K_{hp} E_F$ is the projection based interpolation of solution E_F over the K-th element of the hp grid.

The choice of the semi-norm depends upon the space in which the solution lives —H^1, $H(\text{curl})$, $H(div)$ or L^2—.

hp-adaptivity

Projection based interpolation

$$\Pi_{hp}^{K} E_F = E_1^{K,hp} + E_2^{K,hp} + E_3^{K,hp}.$$

- $E_1^{K,hp}$ is the "bilinear vertex interpolant" of the K-th element of the hp-grid.
- $E_2^{K,hp}$ is the "projection" of $E_F - E_1^{K,hp}$ over each edge of the K-th element of the hp-grid.
- $E_3^{K,hp}$ is the "projection" of $E_F - E_1^{K,hp} - E_2^{K,hp}$ over the interior of the K-th element of the hp-grid.

The projection depends upon the space in which the solution lives —H^1, $H(\mathrm{curl})$, $H(div)$ or L^2—.

Question: How can we combine energies coming from different norms/spaces?

hp goal oriented adaptivity

Mathematical Formulation (Goal-Oriented Adaptivity)

We consider the following problem (in variational form):

$$\begin{cases} \text{Find } L(E)\text{, where } E \in V \text{ such that :} \\ b(E,\xi) = f(\xi) \quad \forall \xi \in V \ . \end{cases}$$

We define residual $r_e(\xi) = b(e,\xi)$. We seek for solution G of:

$$\begin{cases} \text{Find } G \in V'' \sim V \text{ such that :} \\ G(r_e) = L(e) \ . \end{cases}$$

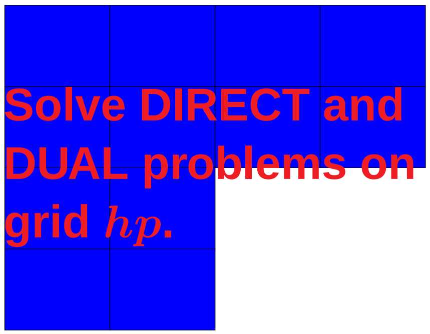
This is necessarily solved if we find the solution of the *dual* problem:

$$\begin{cases} \text{Find } G \in V \text{ such that :} \\ b(E,G) = L(E) \quad \forall E \in V \ . \end{cases}$$

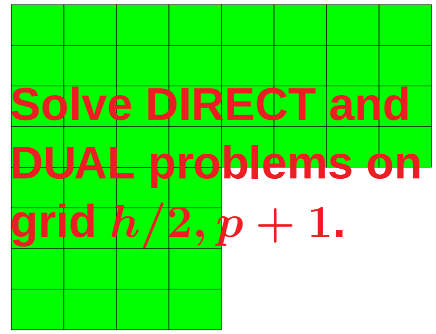
Notice that $L(e) = b(e,G)$.

hp goal oriented adaptivity

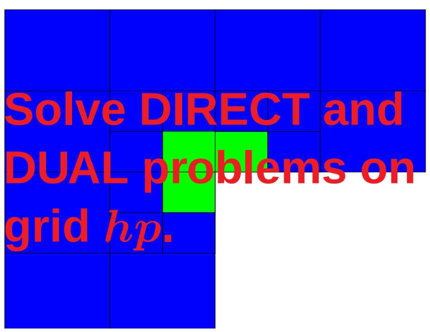
Algorithm for Goal-Oriented Adaptivity



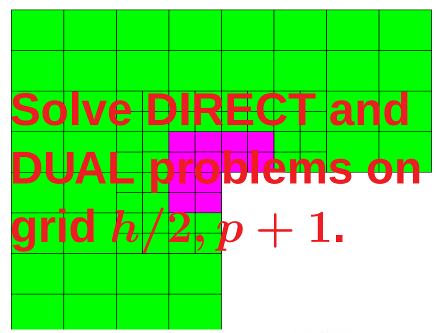
$\longrightarrow$



Compute $e = E_{h/2,p+1} - E_{hp}$, and $\epsilon = G_{h/2,p+1} - G_{hp}$.
Represent the error as: $|L(e)| = |b(e,\epsilon)| \leq \sum_K |b_K(e,\epsilon)|$.
Apply the fully automatic hp-adaptive algorithm.



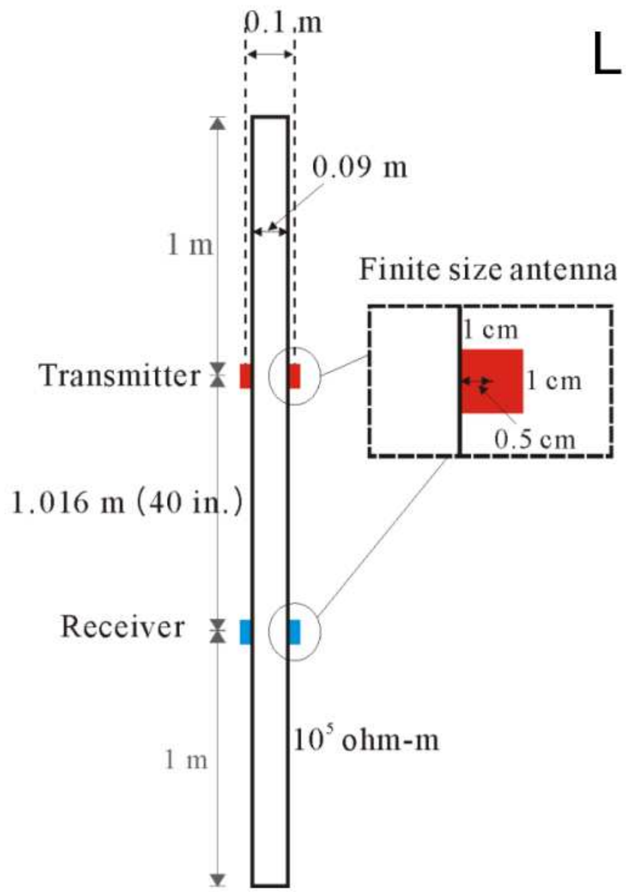
$\longrightarrow$

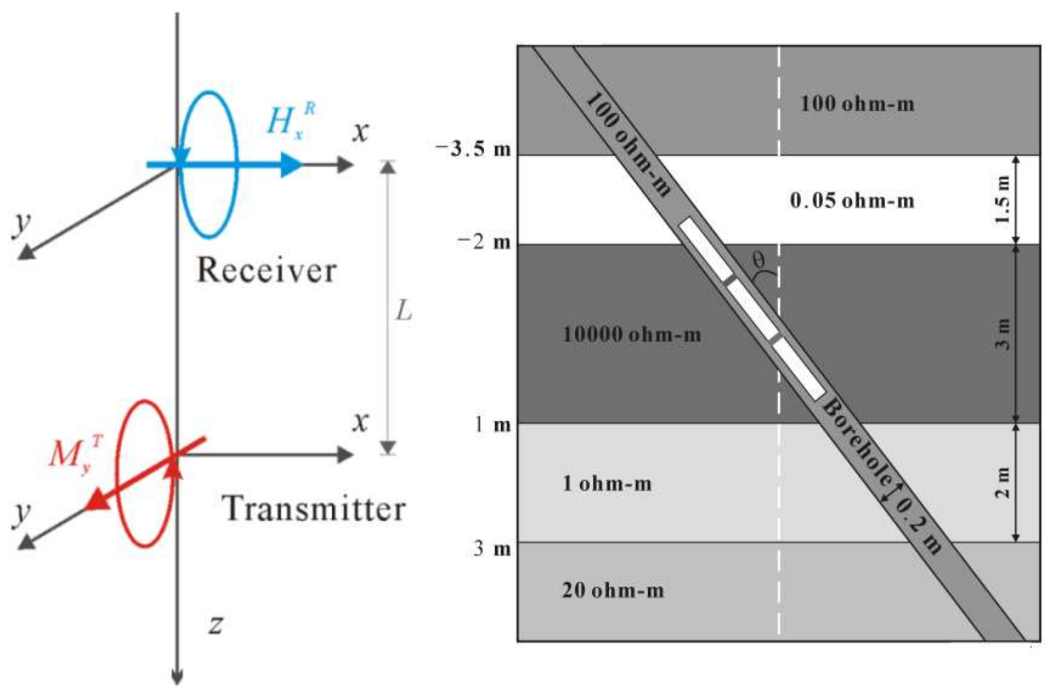


logging electromagnetic applications

Tri-Axial Induction Tool

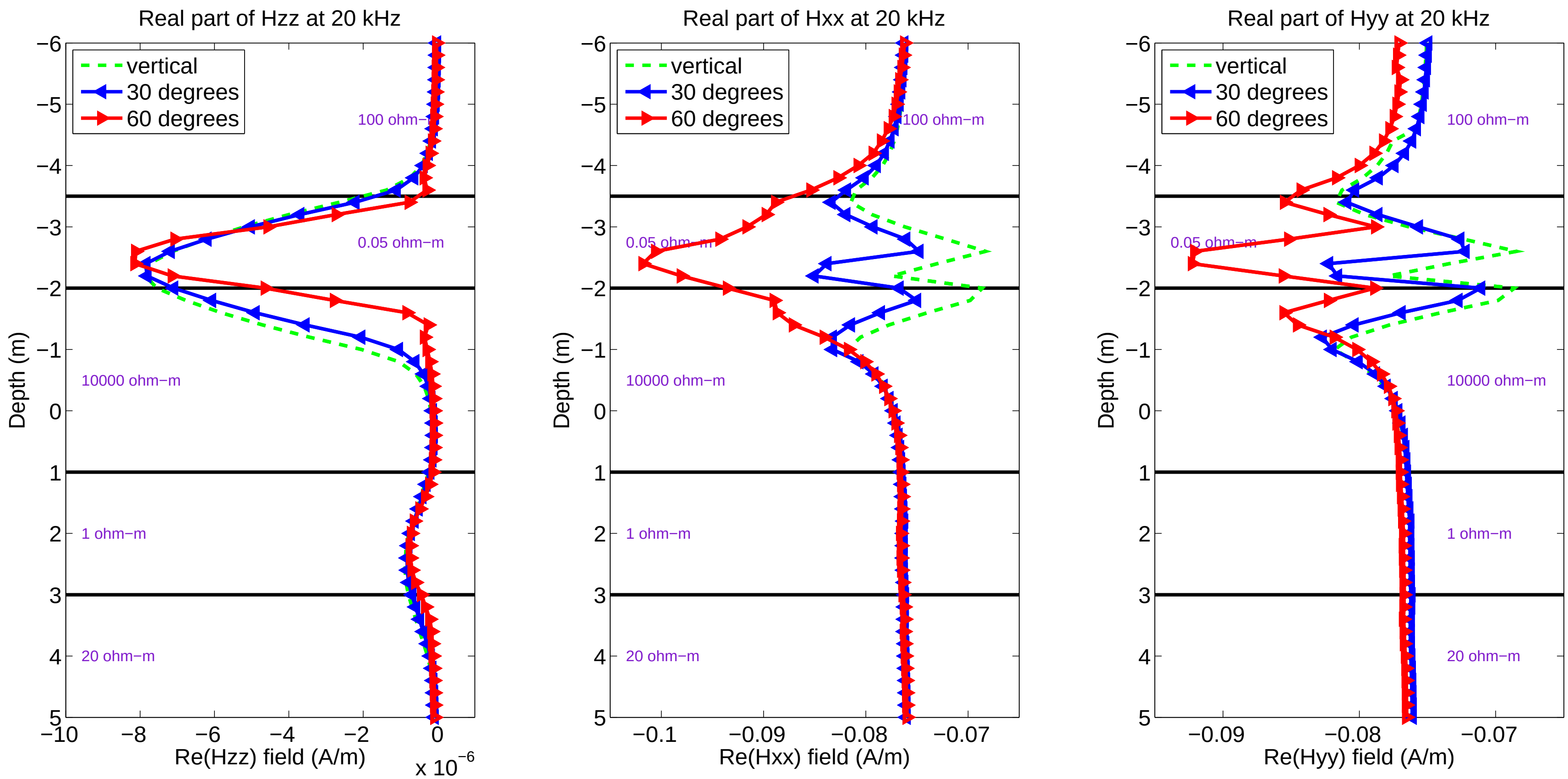
L=40 in, Frequency=20 kHz





logging electromagnetic applications

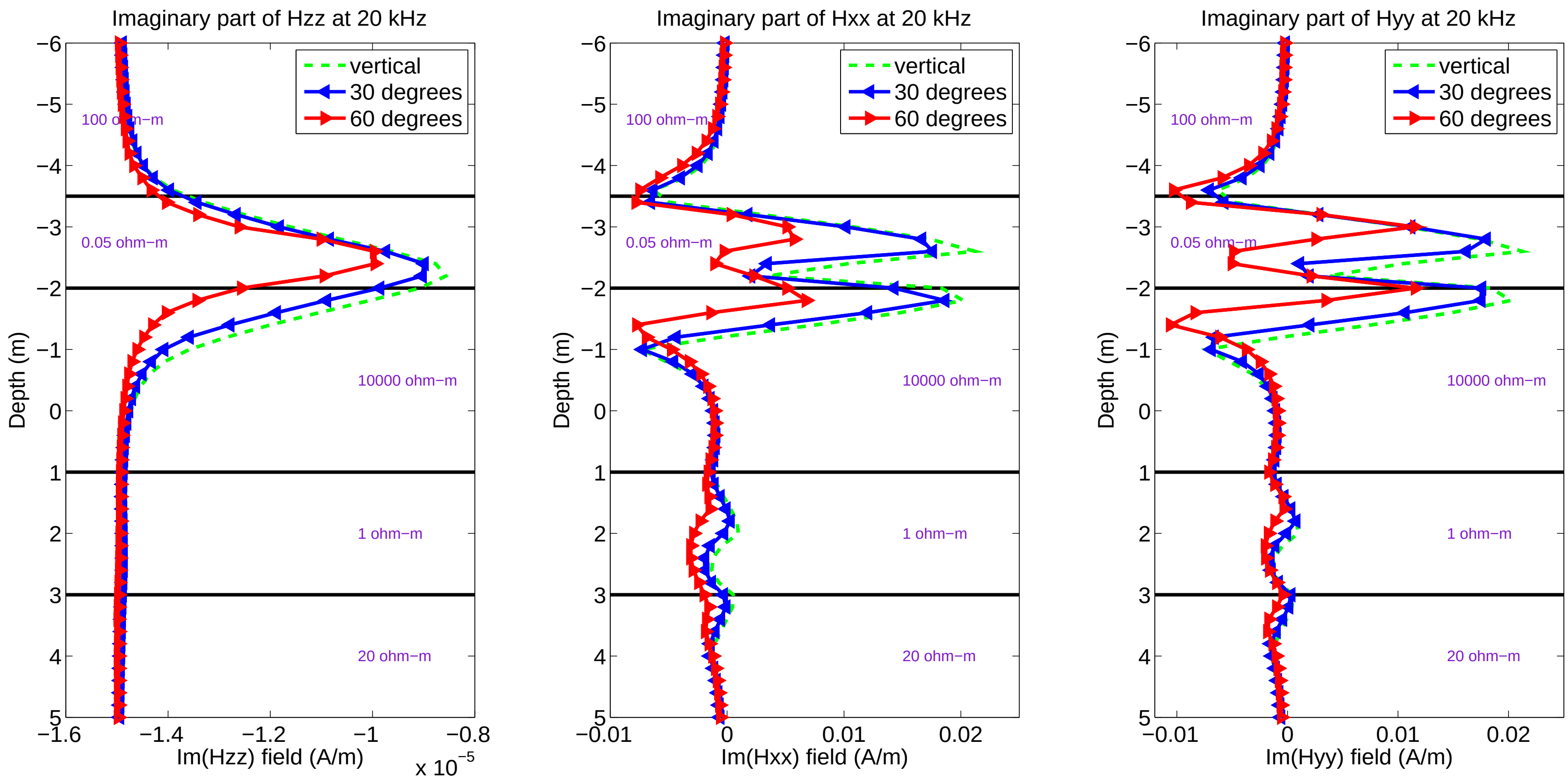
Tri-Axial Induction Tools in Deviated Wells (0, 30, and 60 degrees)



Triaxial tools are more sensitive to dip angle effects

logging electromagnetic applications

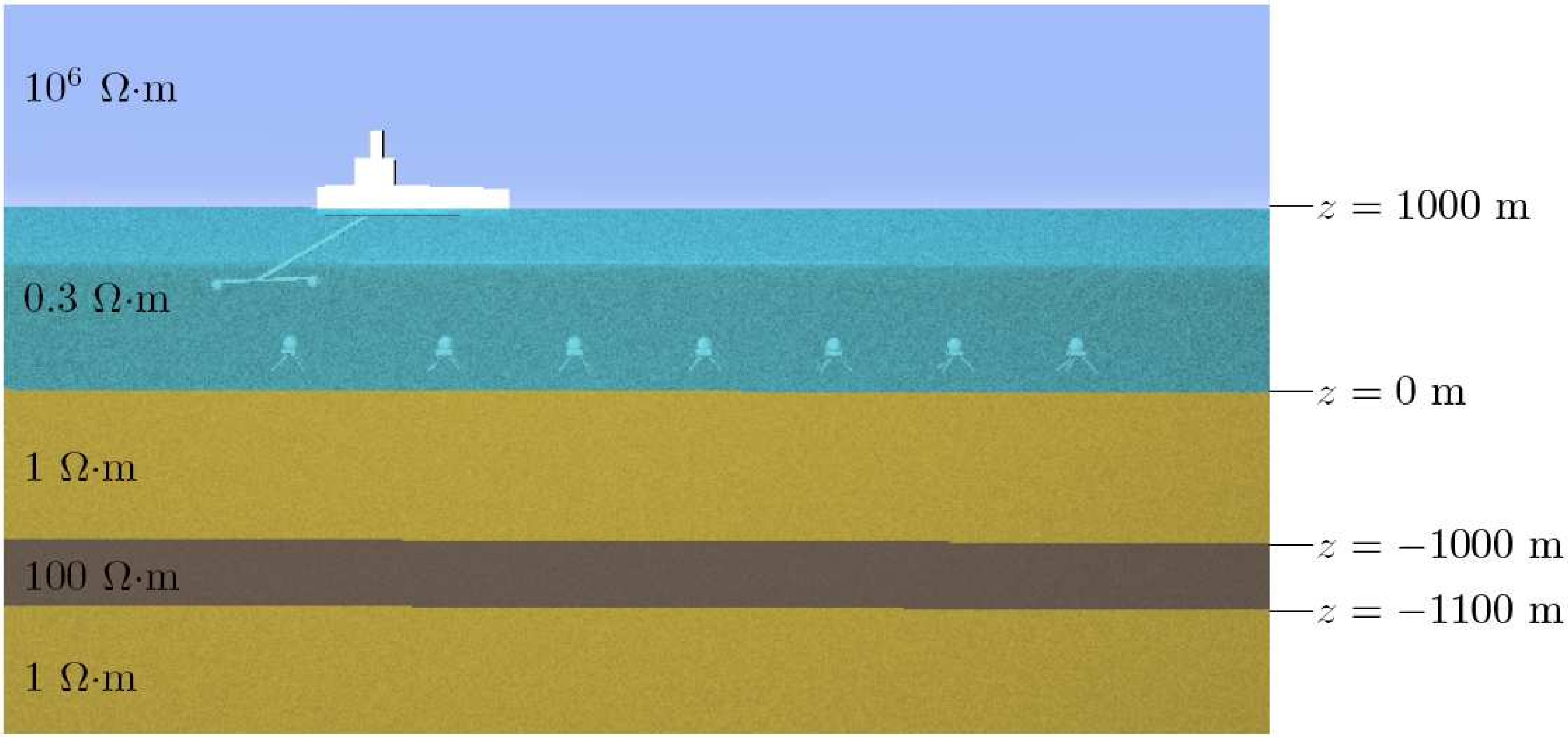
Tri-Axial Induction Tools in Deviated Wells (0, 30, and 60 degrees)



Triaxial tools are more sensitive to dip angle effects

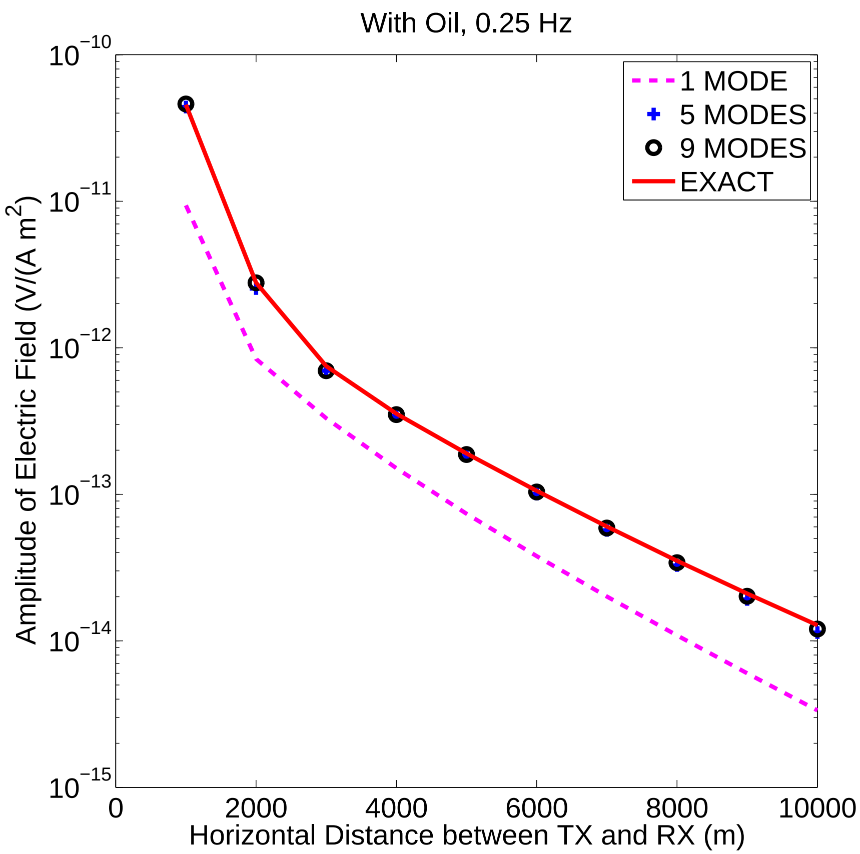
marine CSEM applications

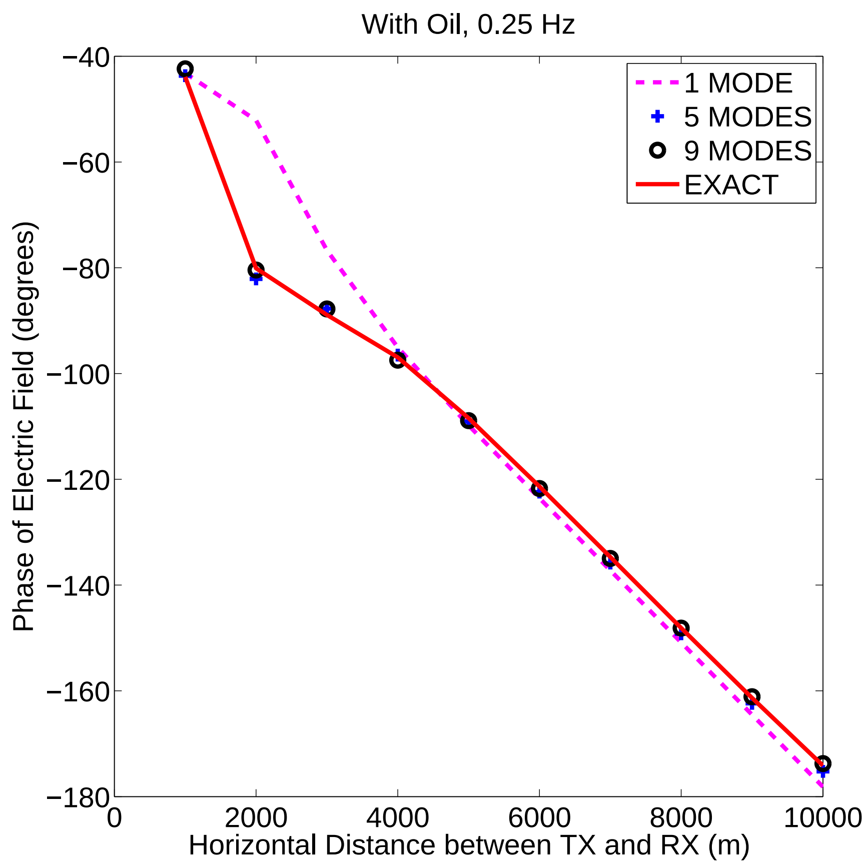
Model Problem I: Marine CSEM Scenario with an Infinite Layer of Oil



marine CSEM applications

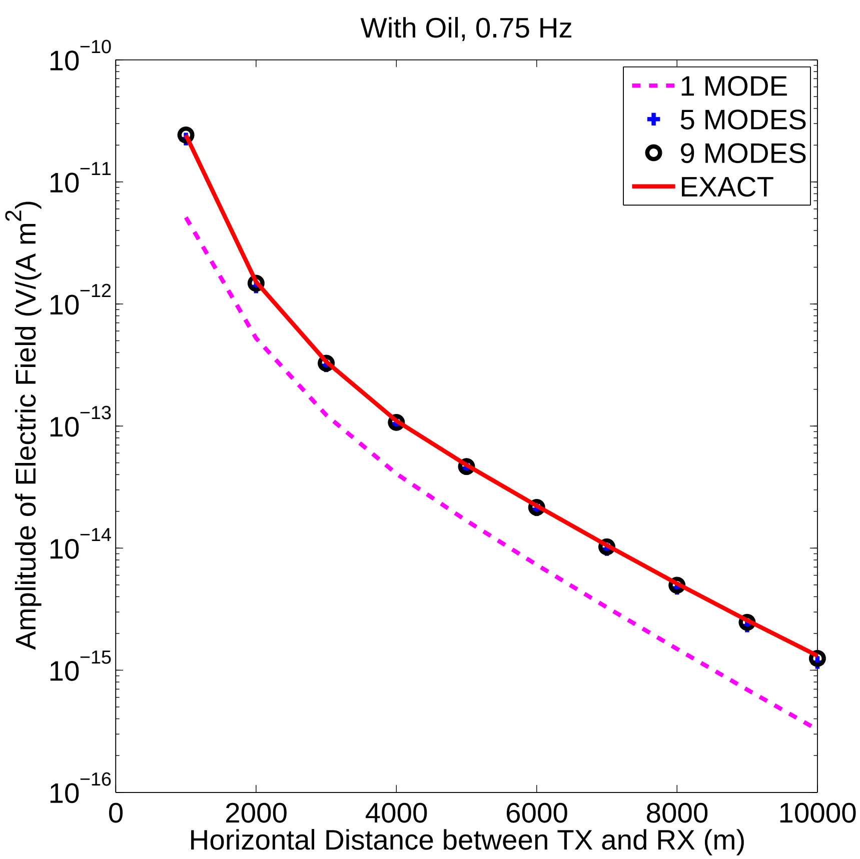
Model Problem I: INFINITE LAYER OF OIL — 0.25 Hz —

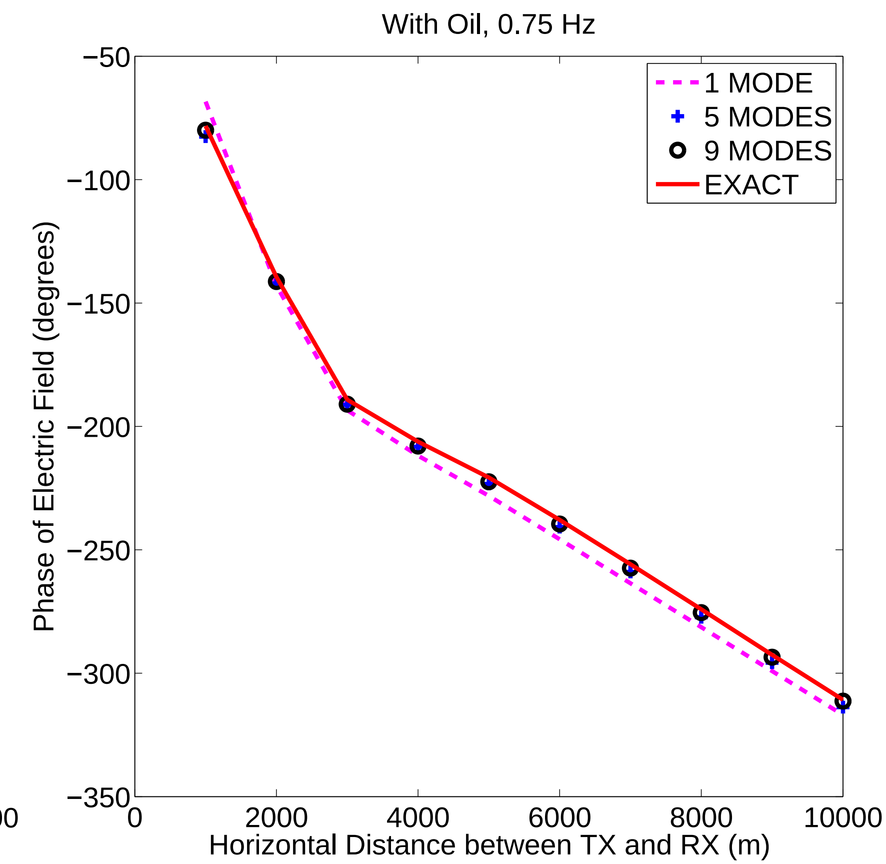




marine CSEM applications

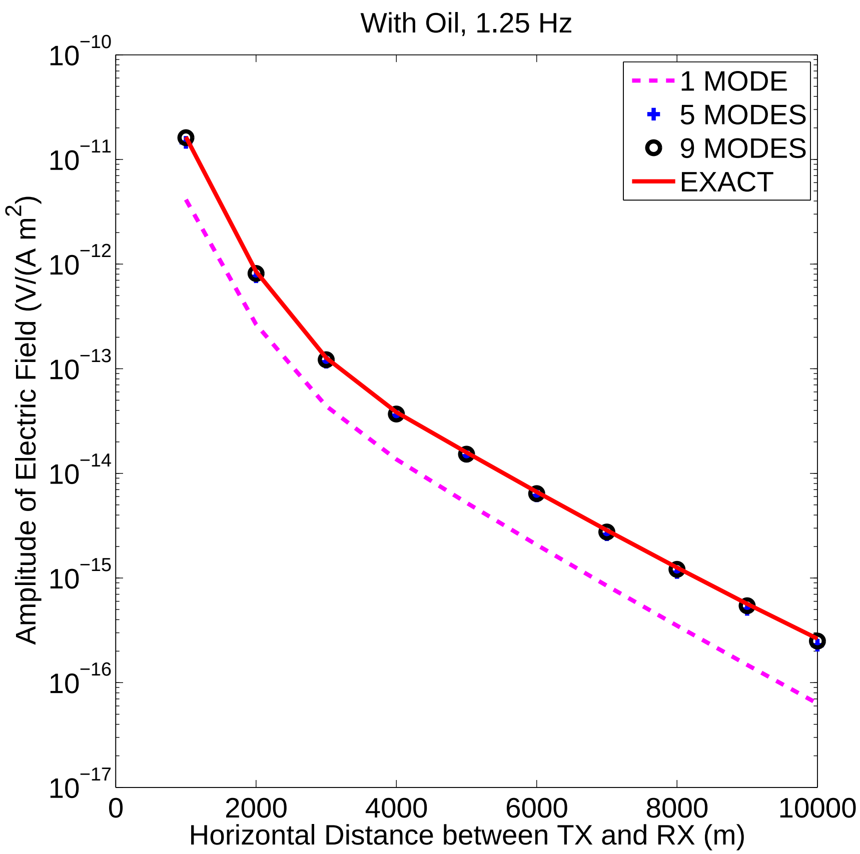
Model Problem I: INFINITE LAYER OF OIL — 0.75 Hz —

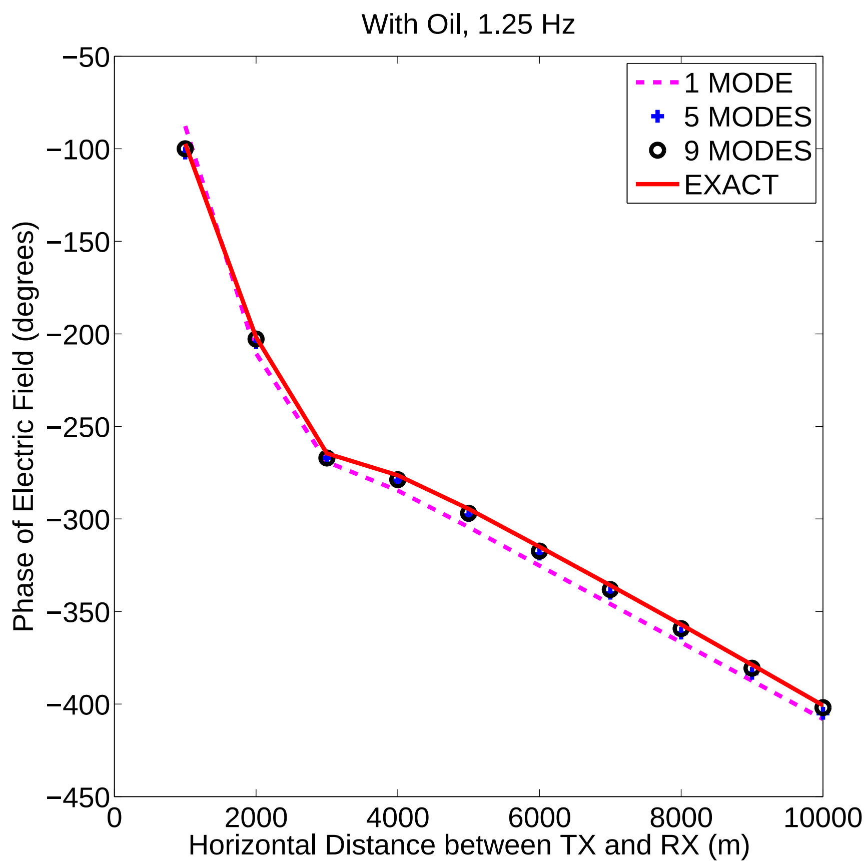




marine CSEM applications

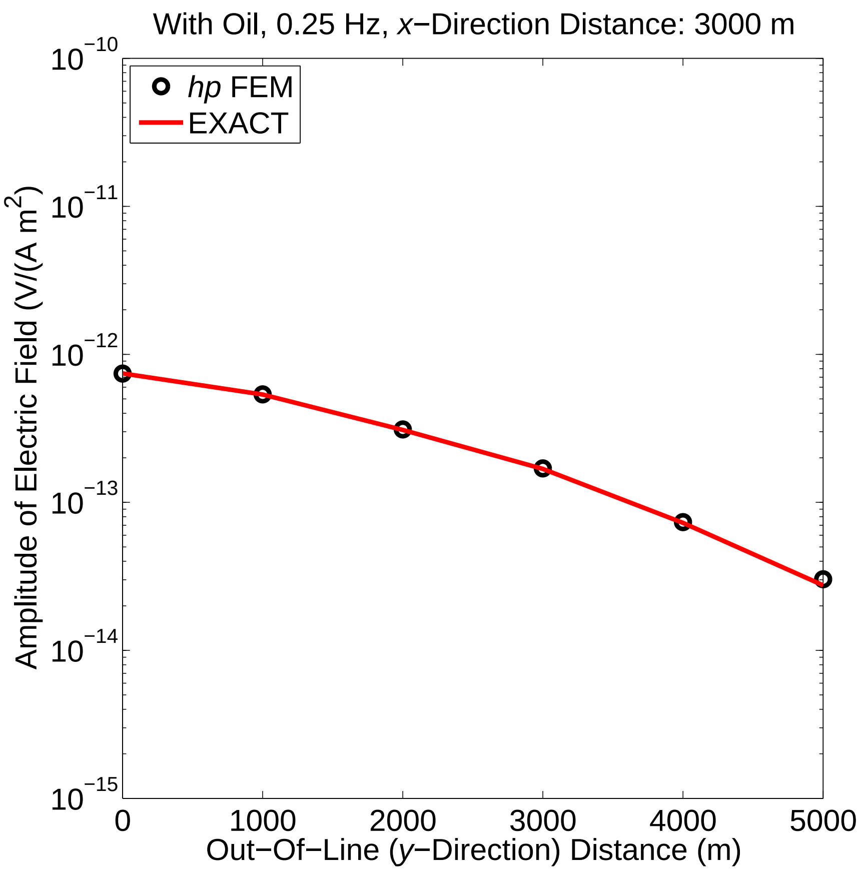
Model Problem I: INFINITE LAYER OF OIL — 1.25 Hz —

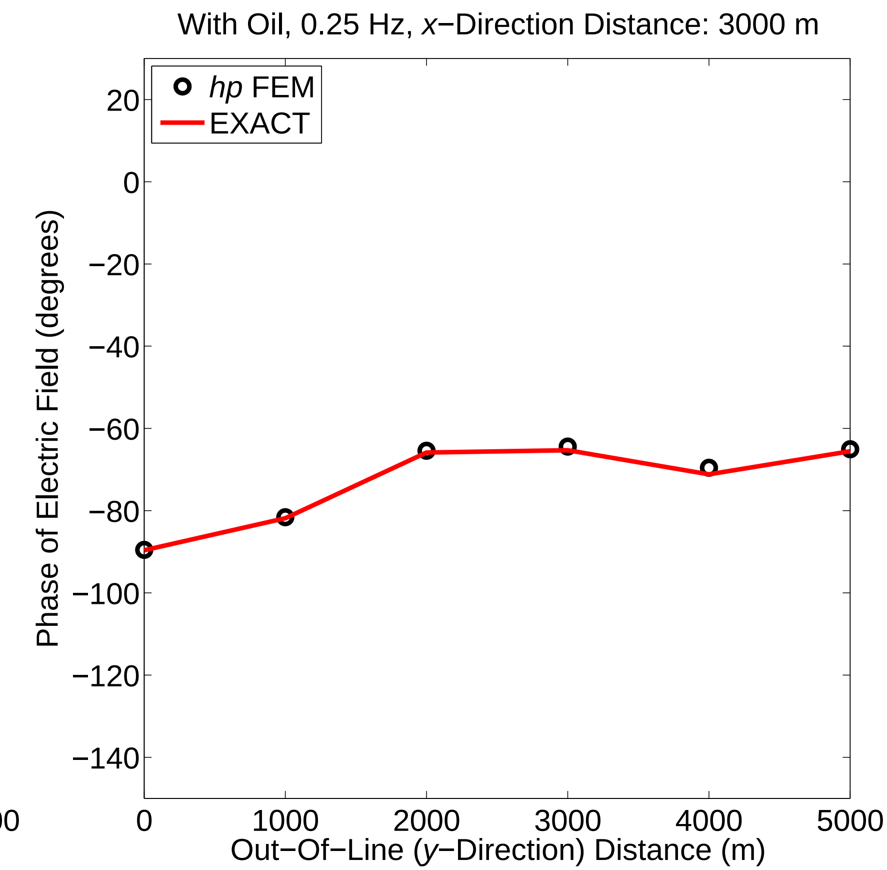




marine CSEM applications

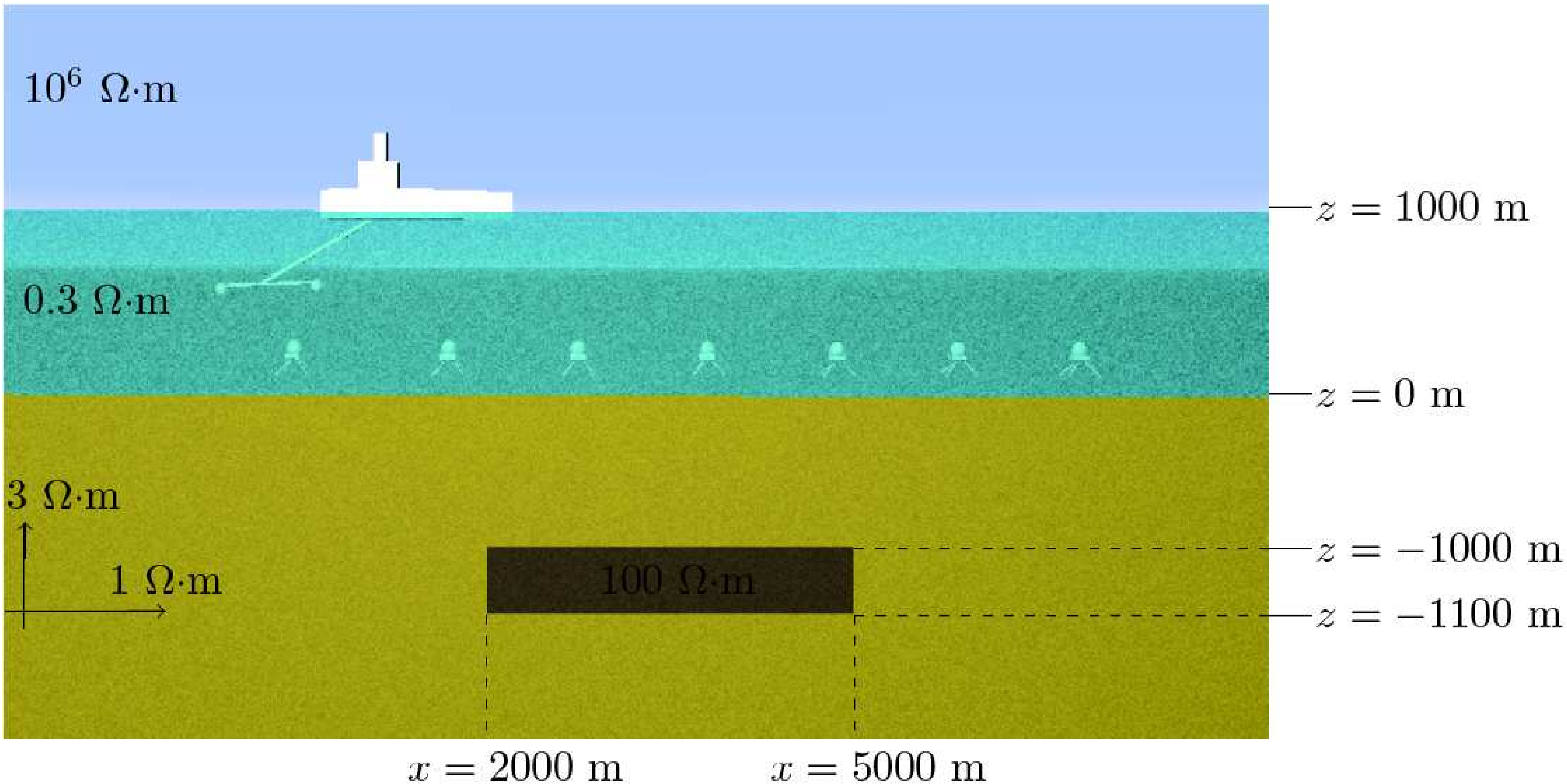
Model Problem I: INFINITE LAYER OF OIL — Out-of-line Receivers —





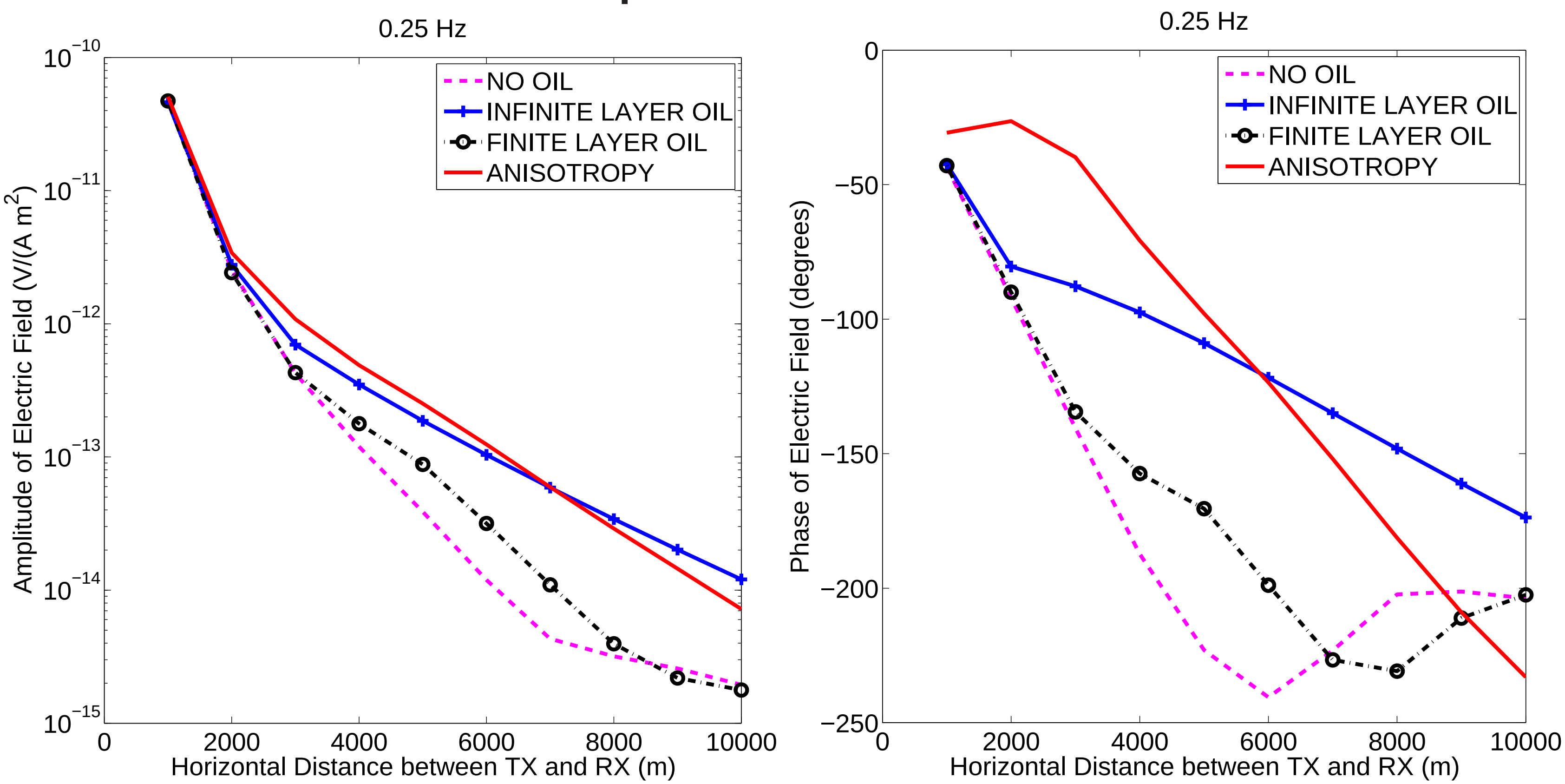
marine CSEM applications

Model Problem I: Marine CSEM Scenario with a Finite Layer of Oil



marine CSEM applications

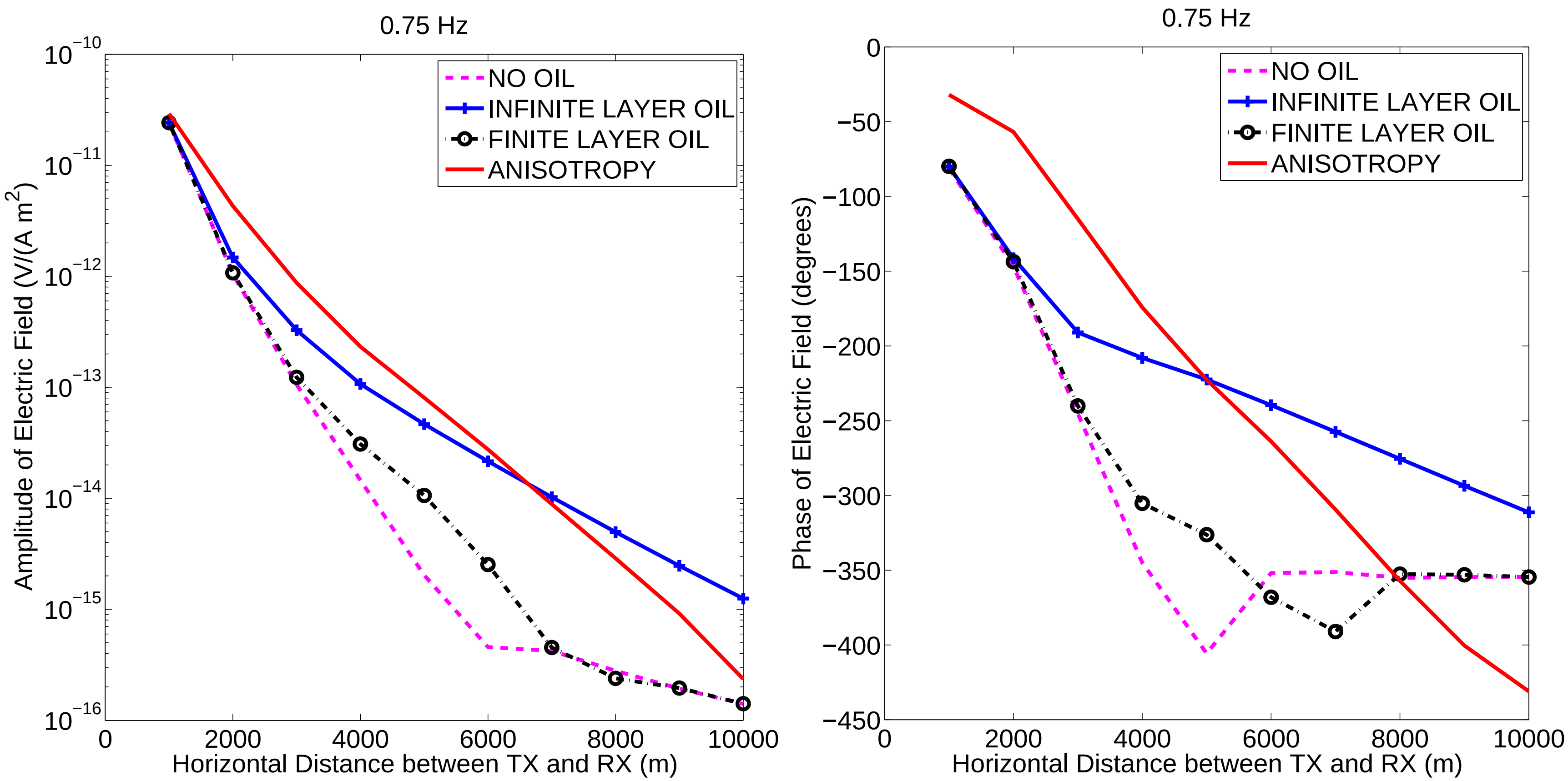
Comparison — 0.25 Hz —



The finite layer of oil is clearly identified, and it is different from the solution for the infinite layer of oil. To consider anisotropy is essential.

marine CSEM applications

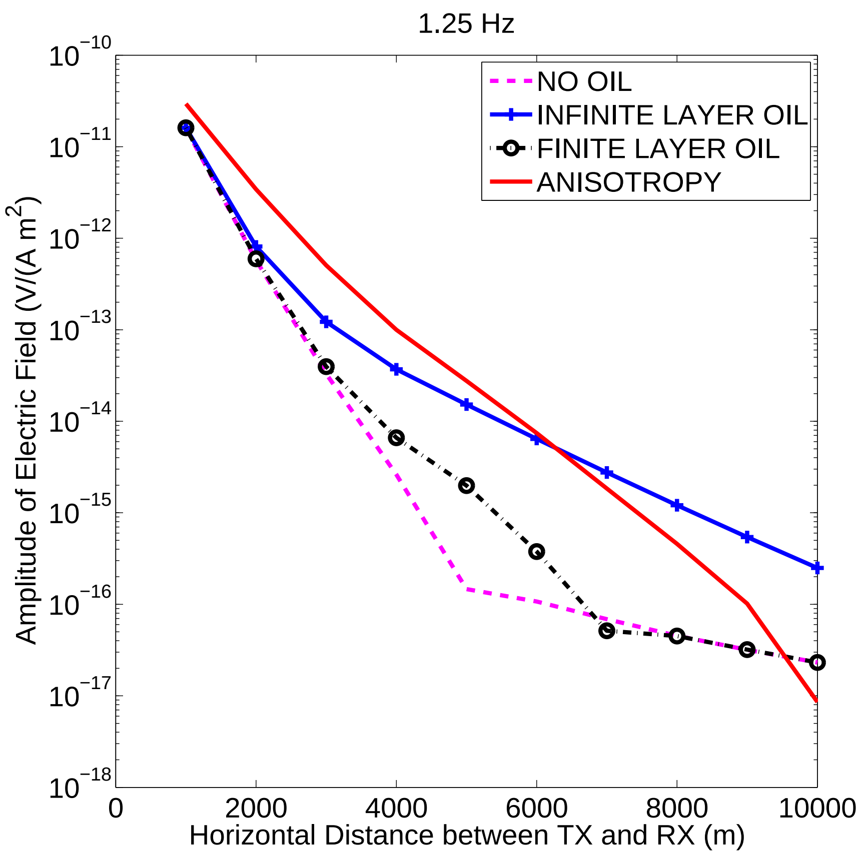
Comparison — 0.75 Hz —

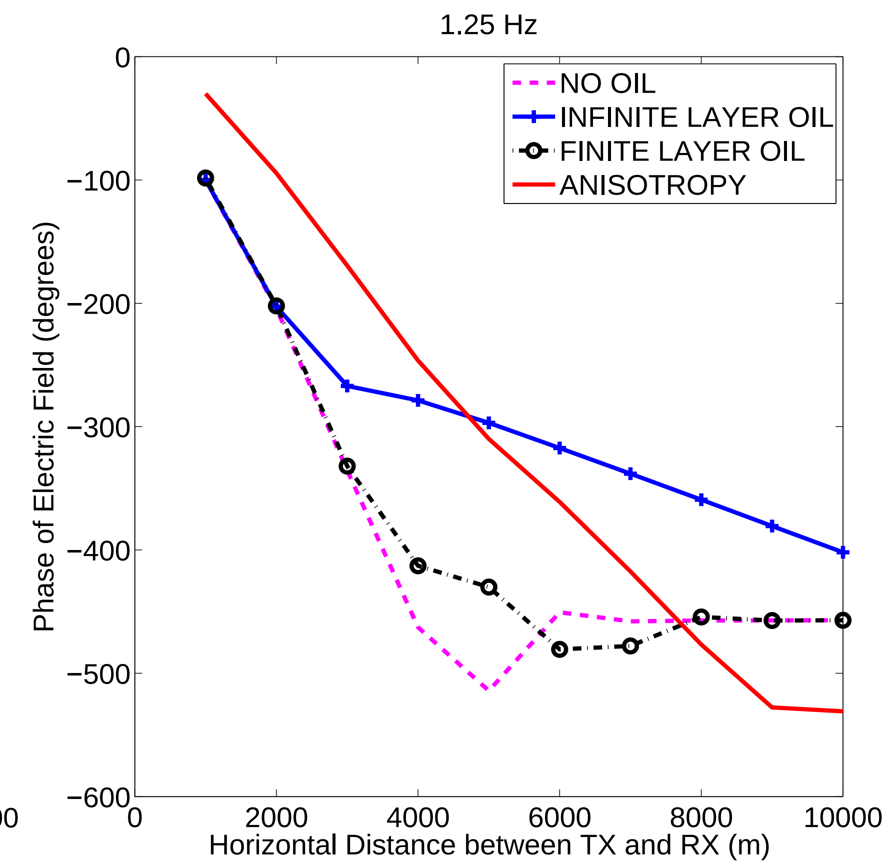


As we increase the frequency, the effect of oil becomes more localized.

marine CSEM applications

Comparison — 1.25 Hz —



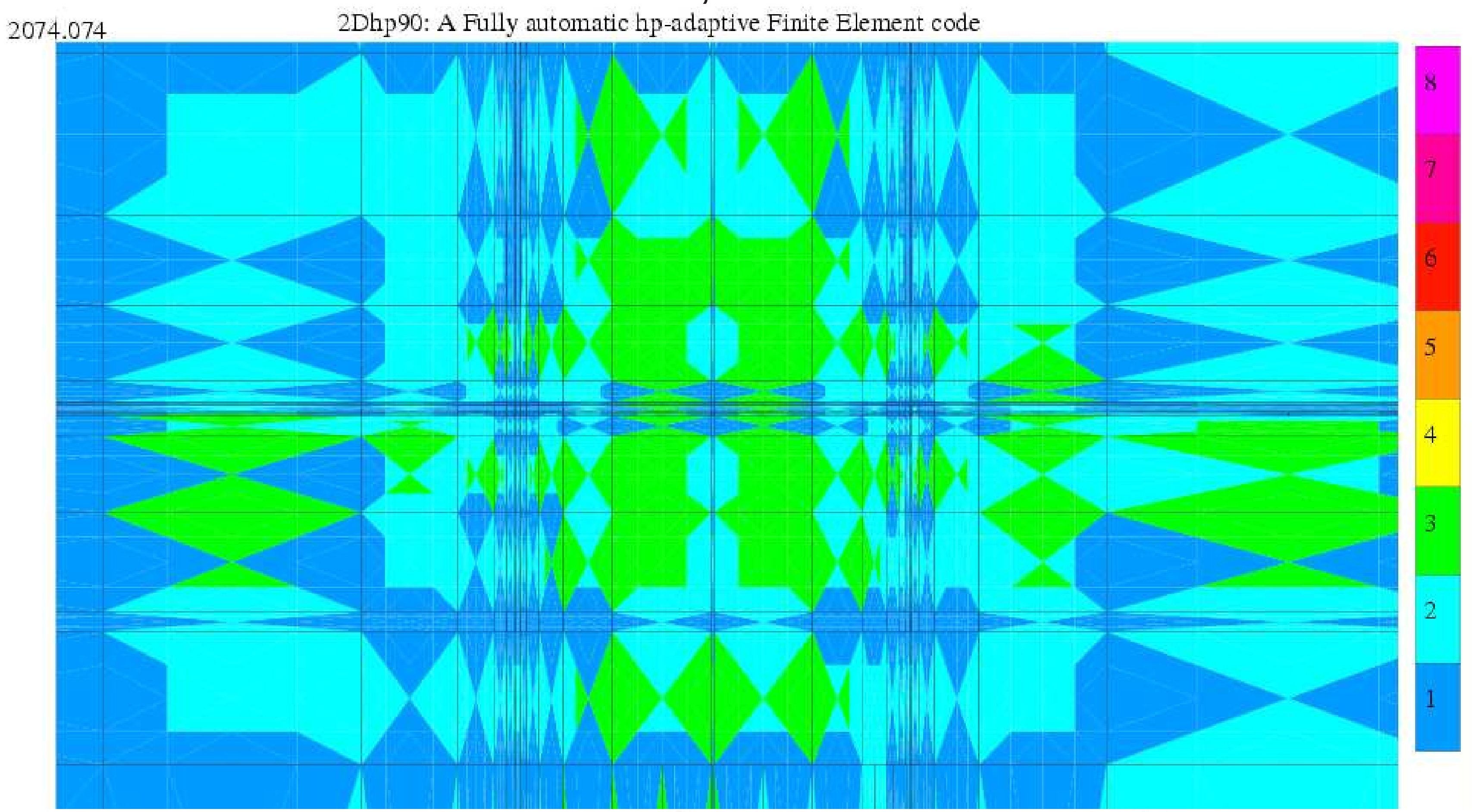


As we increase the frequency, the effect of oil becomes more localized.

marine CSEM applications

0.75 Hz (FINITE LAYER OF OIL)

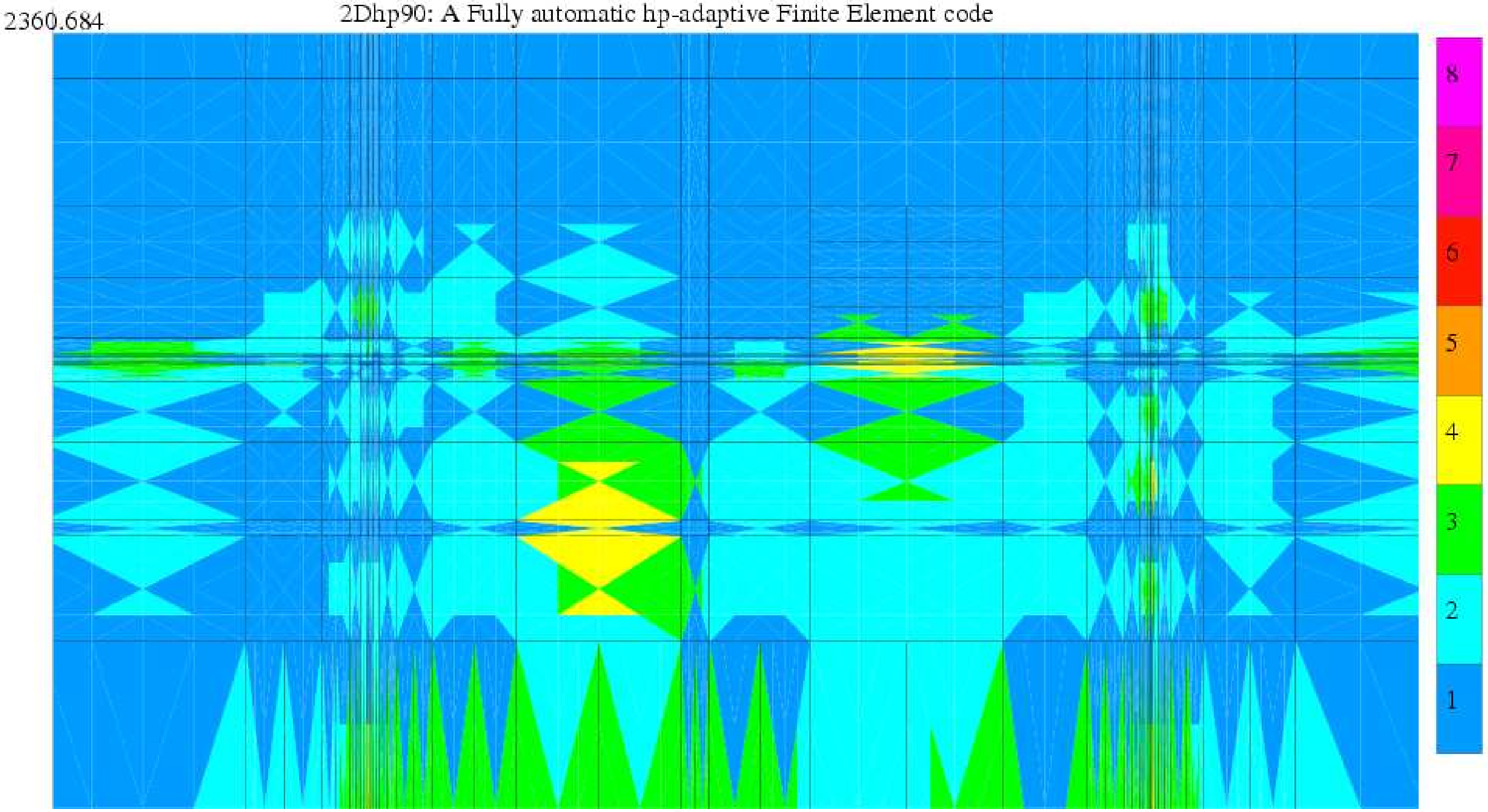
TX: x = 0 m ; RX: x = 2000 m.



marine CSEM applications

0.75 Hz (FINITE LAYER OF OIL)

TX: x = 0 m ; RX: x = 5000 m.



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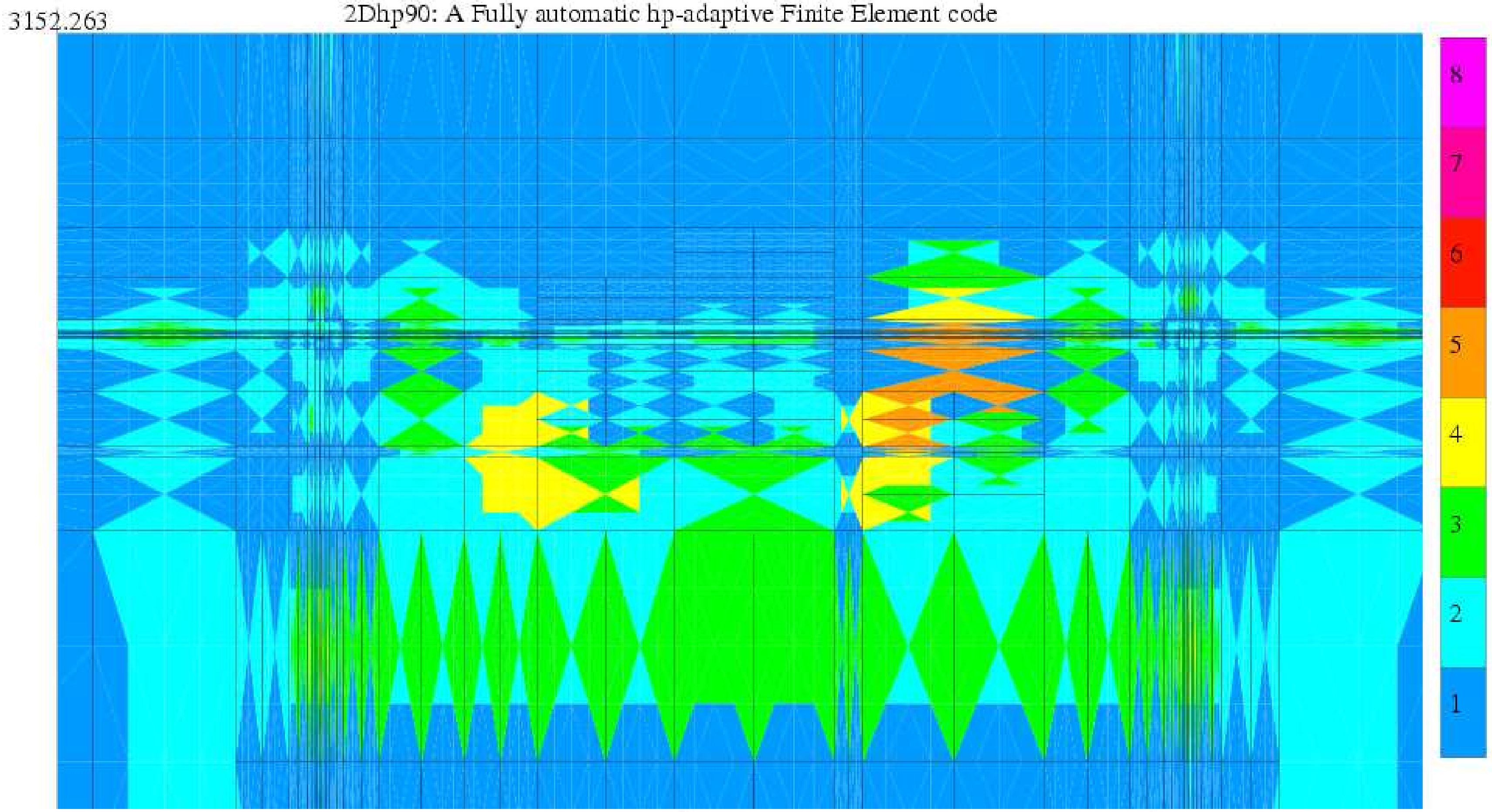
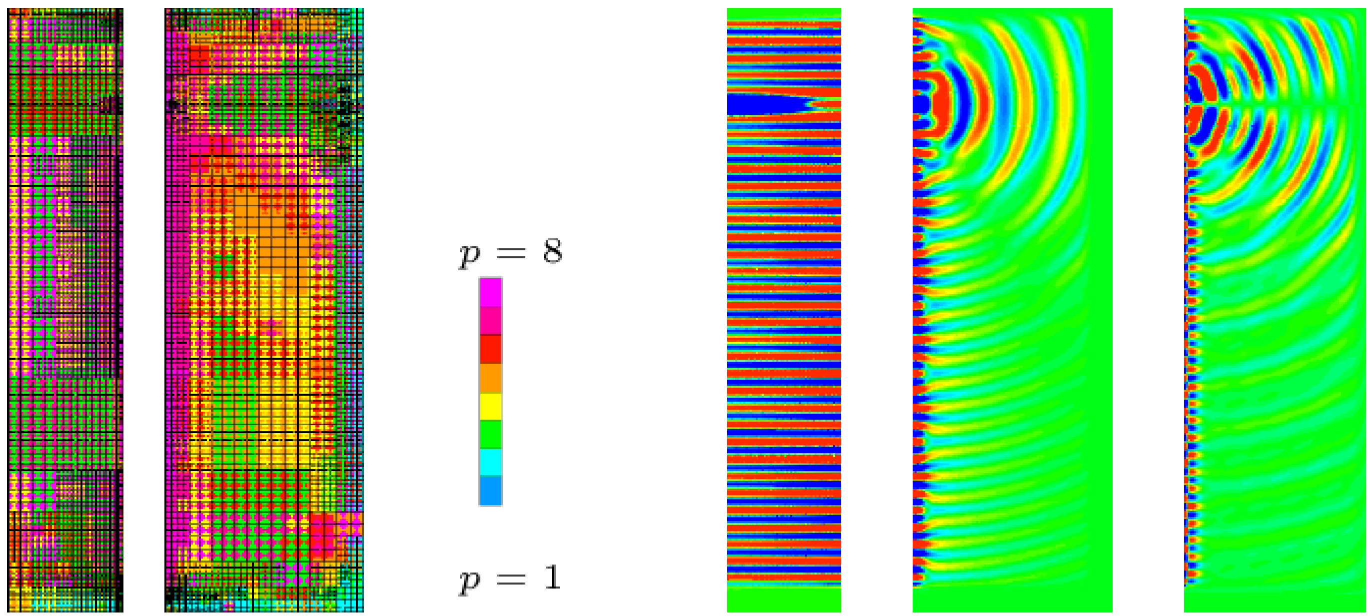
0.75 Hz (FINITE LAYER OF OIL)

TX: x = 0 m ; RX: x = 8000 m.



acoustic applications

Final hp-grid and solution



acoustic hp-mesh | elastic hp-mesh | acoustic p_{acoust} | elastic u_r | elastic u_z

8 KHz, acoustics, open borehole setting (no logging instrument).

conclusions

- We have described an efficient numerical method based on a parallel self-adaptive goal-oriented hp refinement strategy and a Fourier-Finite-Element method.

- The method has been successfully used to simulate the acquisition of logging measurements and marine controlled-source electromagnetic (CSEM) problems.

- Our main objective is to create a software infrastructure enabling solution of challenging multiphysics inverse problems with applications to geophysics (hydrocarbon detection and monitoring, etc.), aeronautics and medicine.

future work

I. Garay

Postdoctoral Fellow
(Since Mar 09)

Acoustic-elastic problems.

A.G. Saint-Guirons

Postdoctoral Fellow
(Since Sep 09)

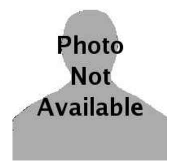
Inversion algorithms.

A. Galdrán

Ph.D. Student
(Since Sep 09)

Fourier-Finite-Element adaptivity.

J. Álvarez



Ph.D. Student
(Since Sep 09)

Dimension reduction algorithms.

future work

I. Andonegui

Technician (Engineer)
(Since May 09)

Visualization.

M.J. Nam

Collaborator

Resistivity logging instruments.

M. Paszynski

Collaborator

Parallel computations.

F. de la Hoz

Collaborator

Fast iterative solvers.

future work

L.E. García-Castillo

Collaborator

Electromagnetic computations.

C. Torres-Verdín

Collaborator

Contacts with the oil industry.

I. Gómez

Collaborator

Three-dimensional computations.