Summer Project. Overview.

An hp-Adaptive Finite Element (FE) Method for Solving Electromagnetic (EM) Problems with Special Emphasis in Petroleum Engineering Applications.

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Baker-Atlas

OVERVIEW

Petroleum Engineering EM applications

Maxwell's Equations

Variational Formulation

The hp-Finite Element Method

hp-Adaptivity

Numerical Results

Conclusions

Future Work

PETROLEUM ENGINEERING EM APPLICATIONS

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MAXWELL'S EQUATIONS

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MAXWELL'S EQUATIONS

Time Harmonic Maxwell's Equations:

$$\nabla \times \mathbf{E} = -j\mu\omega\mathbf{H}$$
$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E} + \sigma\mathbf{E} + \mathbf{J}^{imp}$$

Reduced Wave Equation:

$$\boldsymbol{\nabla} \times \left(\frac{1}{\mu}\boldsymbol{\nabla} \times \boldsymbol{E}\right) - (\omega^2\epsilon - j\omega\sigma)\boldsymbol{E} = -j\omega\boldsymbol{J}^{imp} \,,$$

Electrostatics and Magnetostatics —> Easier eq. to solve
Axisymmetric problems —> Dimension reduction
Harmonic problems in spatial variable z —> Dimension reduction

VARIATIONAL FORMULATION

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VARIATIONAL FORMULATION

Derivation of the Variational Formulation

First, we multiply the reduced wave equation by the complex conjugate of test function $\mathbf{F}$. Then, we integrate over computational domain Ω. Finally, integration by parts yields to the following formula:

$$\int_{\Omega} \{\frac{1}{\mu}(\nabla \times \mathbf{E})(\nabla \times \bar{\boldsymbol{F}})\} dx + \int_{\partial\Omega} \mathbf{n} \times (\nabla \times \mathbf{E}) \bar{\boldsymbol{F}}_t dS = -j\omega \int_{\Omega} \boldsymbol{J}^{imp} \bar{\boldsymbol{F}} dx$$

Observations:

- **The theory of Sobolev Spaces can be substituted by the statement "everything makes sense", and viceversa.**
- **Special features of a problem may and should be incorporated into the variational formulation.**

THE hp-FINITE ELEMENT METHOD

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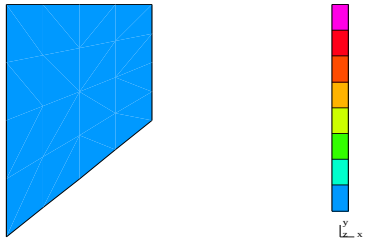
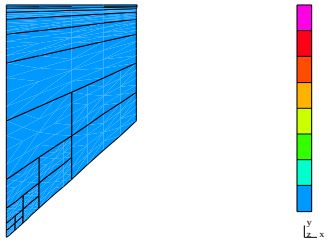
THE *hp*-FINITE ELEMENT METHOD

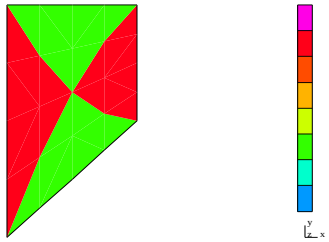
Different refinement strategies for finite elements:



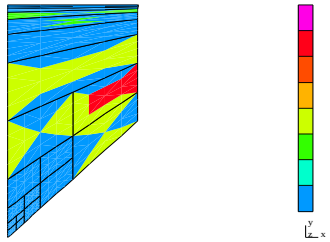
Given initial grid



***h*-refined grid**



***p*-refined grid**



***hp*-refined grid**

THE *hp*-FINITE ELEMENT METHOD

Exponential convergence rates

for a number of regular and SINGULAR problems

for optimal hp-grids
in the asymptotic range (theoretical and numerical results), and
in the pre-asymptotic range (numerical results).

Smaller dispersion (pollution) error

as *p* increases.

More geometrical details captured

as *h* decreases.

A FULLY AUTOMATIC hp-ADAPTIVE STRATEGY

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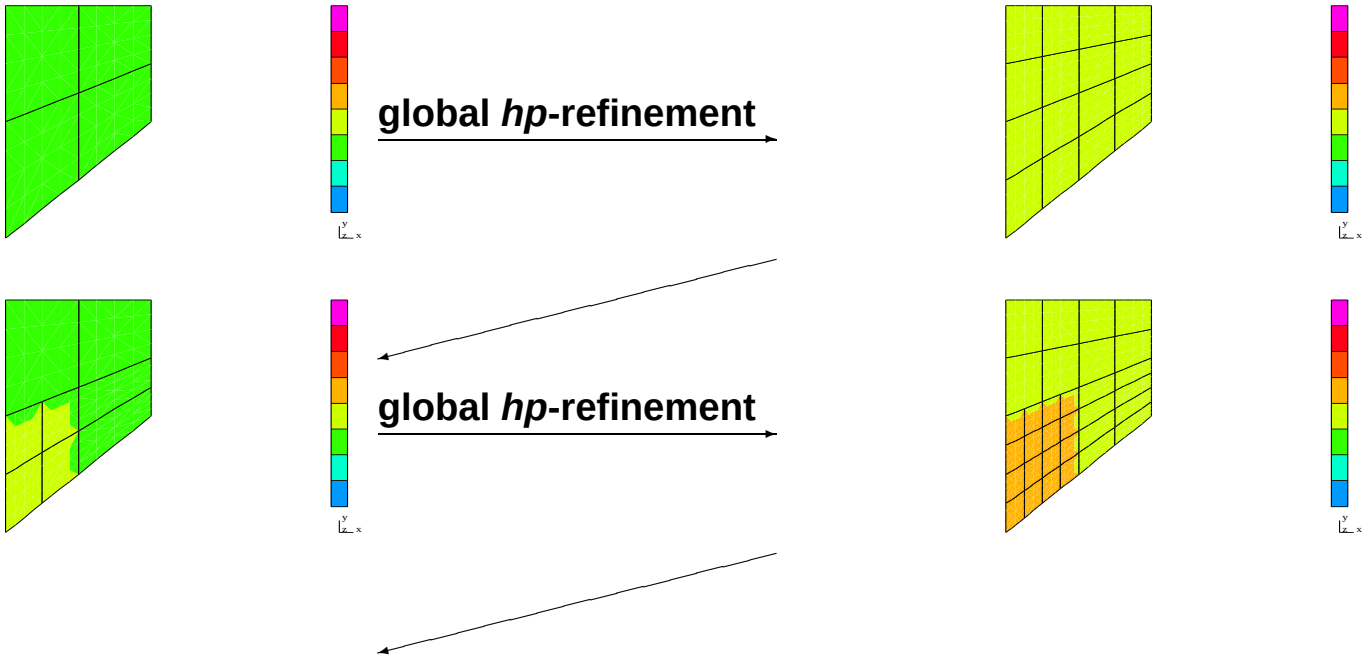
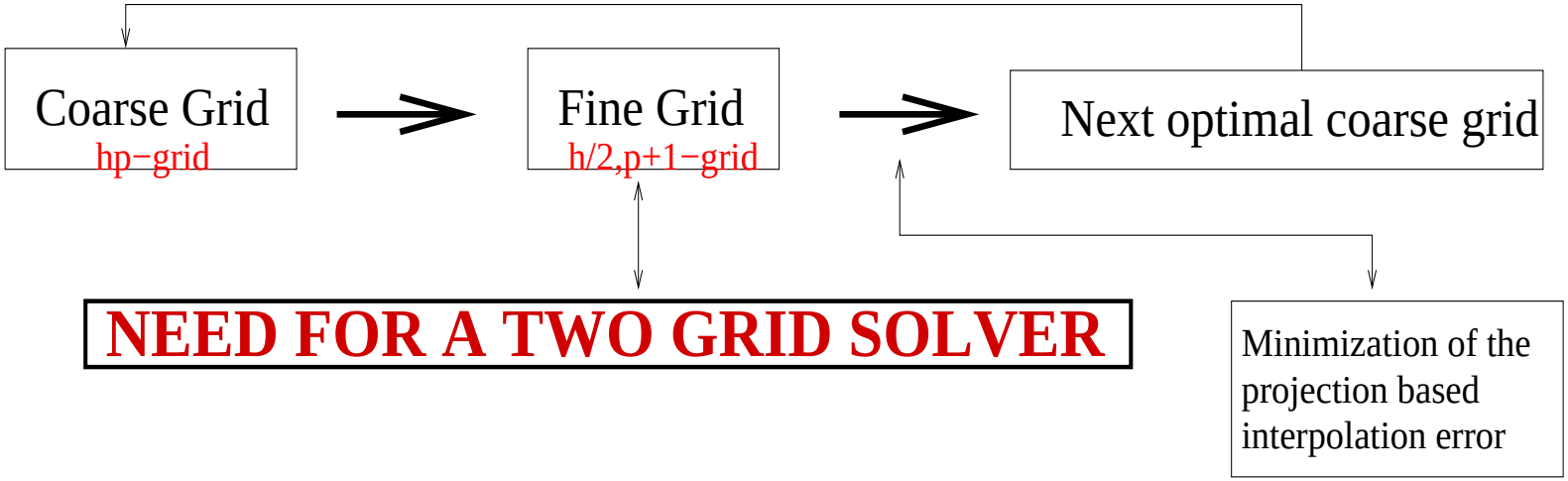
A FULLY AUTOMATIC hp-ADAPTIVE STRATEGY



A FULLY AUTOMATIC hp-ADAPTIVE STRATEGY

Automatic *hp*-adaptivity delivers exponential convergence and enables solution of challenging EM problems



PRELIMINARY NUMERICAL RESULTS

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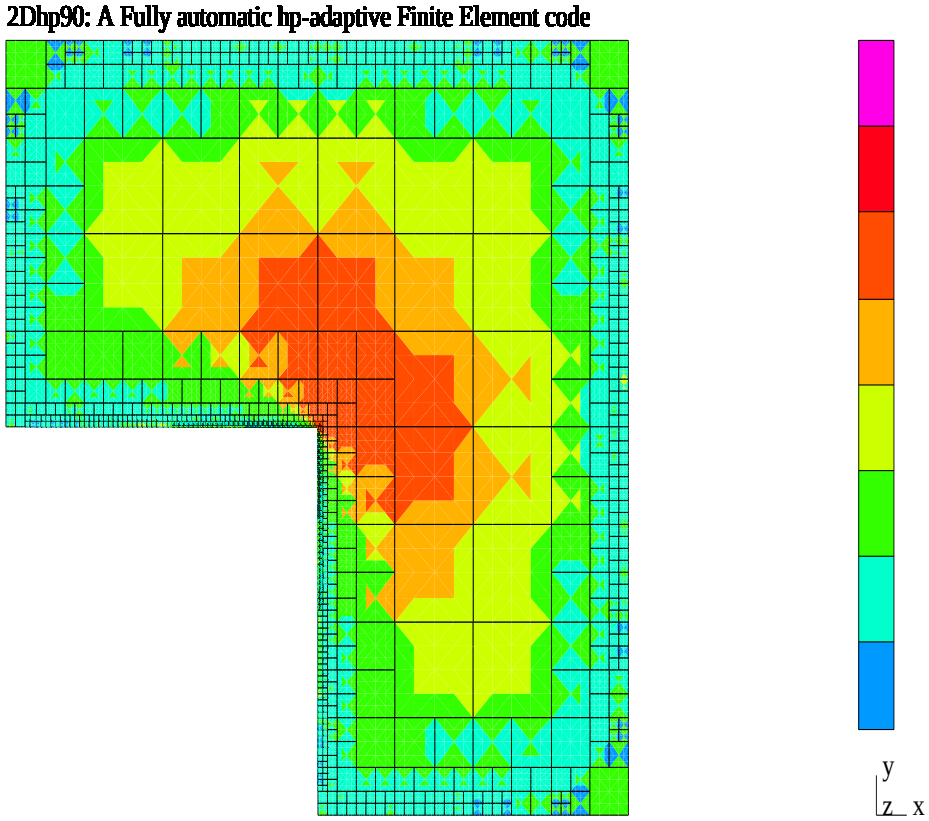
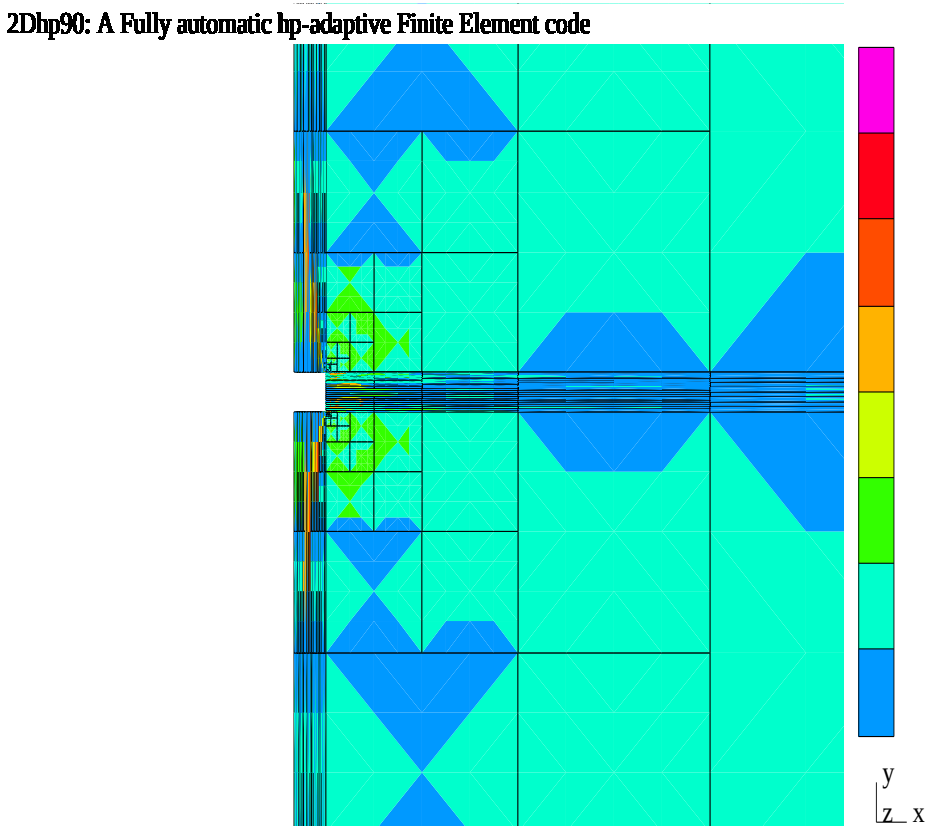
PRELIMINARY NUMERICAL RESULTS

Edge diffraction example: final *hp*-grid, Zoom = 1



PRELIMINARY NUMERICAL RESULTS

Battery example: final *hp*-grid, Zoom = 10



CONCLUSIONS

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CONCLUSION

The fully automatic hp-adaptive strategy produces a grid which resolves singularities.

FUTURE WORK

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FUTURE WORK

Implement a goal-oriented hp-adaptive strategy.

Implement a robust hp-two grid solver.