A Progress Report at Baker-Hughes on:

The Development of a 3D Self-Adaptive Goal-Oriented hp-Finite Element Software for Simulations of DC Resistivity Logging Instruments

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Collaborators: Science Department of Baker-Hughes, C. Michler,
L.E. Garcia-Castillo, A. Zdunek, W. Rachowicz

June 23, 2006

Department of Petroleum and Geosystems Engineering, and
Institute for Computational Engineering and Sciences (ICES)

THE UNIVERSITY OF TEXAS AT AUSTIN

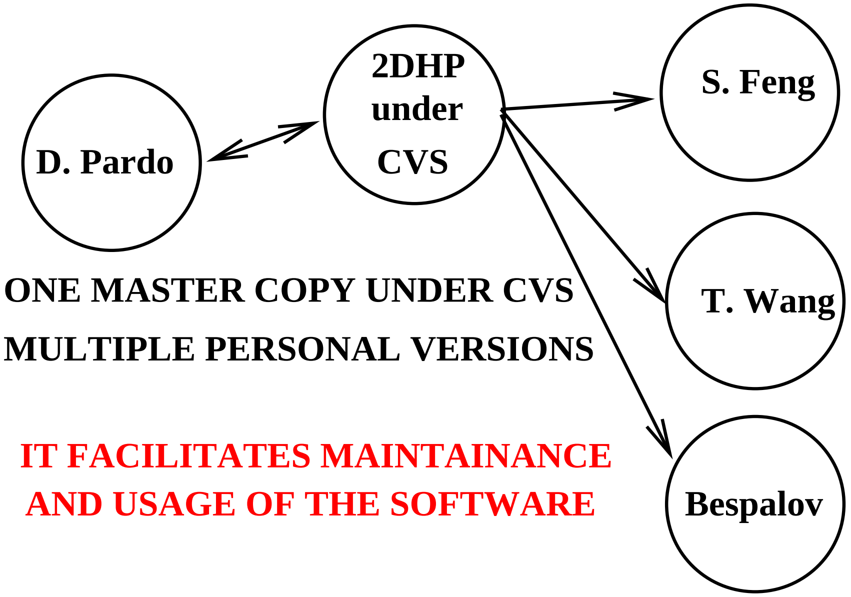
OVERVIEW

1. Recent Advances in the 2D High Performance FE Software
   - Toward a User-Friendly Interface.
   - Perfectly Matched Layers (PML).
2. Current Stage of the 3D High Performance FE Software
   - User Interface.
   - Goal-Oriented Automatic Adaptivity.
   - Geometry and Graphics.
   - Iterative Solver.
   - Parallel Implementation.
3. Preliminary Results
4. Conclusions and Future Work

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Toward a User-Friendly Interface

- **Concurrent Version System (CVS).**



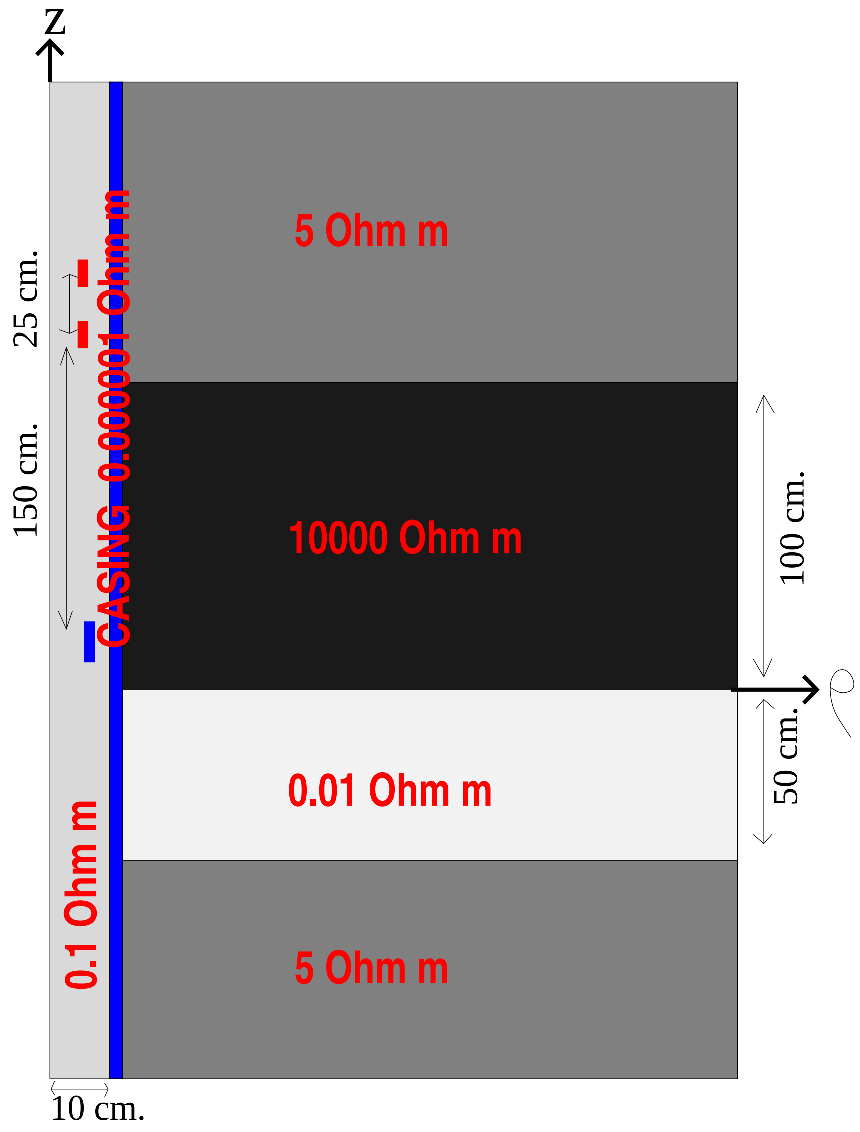
ONE MASTER COPY UNDER CVS

MULTIPLE PERSONAL VERSIONS

IT FACILITATES MAINTAINANCE AND USAGE OF THE SOFTWARE

- **Windows Compatible.**
- **Module-oriented.**
- **A user-friendly interface module is under development.**

PERFECTLY MATCHED LAYER (PML)



Axisymmetric 3D problem.

Six different materials.

Through casing resistivity instrument.

Varying coefficients by up to 10 orders of magnitude.

PERFECTLY MATCHED LAYER (PML)

Perfectly Matched Layer (PML) Formulation

The PML is composed of the following anisotropic materials:

$$\begin{cases} \bar{\bar{\sigma}}_{PML} = \bar{\bar{\Lambda}}\bar{\bar{\sigma}} \\ \bar{\bar{\epsilon}}_{PML} = \bar{\bar{\Lambda}}\bar{\bar{\epsilon}} \\ \bar{\bar{\mu}}_{PML} = \bar{\bar{\Lambda}}\bar{\bar{\mu}} \end{cases} \quad ; \quad \bar{\bar{\Lambda}} = \begin{bmatrix} \frac{\tilde{\rho}}{\rho}\frac{s_z}{s_\rho} & 0 & 0 \\ 0 & \frac{\rho}{\tilde{\rho}} s_z s_\rho & 0 \\ 0 & 0 & \frac{\tilde{\rho}}{\rho}\frac{s_\rho}{s_z} \end{bmatrix} \quad ; \quad \tilde{\rho} = \int_0^\rho s_\rho(\rho') d\rho'$$

s_ρ, s_ϕ, **and** s_z **are the stretching coordinate functions. We define:**

$$s_\rho = s_\phi = s_z = 1 + \phi - j\phi$$

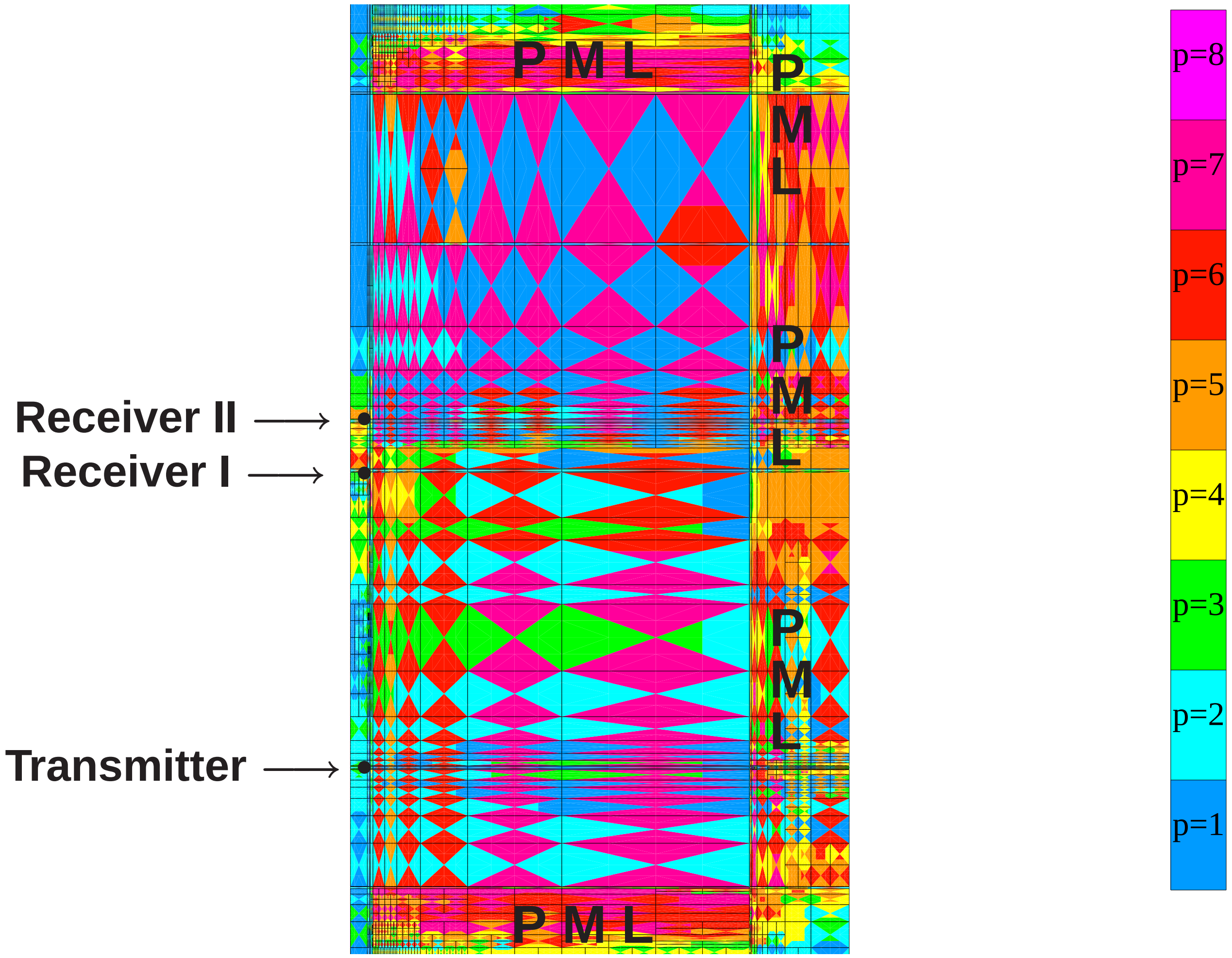
We consider three different PML's by defining three different functions $\phi(x)$**:**

$$\phi(x) = \begin{cases} \phi_1(x) = \left[2\left(\frac{x - x_0}{x_1 - x_0}\right)\right]^{17} & \textbf{PML 1,} \\ \phi_2(x) = 20000\left(\frac{x - x_0}{x_1 - x_0}\right) & \textbf{PML 2,} \\ \phi_3(x) = 10000 & \textbf{PML 3.} \end{cases} \quad x \in (x_0, x_1)$$

Within the PML, both propagating and evanescent waves become arbitrarily fast evanescent waves.

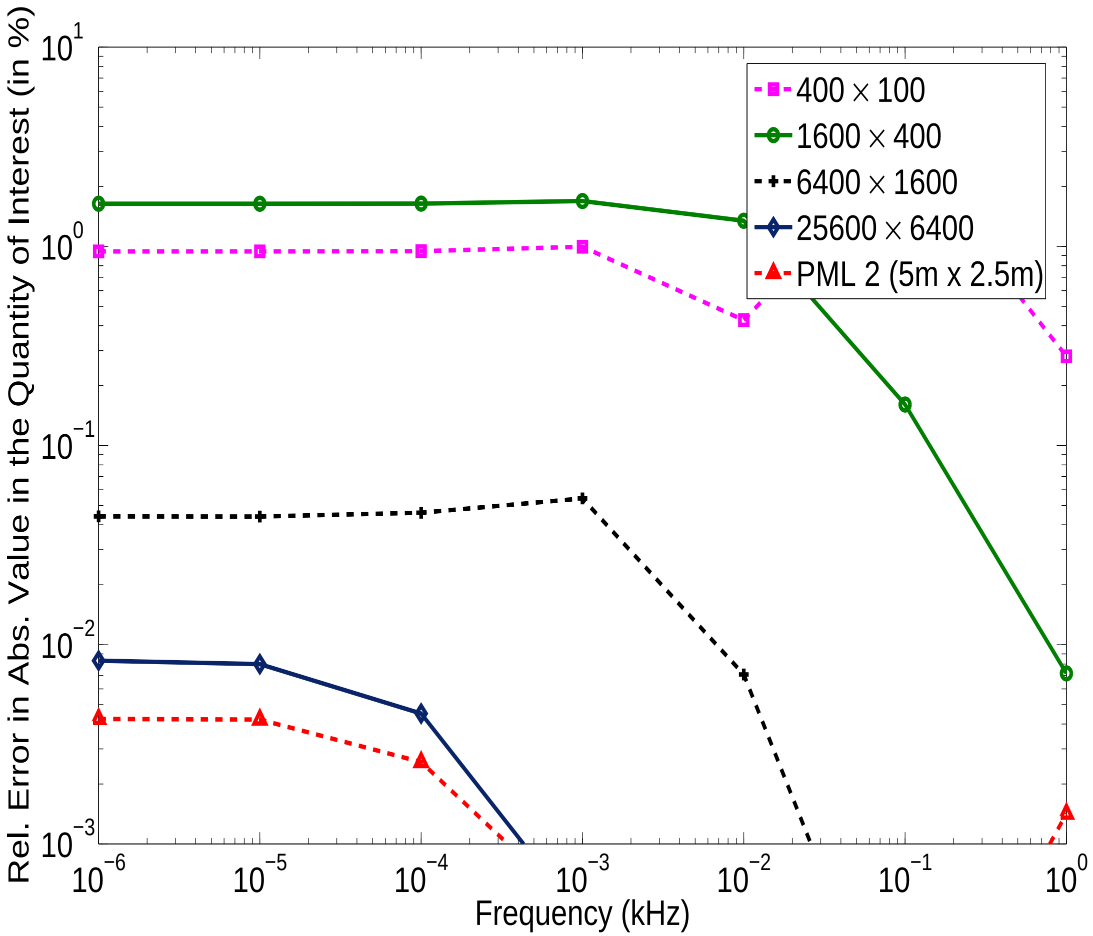
PERFECTLY MATCHED LAYER (PML)

Final hp-Grid with a 0.5 m Thick PML.



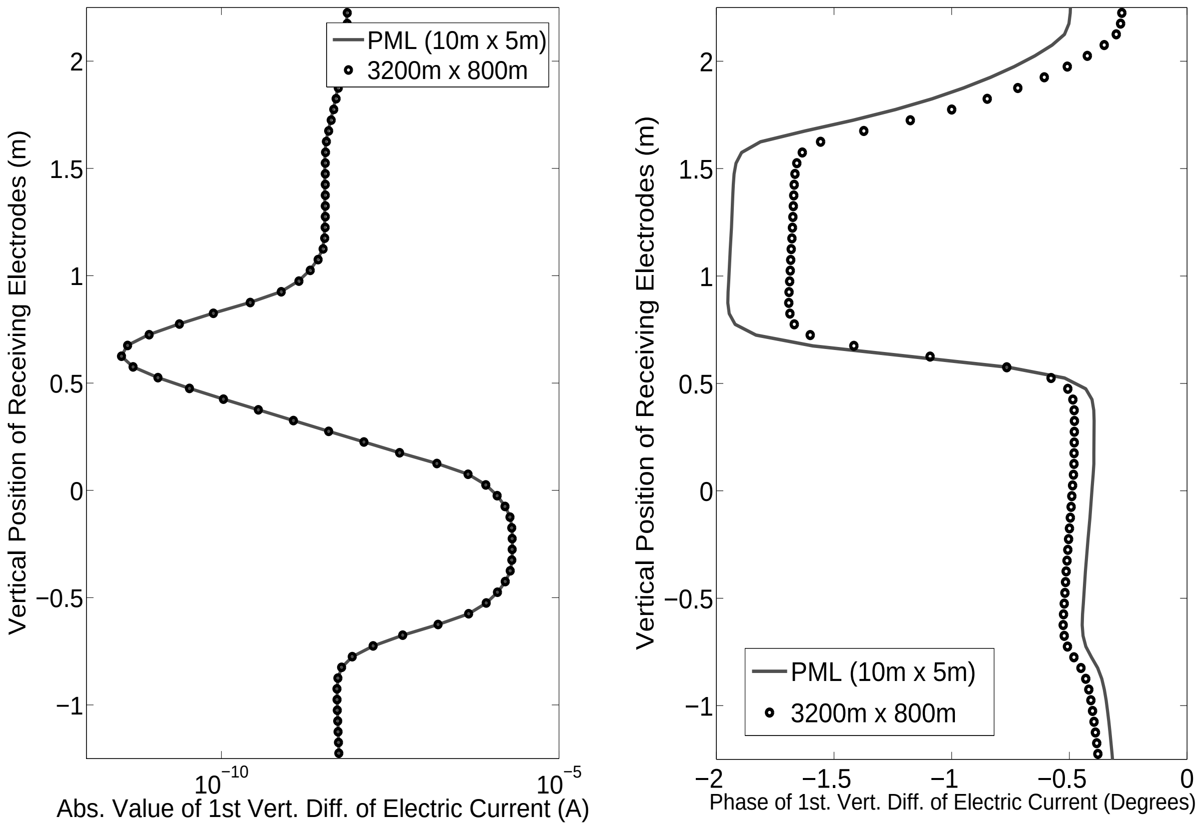
PERFECTLY MATCHED LAYER (PML)

Reference Solution: PML 1 (5 m x 2.5 m)



PMLs provide accurate solutions without reflections from the boundary

PERFECTLY MATCHED LAYER (PML)



If we compute the phase, a computational domain of 3200 m x 800 m is not large enough.

PERFECTLY MATCHED LAYER (PML)

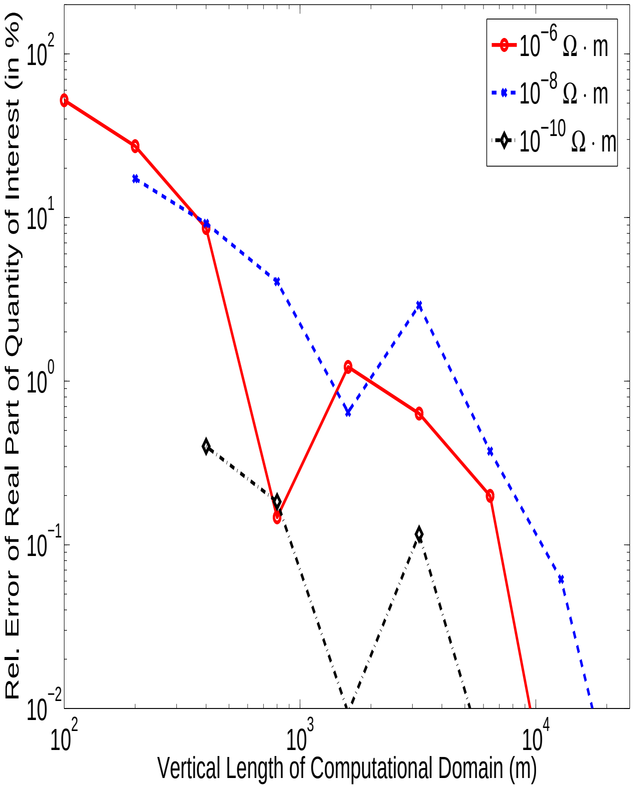
Number of unknowns employed by the self-adaptive goal-oriented hp-FE method as a function of the size of computational domain and presence of a PML

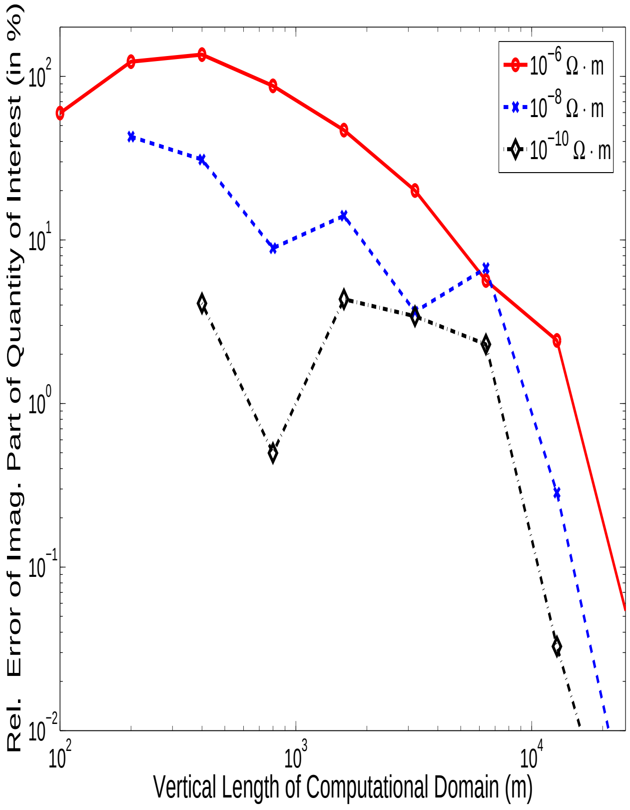
Domain Size (m)	Nr. Unknowns ($\approx 1\%$ error)	Nr. Unknowns ($\approx 0.01\%$ error)
PML 1 (5 x 2.5)	19541 (0.083%)	24886 (0.037%)
PML 2 (5 x 2.5)	7095 (0.29%)	13345 (0.006%)
PML 3 (5 x 2.5)	8679 (1.04%)	19640 (0.009%)
6400 x 1600	12327 (0.43%)	18850 (0.014%)
12800 x 3200	12964 (0.43%)	18892 (0.014%)
25600 x 6400	12099 (1.22%)	19828 (0.037%)

PML 2 provides considerable savings

PERFECTLY MATCHED LAYER (PML)

Reference Solution: PML 1 (5 m x 2.5 m) - 1 Hz -.

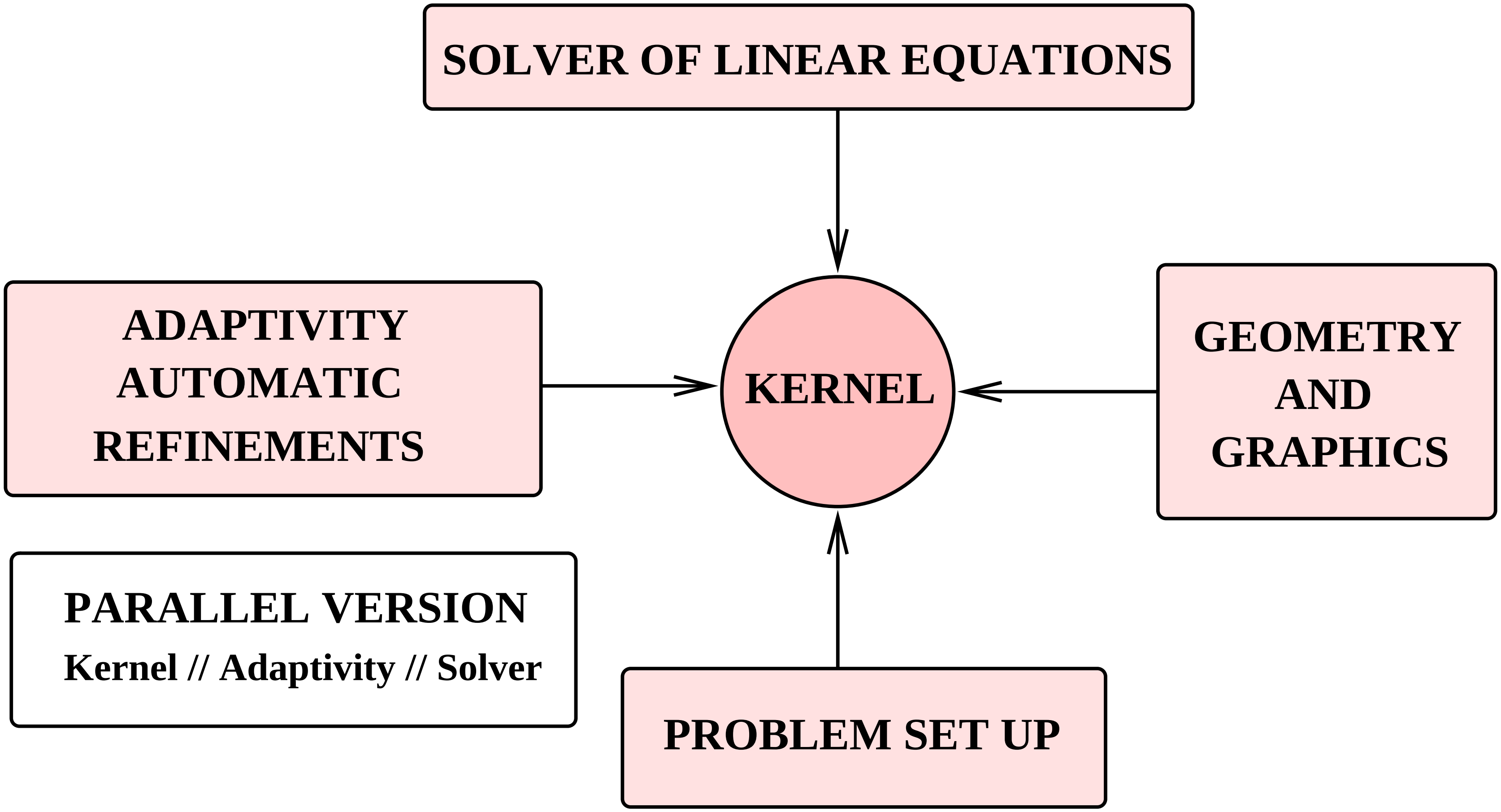




If a PML is not used, we need to consider a computational domain with several thousand meters in the vertical direction.

CURRENT STAGE OF THE 3D hp-FE SOFTWARE

3Dhp-log Contains Several Packages:



SOLVER OF LINEAR EQUATIONS

ADAPTIVITY
AUTOMATIC
REFINEMENTS

KERNEL

GEOMETRY
AND
GRAPHICS

PARALLEL VERSION
Kernel // Adaptivity // Solver

PROBLEM SET UP

CURRENT STAGE OF THE 3D hp-FE SOFTWARE

Main Components of the Software

• Goal-oriented adaptivity.	β-Version & Presented
• Problem Set-up and user-interface.	β-Version & Presented
• Geometry. Iso-parametric elements used for adaptivity, and exact geometry elements used for computation of solutions.	β-Version & Presented
• Direct solver based on the package MUMPS.	Completed & Presented
• Graphics. New graphics based on package VTK.	α-Version
• Iterative solver: Multigrid goal-oriented solver with edge-based preconditioning.	α-Version
• Parallel version.	α-Version
• Friendly user-interface.	Under development
• Parallel iterative solver.	Under development

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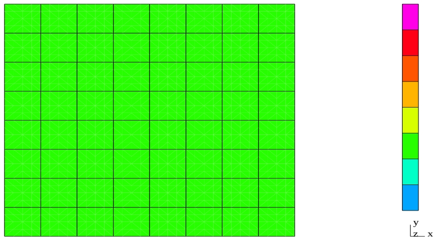
CURRENT STAGE OF THE 3D hp-FE SOFTWARE

Multigrid (two-grid) Solver (Ax=b)

Fine Grid Smoothing (Sol. Local Problems)



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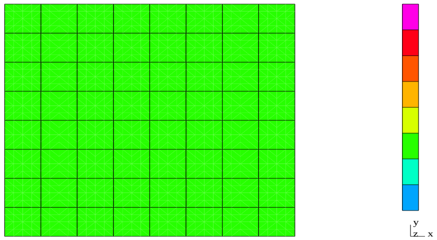
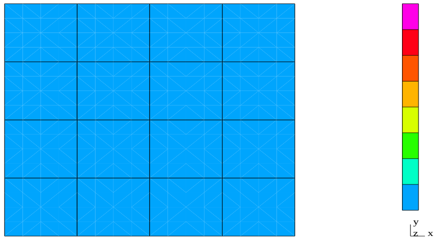


Coarse Grid Correction (Sol. Global Problem)



V-cycle

CURRENT STAGE OF THE 3D hp-FE SOFTWARE

We seek x such that $Ax = b$. Consider the following iterative scheme:

$$\boxed{\begin{aligned} \boldsymbol{r}^{(n+1)} &= [\boldsymbol{I} - \boldsymbol{\alpha}^{(n)} \boldsymbol{A}\boldsymbol{S}]\boldsymbol{r}^{(n)} \\ \boldsymbol{x}^{(n+1)} &= \boldsymbol{x}^{(n)} + \boldsymbol{\alpha}^{(n)} \boldsymbol{S}\boldsymbol{r}^{(n)} \end{aligned}}$$

where S is a matrix, and $\alpha^{(n)}$ is a relaxation parameter. $\alpha^{(n)}$ *optimal* if:

$$\boxed{\boldsymbol{\alpha}^{(n)} = \arg\ \min \parallel \boldsymbol{x}^{(n+1)} - \boldsymbol{x} \parallel_A = \frac{(\boldsymbol{A}^{-1}\boldsymbol{r}^{(n)}, \boldsymbol{S}\boldsymbol{r}^{(n)})_A}{(\boldsymbol{S}\boldsymbol{r}^{(n)}, \boldsymbol{S}\boldsymbol{r}^{(n)})_A}}$$

Then, we define our two grid solver as:

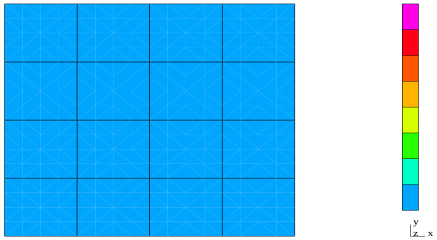
1 iteration with $S = S_F = \sum A_i^{-1}$ $+$

1 iteration with $S = S_C = P_C A_C^{-1} R_C$

CURRENT STAGE OF THE 3D hp-FE SOFTWARE

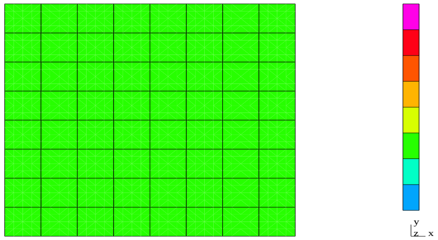
Selection of patches (for block Jacobi smoother)

Coarse Grid

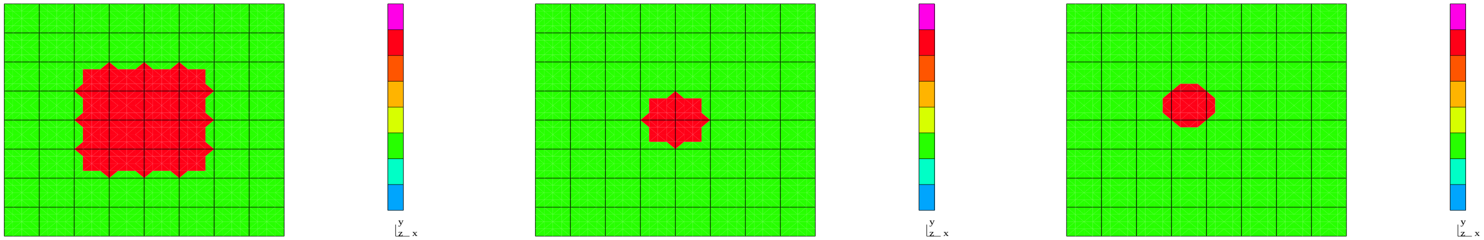


global *hp*-refinement

Fine Grid



Three examples of patches (blocks) for the Block Jacobi smoother:



Example 1: span of basis functions with support contained in the support of a coarse grid vertex node basis function.
Example 2: span of basis functions with support contained in the support of a fine grid vertex node basis function.
Example 3: span of basis functions corresponding to an element stiffness matrix.

CURRENT STAGE OF THE 3D hp-FE SOFTWARE

Error reduction and stopping criteria

Let $e^{(n)} = x^{(n)} - x$ the error at step n, $\tilde{e}^{(n)} = [I - S_C A]e^{(n)} = [I - P_C]e^{(n)}$. Then:

$$\boxed{\frac{\| e^{(n+1)} \|_A^2}{\| e^{(n)} \|_A^2} = 1 - \frac{| (\tilde{e}^{(n)}, S_F A \tilde{e}^{(n)})_A |^2}{\| \tilde{e}^{(n)} \|_A^2 \| S_F A \tilde{e}^{(n)} \|_A^2} = 1 - \frac{| (\tilde{e}^{(n)}, (P_C + S_F A)\tilde{e}^{(n)})_A |^2}{\| \tilde{e}^{(n)} \|_A^2 \| S_F A \tilde{e}^{(n)} \|_A^2}}$$

Then:

$$\boxed{\frac{\| e^{(n+1)} \|_A^2}{\| e^{(n)} \|_A^2} \leq \sup_e [1 - \frac{| (e, (P_C + S_F A)e)_A |^2}{\| e \|_A^2 \| S_F A e \|_A^2}] \leq C < 1 \qquad \text{(Error Reduction)}}$$

For our stopping criteria, we want: Iterative Solver Error $\approx$ Discretization Error. That is:

$$\boxed{\frac{\| e^{(n+1)} \|_A}{\| e^{(0)} \|_A} \leq 0.01 \qquad \text{(Stopping Criteria)}}$$

CURRENT STAGE OF THE 3D hp-FE SOFTWARE

Major Challenges (Iterative Solver)

The use of goal-oriented adaptivity. A new strategy for selecting the optimal relaxation parameter has been implemented. This strategy minimizes the error in the quantity of interest rather than in the energy-norm.

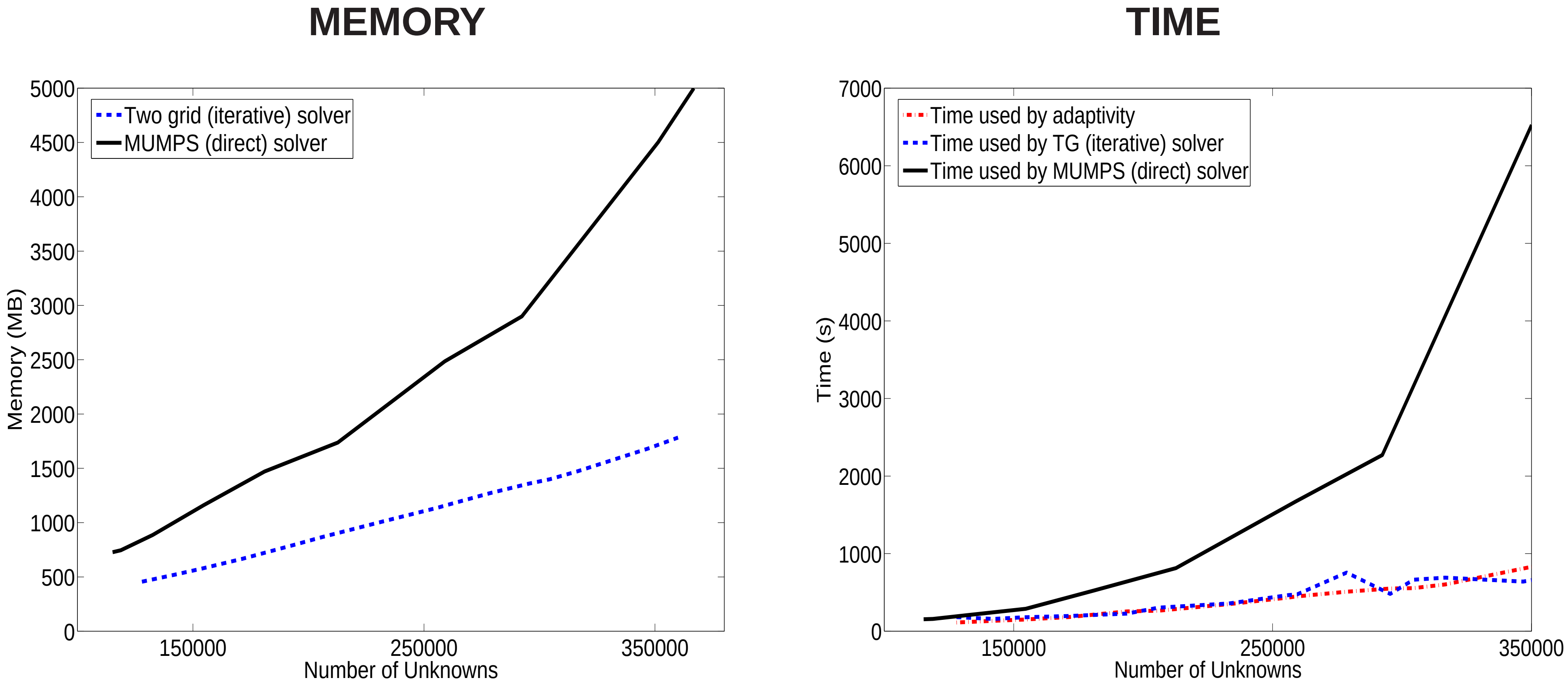
The presence of (arbitrary) elongated elements. An additional edge-based (global) smoother has been implemented. This additional smoother makes the convergence of the iterative solver independent of the aspect ratio of the elements.

Convergence theory for elongated elements. Under development.

CURRENT STAGE OF THE 3D hp-FE SOFTWARE

Axisymmetric Model Problem

MEMORY **TIME**



1.2 Ghz processor

Iterative solvers are needed for simulation of 3D resistivity logging applications

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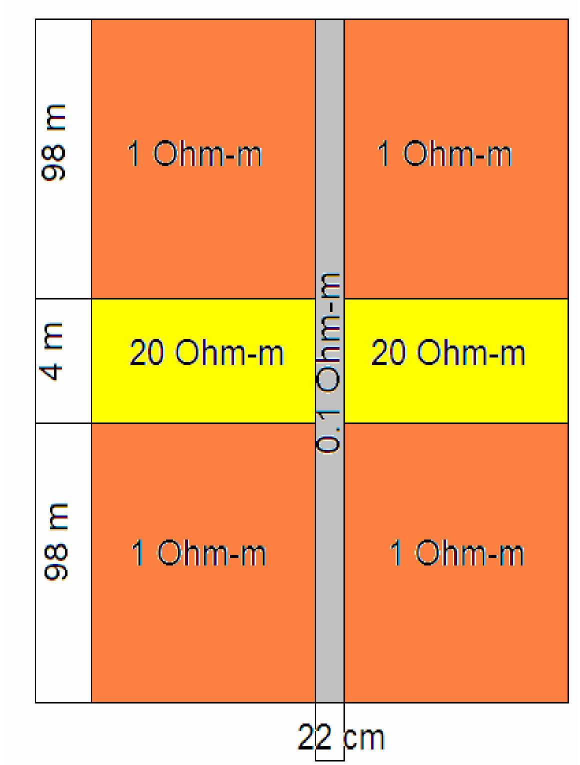
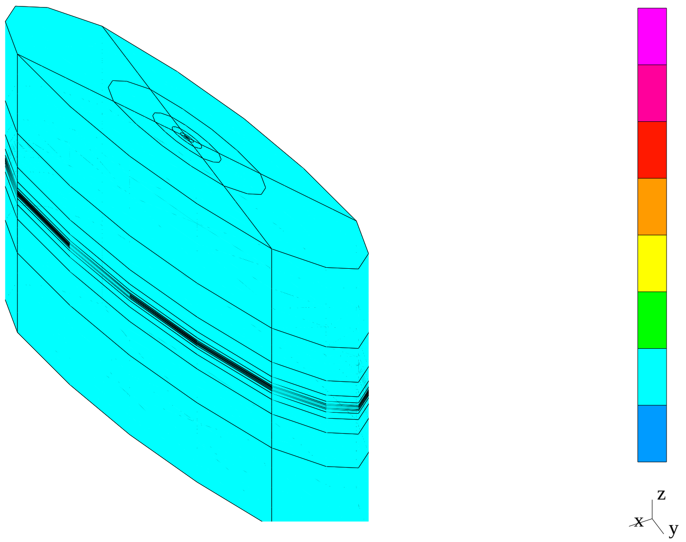
CURRENT STAGE OF THE 3D hp-FE SOFTWARE

Parallel Version

Domain Demcomposition Based Parallel hp-Adaptivity

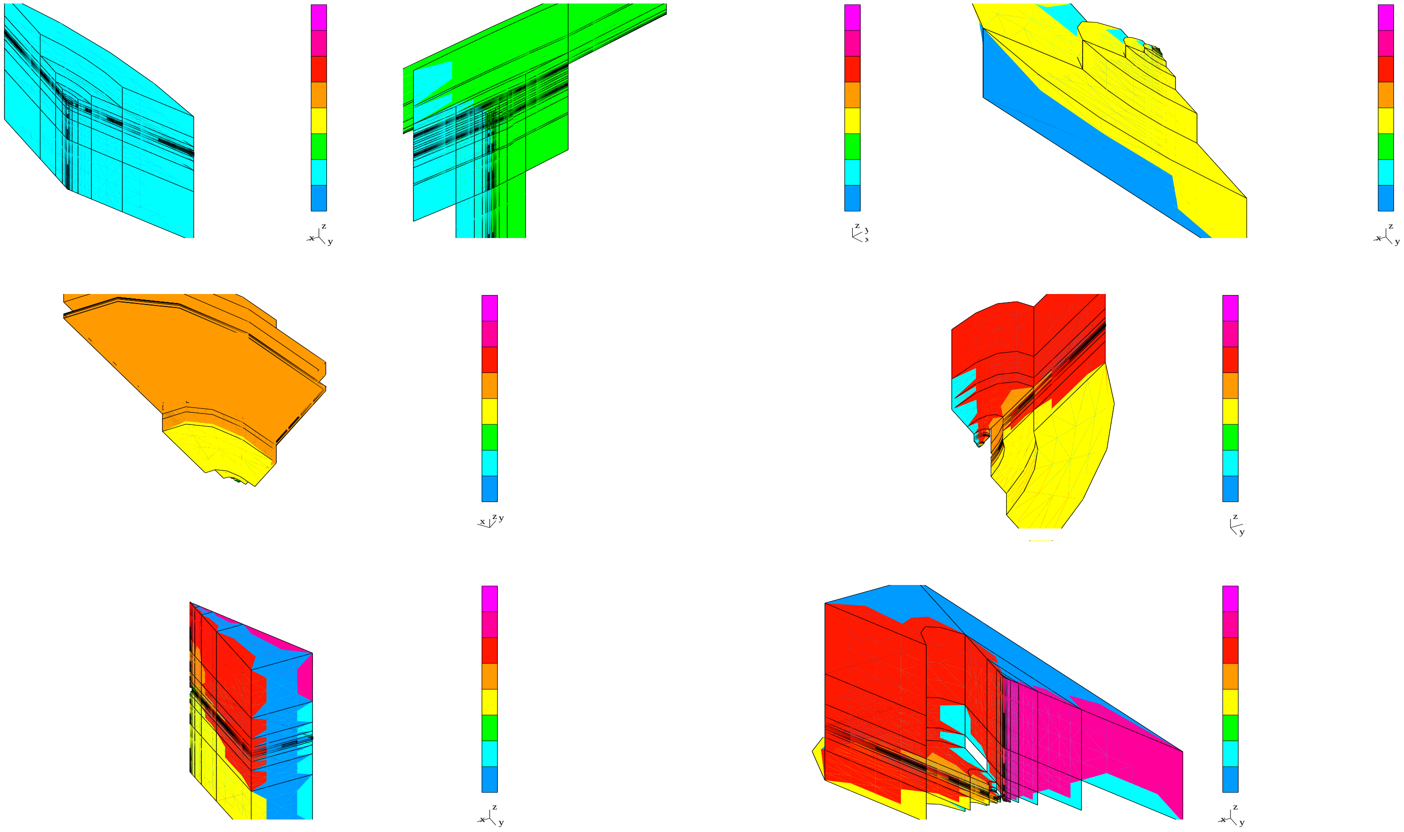




CURRENT STAGE OF THE 3D hp-FE SOFTWARE

Parallel Version

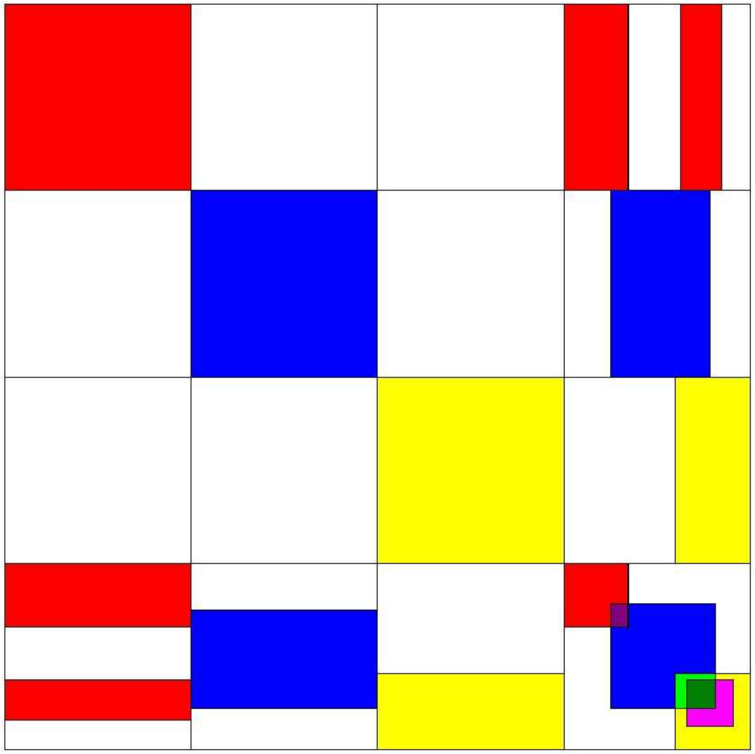
Domain Demcomposition Based Parallel hp-Adaptivity



CURRENT STAGE OF THE 3D hp-FE SOFTWARE

Parallel Direct Solver

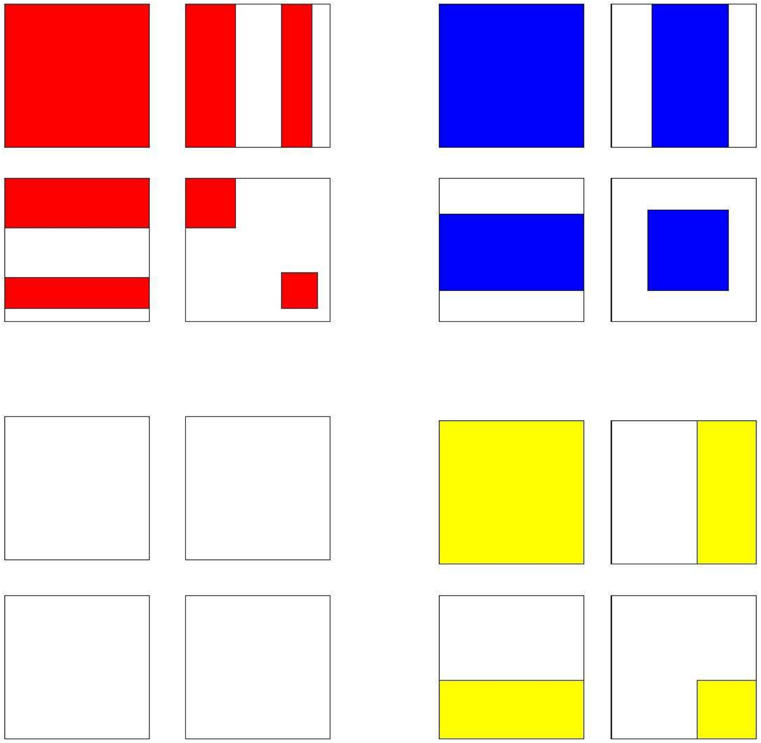
Parallel Domain Decomposition Solver with Serial Direct Solver (MUMPS) Executed Over Each Subdomain in Parallel (I)



CURRENT STAGE OF THE 3D hp-FE SOFTWARE

Parallel Direct Solver

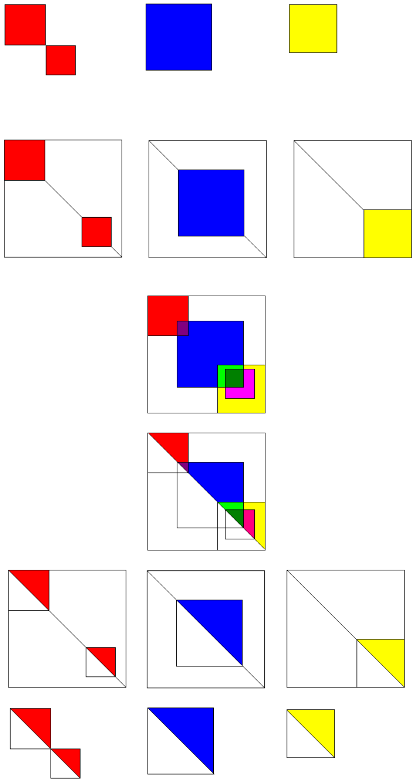
Parallel Domain Decomposition Solver with Serial Direct Solver (MUMPS) Executed Over Each Subdomain in Parallel (II)



CURRENT STAGE OF THE 3D hp-FE SOFTWARE

Parallel Direct Solver

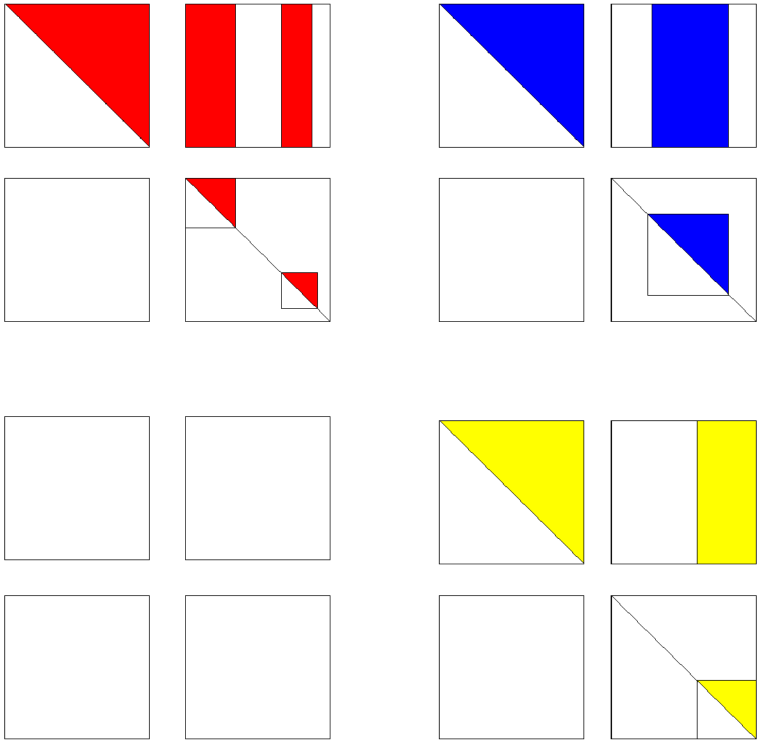
Parallel Domain Decomposition Solver with Serial Direct Solver (MUMPS) Executed Over Each Subdomain in Parallel (III)



CURRENT STAGE OF THE 3D hp-FE SOFTWARE

Parallel Direct Solver

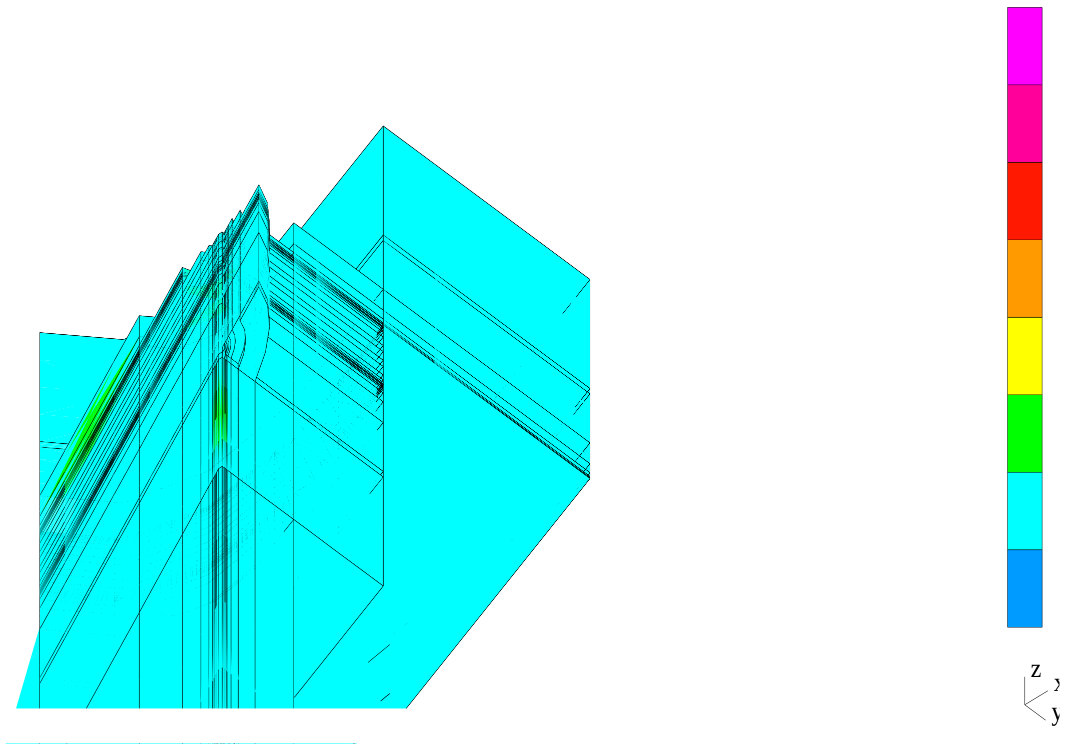
Parallel Domain Decomposition Solver with Serial Direct Solver (MUMPS) Executed Over Each Subdomain in Parallel (IV)



CURRENT STAGE OF THE 3D hp-FE SOFTWARE

Parallel Automatic hp-Adaptivity

Parallel Mesh Optimization (Decisions About Optimal Mesh Refinements Made Fully in Parallel)



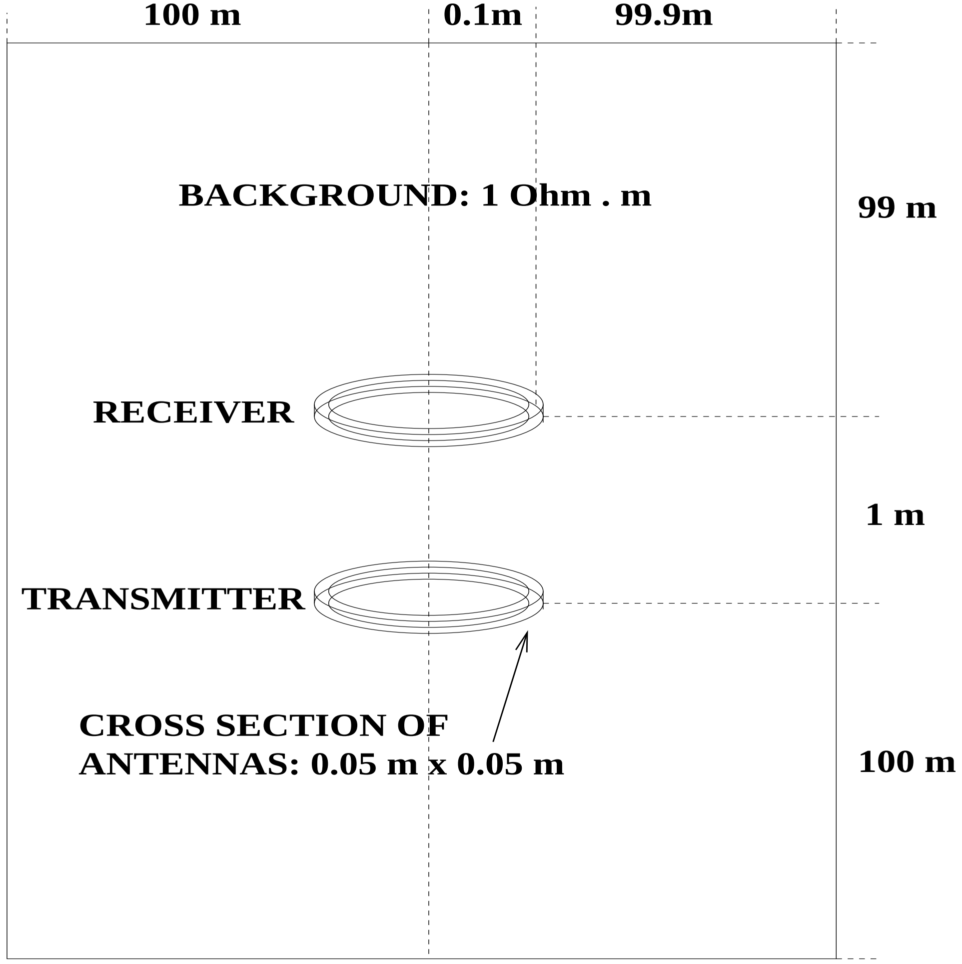
CURRENT STAGE OF THE 3D hp-FE SOFTWARE

Parallel Version. Main challenges.

- **Managing data migration, minimizing interface size (α-Version).**
- **Global numbering of interface nodes (α-Version).**
- **Retrieving Schur complement from serial solver (MUMPS) (α-Version).**
- **Aggregating contributions to Schur complement (global mapping required) (α-Version).**
- **Parallelization of the goal-oriented adaptivity (In testing).**
- **Parallelization of multi-grid iterative solver (Under development).**

PRELIMINARY RESULTS ($-\nabla(\bar{\bar{\sigma}} \cdot \nabla\psi) = \mathrm{f}$)

Electrode Problem



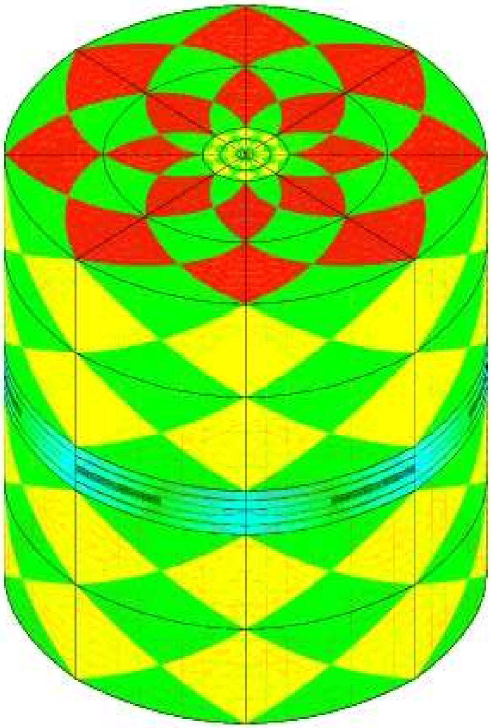
Loop antenna in a homogeneous media at DC.

Computational domain:
100 m x 100 m.

PRELIMINARY RESULTS ($-\nabla(\bar{\bar{\sigma}} \cdot \nabla\psi) = \mathrm{f}$)

Electrode Problem

Final hp-grid



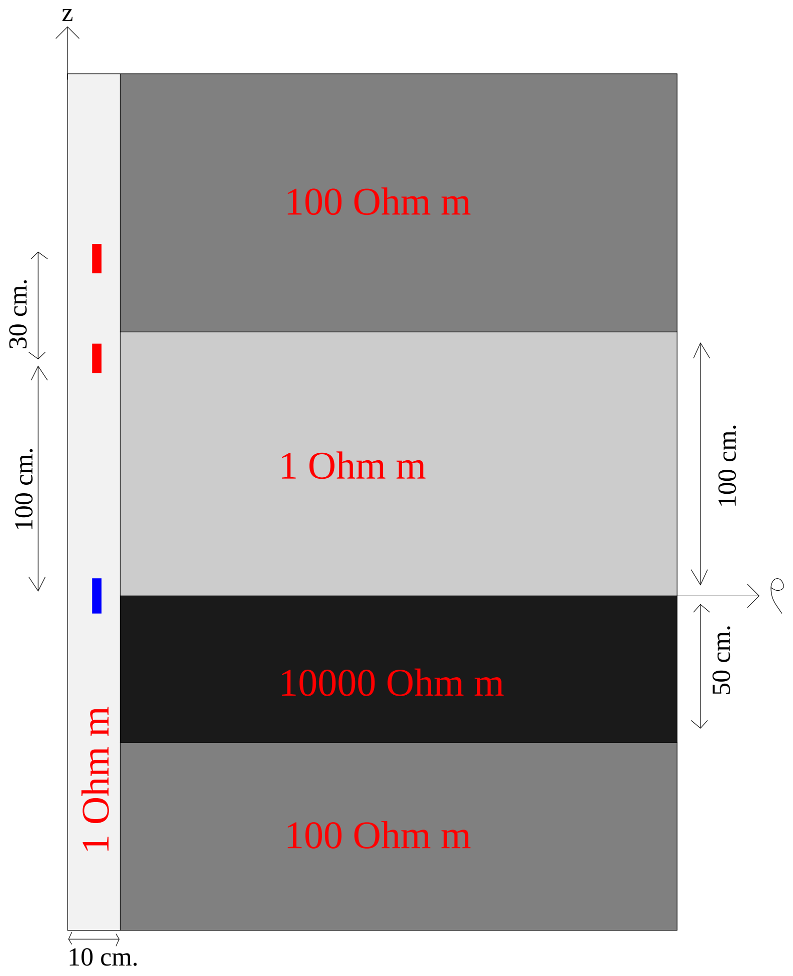
Final solution

2D Solution: 0.078131

3D Solution: 0.078121

PRELIMINARY RESULTS ($-\nabla(\bar{\bar{\sigma}} \cdot \nabla\psi) = \mathrm{f}$)

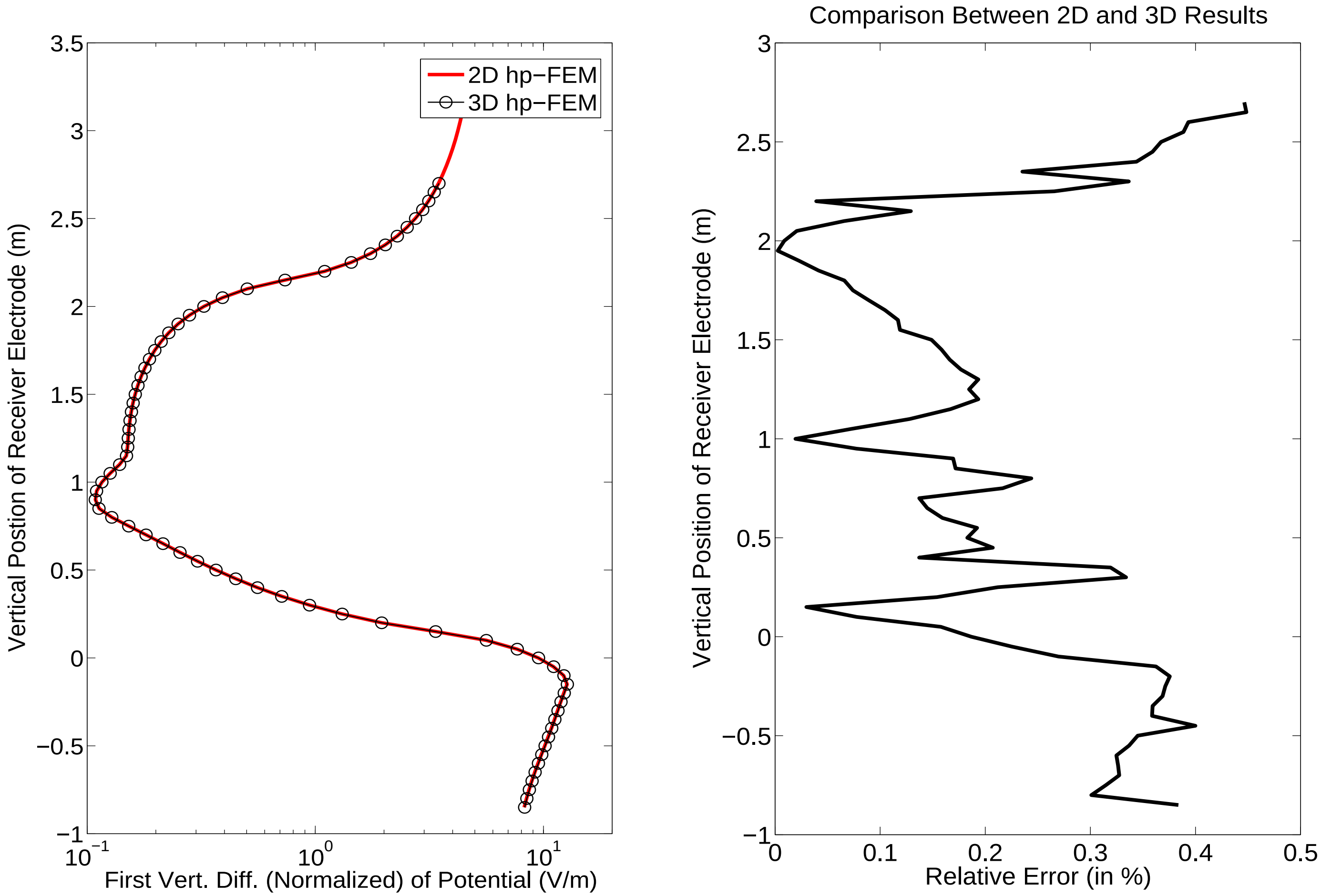
Axisymmetric Model Problem



- **Borehole and four materials on the formation.**
- **Size of computational domain:** $100m \times 100m$.
- **Size of electrode:** $0.05m \times 0.05m$.
- **Objective: Compute First Vertical Difference of Potential.**

PRELIMINARY RESULTS ($-\nabla(\bar{\bar{\sigma}} \cdot \nabla\psi) = \mathrm{f}$)

Axisymmetric Model Problem



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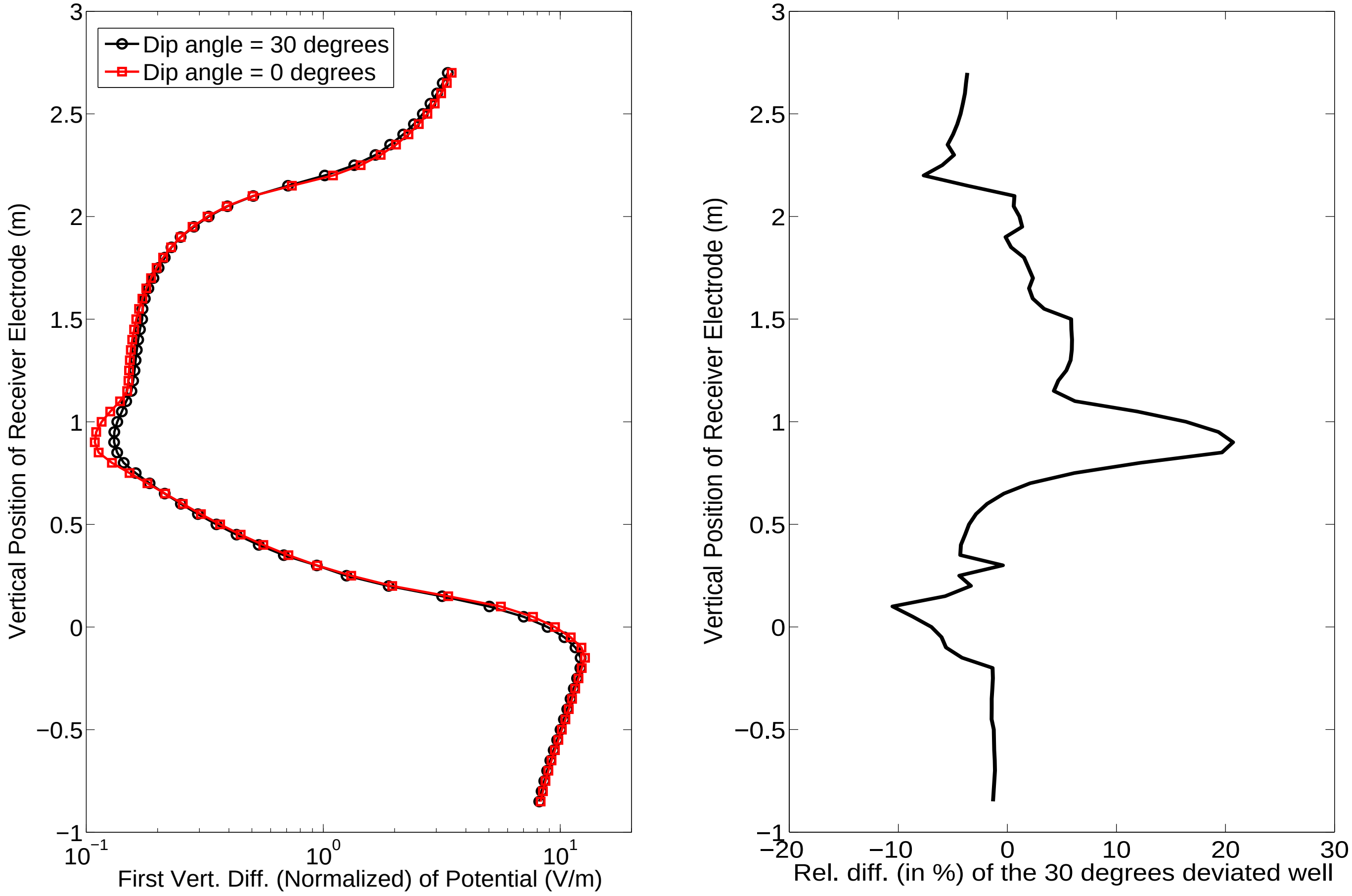

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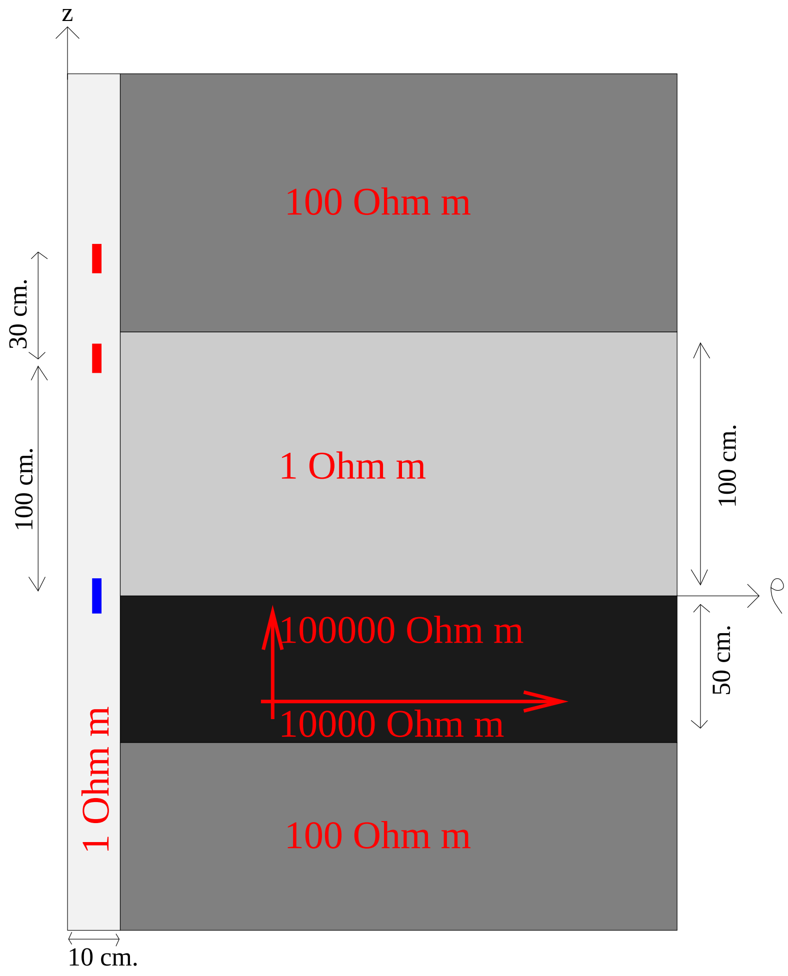

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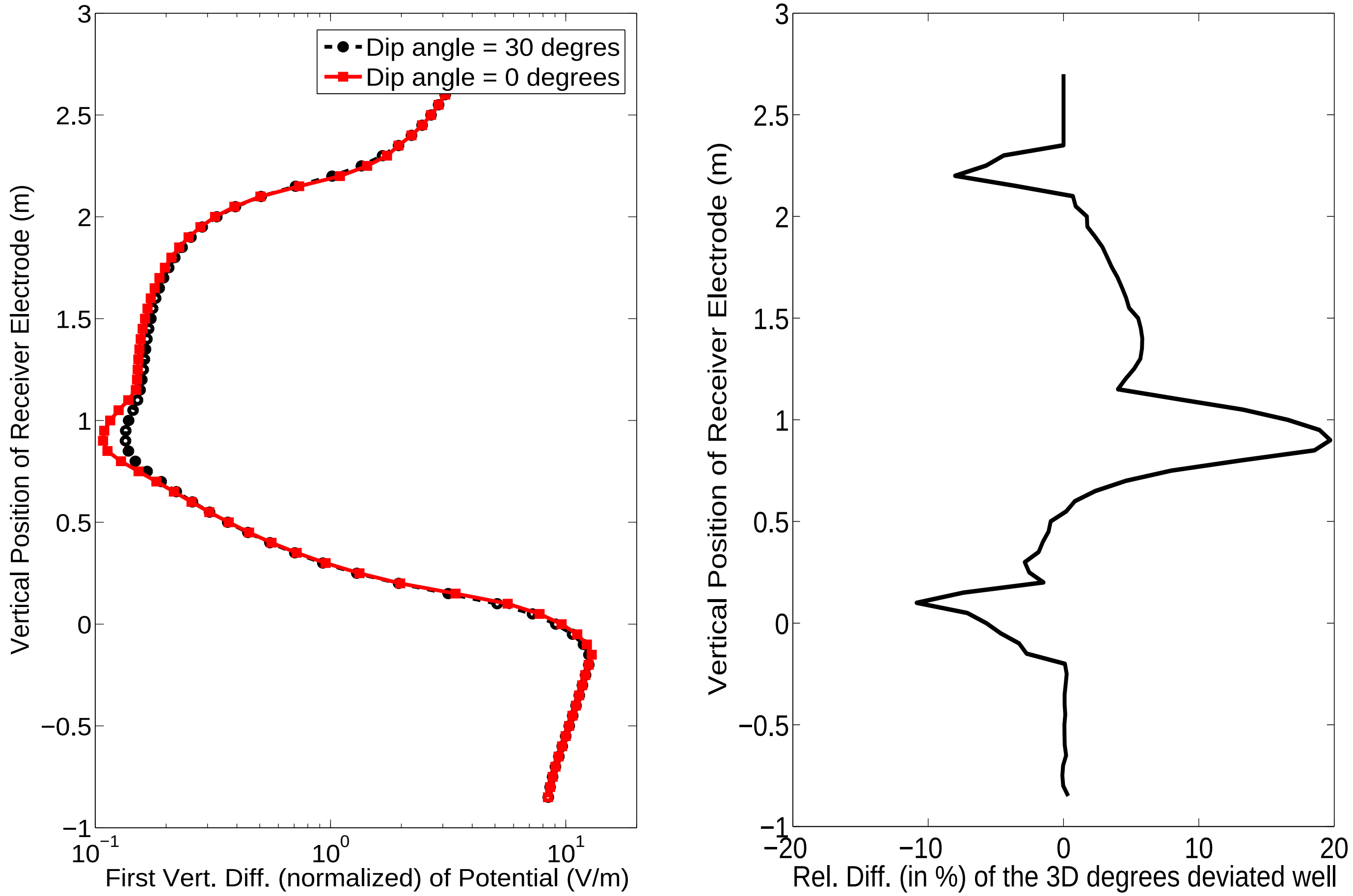
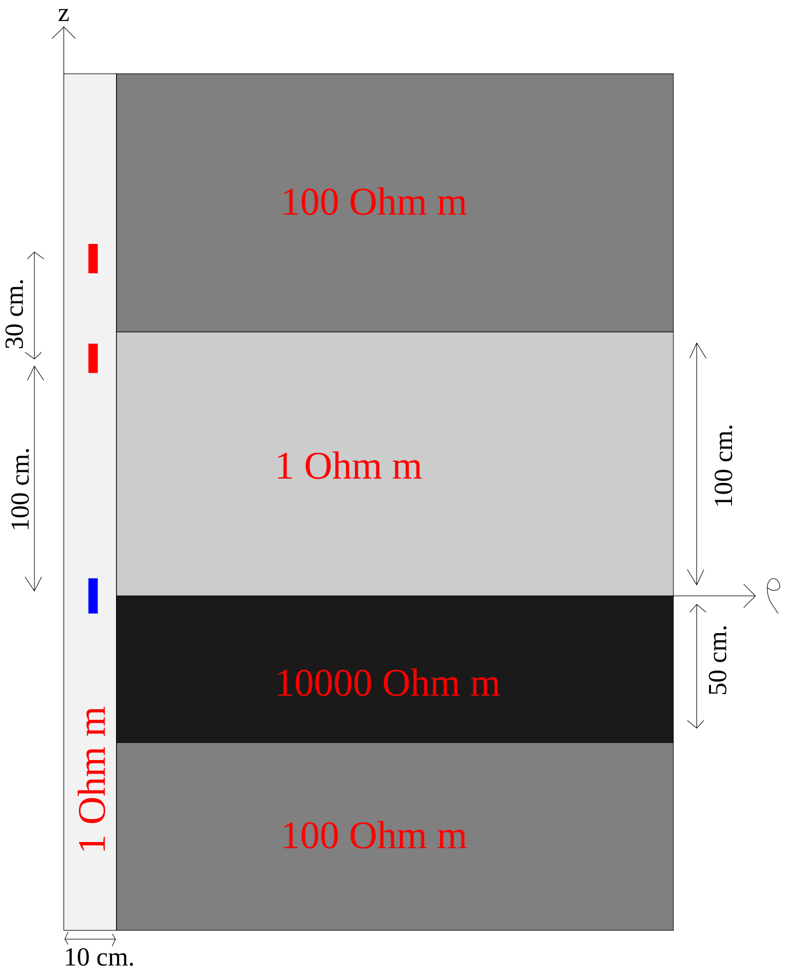
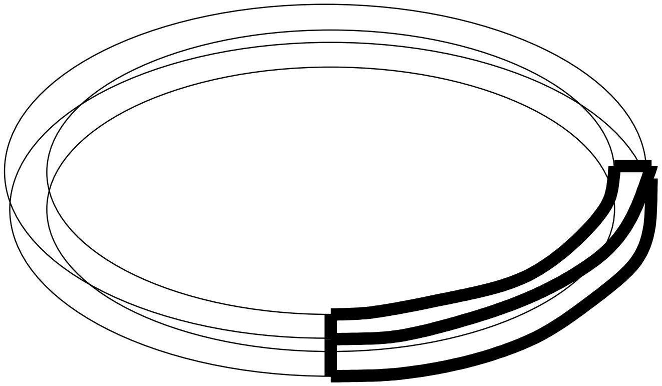


PRELIMINARY RESULTS $(-\nabla(\bar{\bar{\sigma}} \cdot \nabla\psi) = \mathrm{f})$

Axisymmetric Model Problem



Transmitter Electrode: One fourth of a ring.



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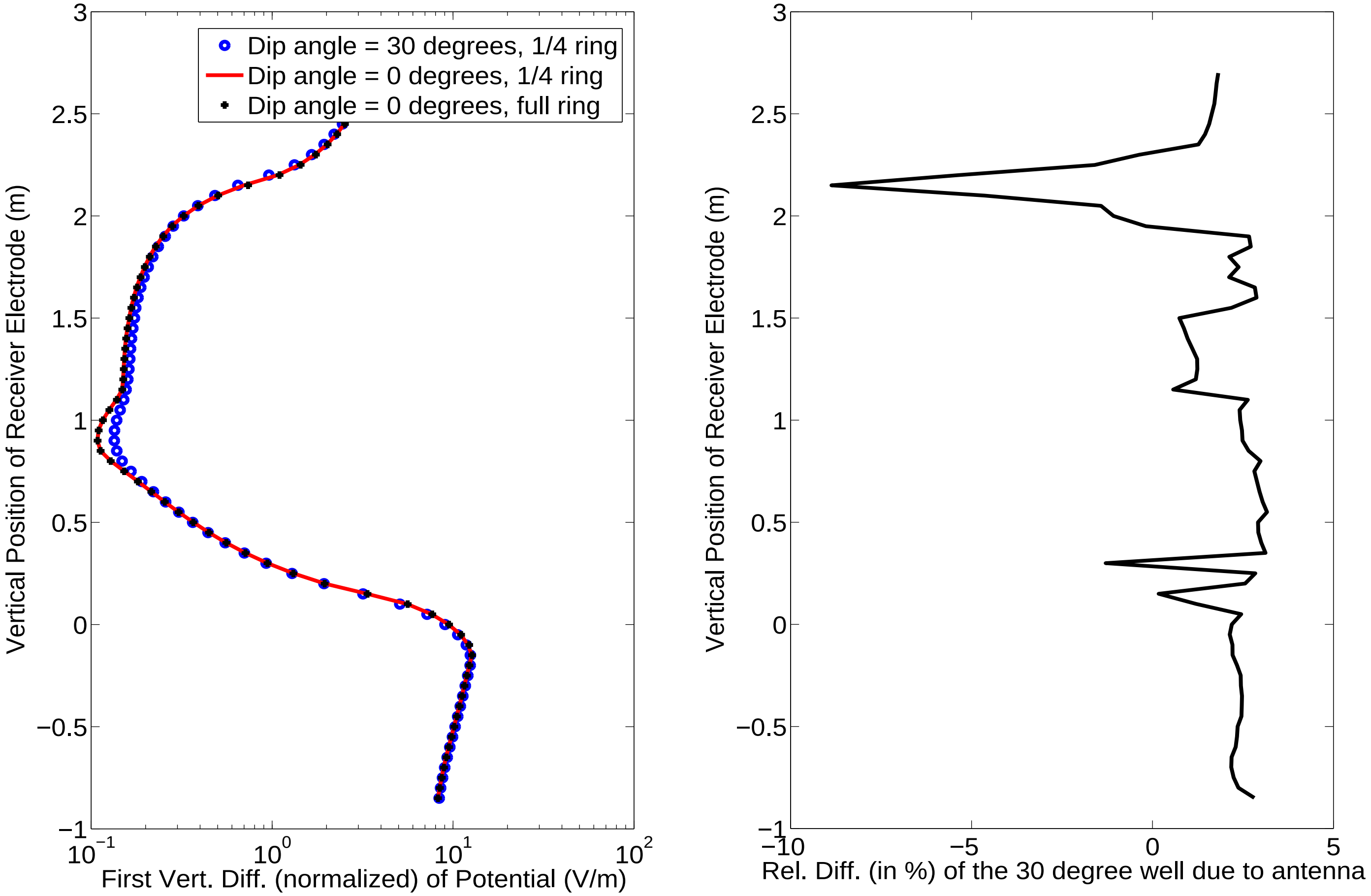

CONCLUSIONS AND FUTURE WORK

- **The self-adaptive goal-oriented hp-adaptive strategy converges exponentially in terms of a user-prescribed quantity of interest vs. the CPU time.**

- **Preliminary results indicate that it shall be possible to simulate a variety of EM logging instruments in deviated wells by using the 3D self-adaptive goal-oriented hp-FEM.**
  - **The software is expected to be suitable for ALL kind of resistivity logging instruments in possibly cased wells.**
  - **Cylindrical geometries can be accurately described by using higher-order elements.**

FUTURE WORK

Tasks and Completion Date

3D DC CODE

- **PHASE I: NEW adaptive package and solver. β-VERSION.**
- **PHASE II: Goal-Oriented Adaptivity. β-VERSION.**
- **PHASE III: Interface for Describing Logging Problems. β-VERSION.**
- **PHASE IV: New 3D Graphics Package. α-VERSION.**
- **PHASE V: Parallel Solver (MUMPS). α-VERSION.**
- **PHASE VI: Iterative Solver. α-VERSION.**
- **PHASE VII: Logging Examples (without casing). PRELIMINARY RESULTS.**
- **PHASE VIII: Parallel Version of 3D code. 20 Jul 2006.**
- **PHASE IX: Through Casing Resistivity Instruments. 20 Aug 2006.**
- **PHASE X: 3D Perfectly Matched Layer (PML). 20 Oct 2006.**

FUTURE WORK

Tasks and Completion Date

3D AC CODE

- **PHASE I: NEW adaptive package and solver. 1 Aug 2006.**
- **PHASE II: Edge-element formulation. 1 Sep 2006.**
- **PHASE III: Goal-Oriented Adaptivity. 1 Oct 2006.**
- **PHASE IV: Interface for Describing Logging Problems. 1 Nov 2006.**
- **PHASE V: Parallel Solver (MUMPS). 1 Nov 2006.**
- **PHASE VI: Iterative Solver. 1 Feb 2007.**
- **PHASE VII: Logging Examples. 1 May 2007.**
- **PHASE VIII: 3D AC Perfectly Matched Layer (PML). 1 Jun 2007.**
- **PHASE IX: Parallel Version of 3D code. 1 Jul 2007.**
- **PHASE X: Through Casing Resistivity Instruments. 1 Sep 2007.**

FUTURE WORK

Tasks and Completion Date

Outcome of the project: A software for simulation of the following types of resistivity logging problems.

- **Induction instruments.**
- **Laterolog instruments.**
- **Deviated wells.**
- **Eccentric wells.**
- **Anisotropy effects.**
- **Patch antennas/electrodes.**
- **Through casing resistivity tools.**
- **Different frequencies.**

All of the above (when applicable) can be combined in one problem. For example, a laterolog instrument in a deviated well with eccentricity, and with a patch antenna.

APPENDIX: SHOULD WE USE CYLINDRICAL GRIDS?

The possibility of using cylindrical grids

MAIN ADVANTAGES

Simpler geometries.

Possibly less elements needed on the azimuthal direction.

MAIN DISADVANTAGES

Advantages mentioned above are not clear in the case of deviated wells.

Extra boundary condition needed ($\Psi(0) = \Psi(2\pi)$).

Integration becomes not exact.

New partial differential equations need to be implemented.

Continuous elements and Nedelec elements are based on cartesian geometries. At $\rho = 0$ degenerated elements may be needed.

FOR DEVIATED WELLS, IT IS NOT CLEAR THAT THE USE OF CYLINDRICAL GRIDS (AS OPPOSED TO CARTESIAN GRIDS) BECOMES MORE ADEQUATE