Joint Industry Research Consortium on Formation Evaluation

Integrated Approach Toward Formation Evaluation Using an hp-Adaptive Goal-Oriented Finite Element Formulation

D. Pardo, M. Paszynski, C. Michler, C. Torres-Verdín, L. Demkowicz

Collaborators: J. Kurtz, W. Rachowicz, A. Zdunek, L.E. García-Castillo

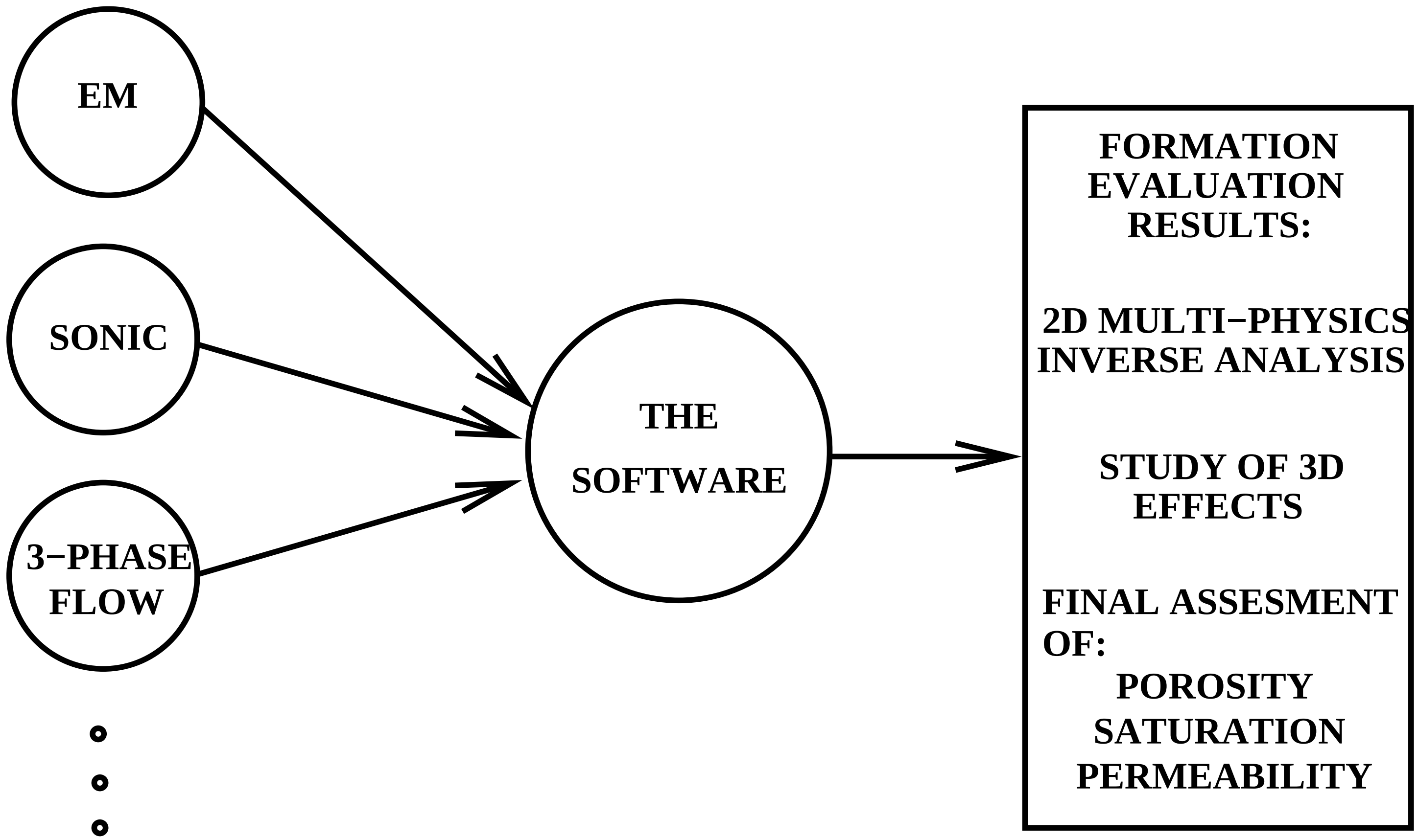
August 17, 2006

Department of Petroleum and Geosystems Engineering, and
Institute for Computational Engineering and Sciences (ICES)

THE UNIVERSITY OF TEXAS AT AUSTIN

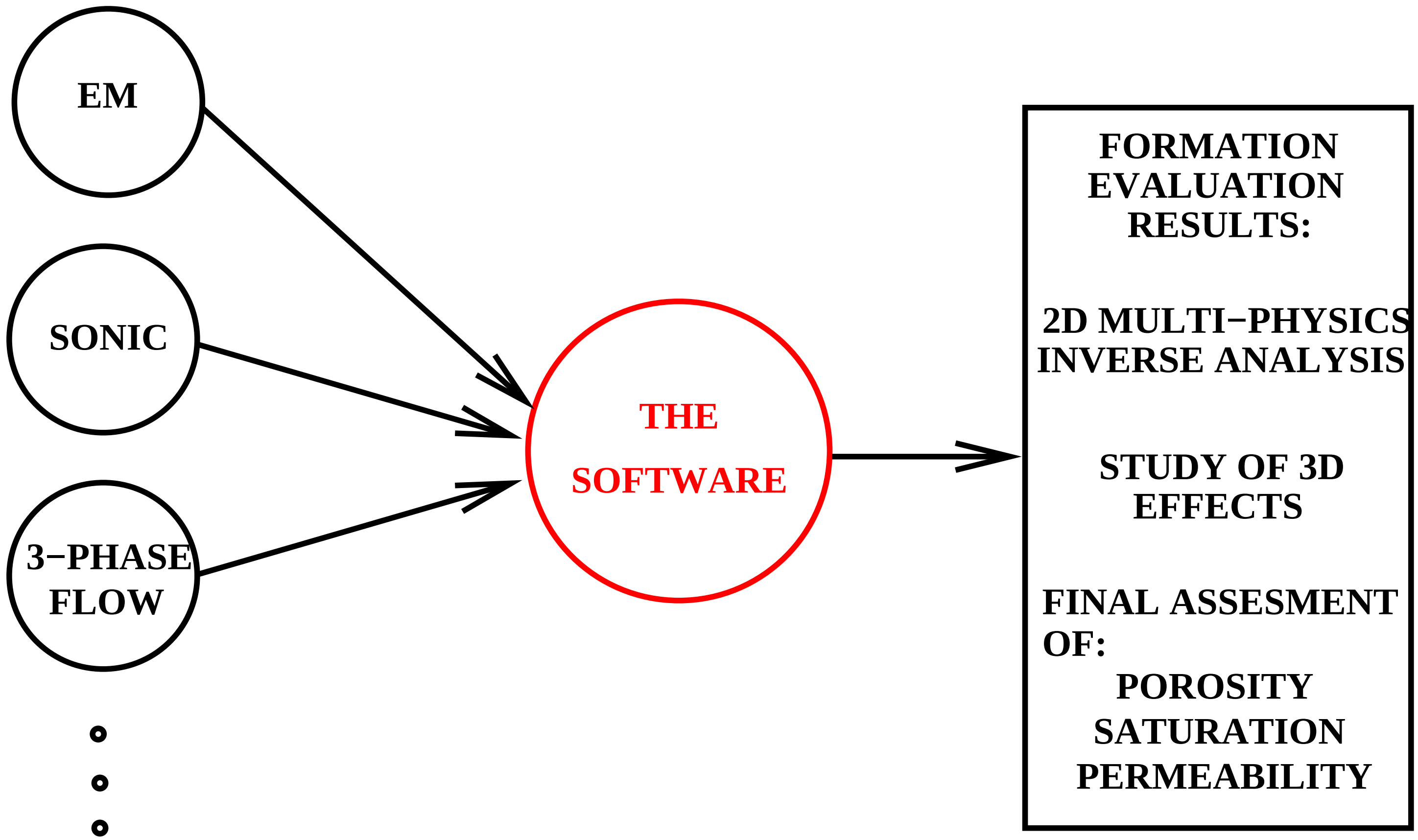
INTEGRATED APPROACH TOWARD FORMATION EVALUATION

ONE software FOR ALL the formation evaluation



INTEGRATED APPROACH TOWARD FORMATION EVALUATION

What software can we use?



INTEGRATED APPROACH TOWARD FORMATION EVALUATION

What software can we use?

A Self-Adaptive Goal-Oriented hp-Finite Element Method

MAIN ADVANTAGES

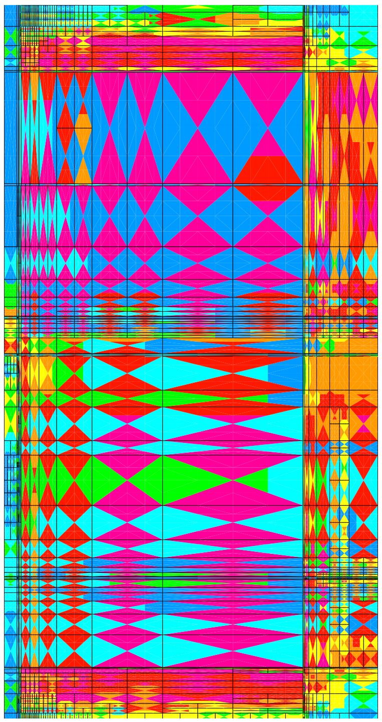
1. It supports multi-physics.
2. It supports 2D and 3D computations.
3. It automatically generates optimal grids with few unknowns.
4. It provides error estimation *guaranteed accuracy*.
5. It is suitable for high-contrast problems.
6. Integrated effort.

MAIN DRAWBACKS

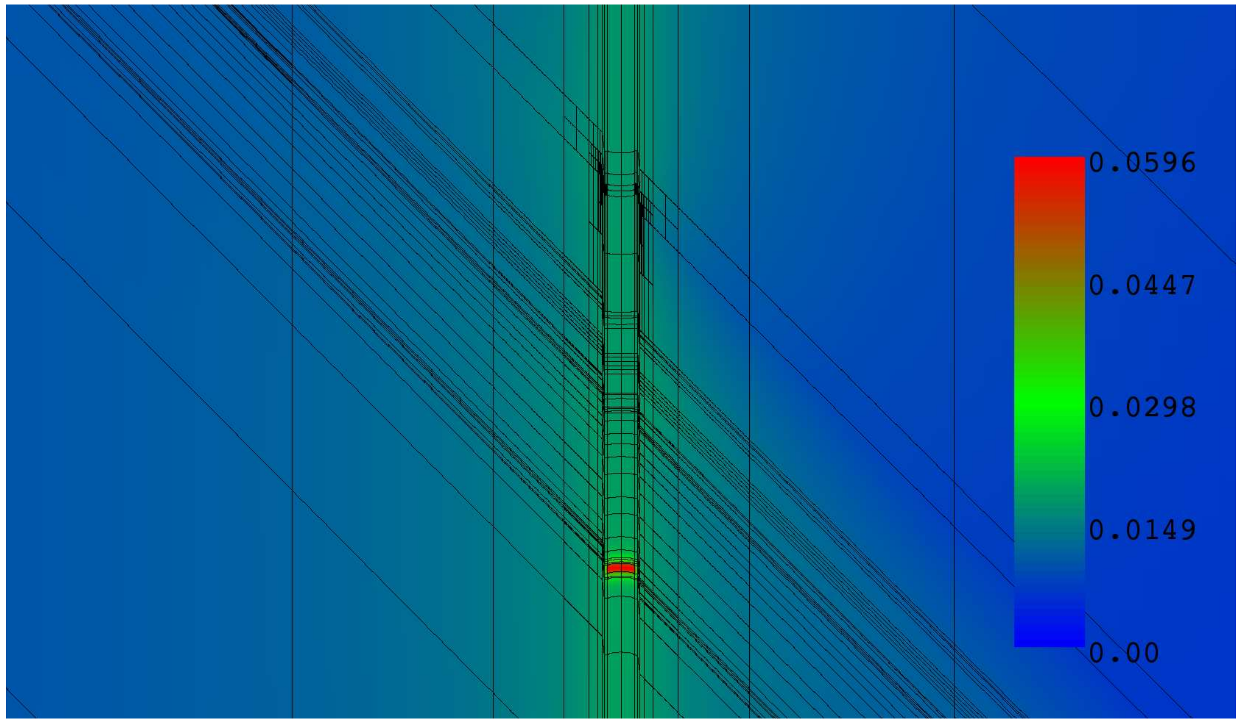
1. It is complex (mathematically involved).
2. It is large ($>$ 100.000 lines of code).
3. 3D computations are time and memory consuming.

INTEGRATED APPROACH TOWARD FORMATION EVALUATION

2D and 3D Resistivity Logging (Electromagnetics)



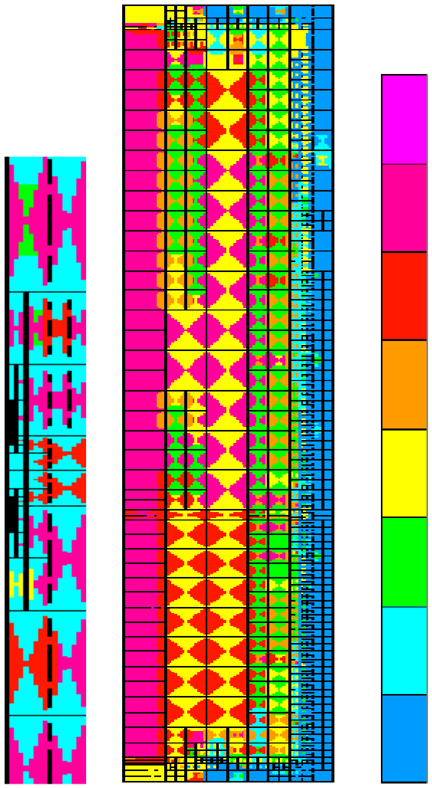
2D hp-mesh



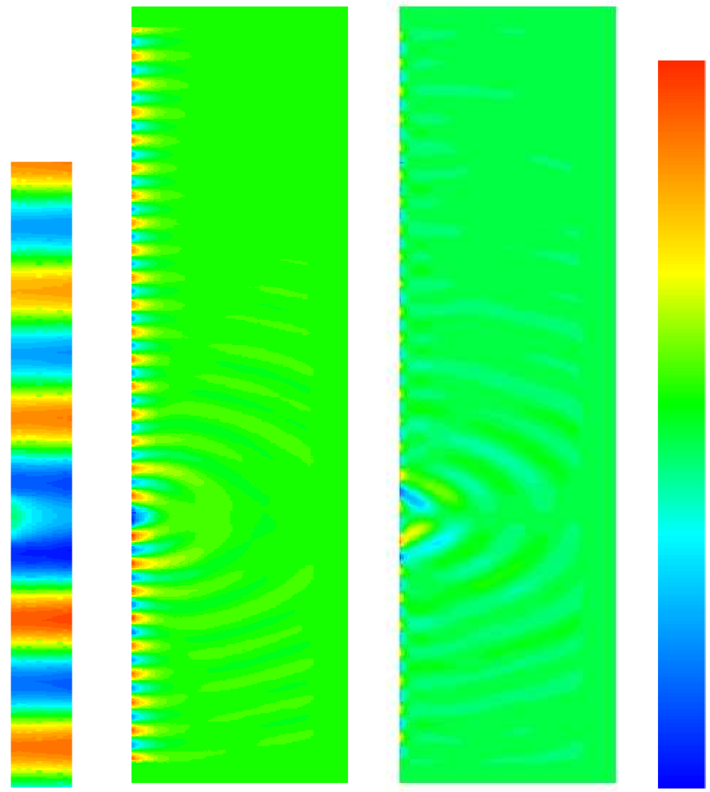
3D TCRT Solution (Dip Angle = 60 degrees)

INTEGRATED APPROACH TOWARD FORMATION EVALUATION

2D Sonic Logging (Acoustics/Elasticity)



Final hp-mesh



Final Solution

INTEGRATED APPROACH TOWARD FORMATION EVALUATION

Work Plan

1. 2D DC simulator (2004).
2. 2D AC simulator (2005).
3. 3D parallel DC simulator (2006).
4. 2D sonic simulator (2006).
5. 3D parallel AC simulator (2007).
6. 2D inverse DC-AC simulator (2007-2008).
7. 2D AC-sonic inverse simulator (2008).

INTEGRATED APPROACH TOWARD FORMATION EVALUATION

Type of Problems We Can Solve with 2Dhp90 (v. 8.0)

Physical Devices	Magnetic Buffers	Insulators	Displacement Currents
	Casing	Casing Imperfections	Combination of all
Materials	Isotropic	Anisotropic*	
Sources	Toroidal Antennas	Solenoidal Antennas	Dipoles in Any Direction
	Electrodes	Finite Size Antennas	Combination of All
Logging Instruments	LWD/MWD	Laterolog	Normal
	Induction	Dielectric Instruments	Cross-well Marine EM
Frequency	0-10 Ghz.		
Invasion	Water	Oil	etc.

ALL AXISYMMETRIC RESISTIVITY LOGGING PROBLEMS

Joint Industry Research Consortium on Formation Evaluation

Numerical Simulation of 3D DC Borehole Resistivity Measurements Using an hp-Adaptive Goal-Oriented Finite Element Formulation

D. Pardo, M. Paszynski, C. Torres-Verdín, L. Demkowicz

Collaborators: Science Department of Baker-Hughes, C. Michler, J. Kurtz, W. Rachowicz, A. Zdunek, L.E. García-Castillo

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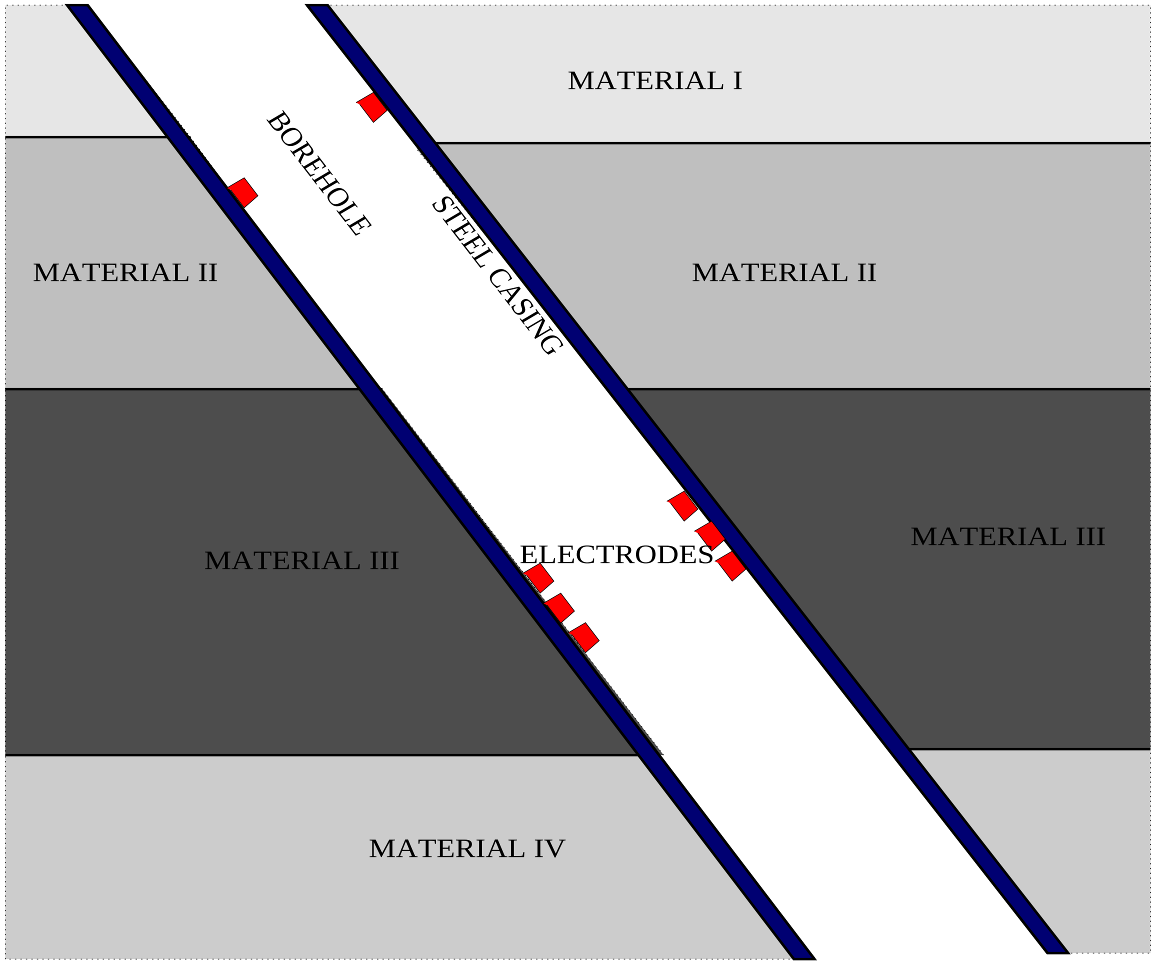
OVERVIEW

1. **Numerical Methodology**
   - hp-Finite Elements (Exponential convergence)
   - Automatic Goal-Oriented Refinements (in the quantity of interest)
   - Multi-grid (iterative) solver of linear equations.
2. **Current Stage of the 3D DC hp-FE Software**
3. **Numerical Simulations of 3D DC:**
   - Laterolog Measurements
   - LWD Measurements (at DC)
4. **Conclusions and Future Work**

THROUGH CASING RESISTIVITY INSTRUMENTS

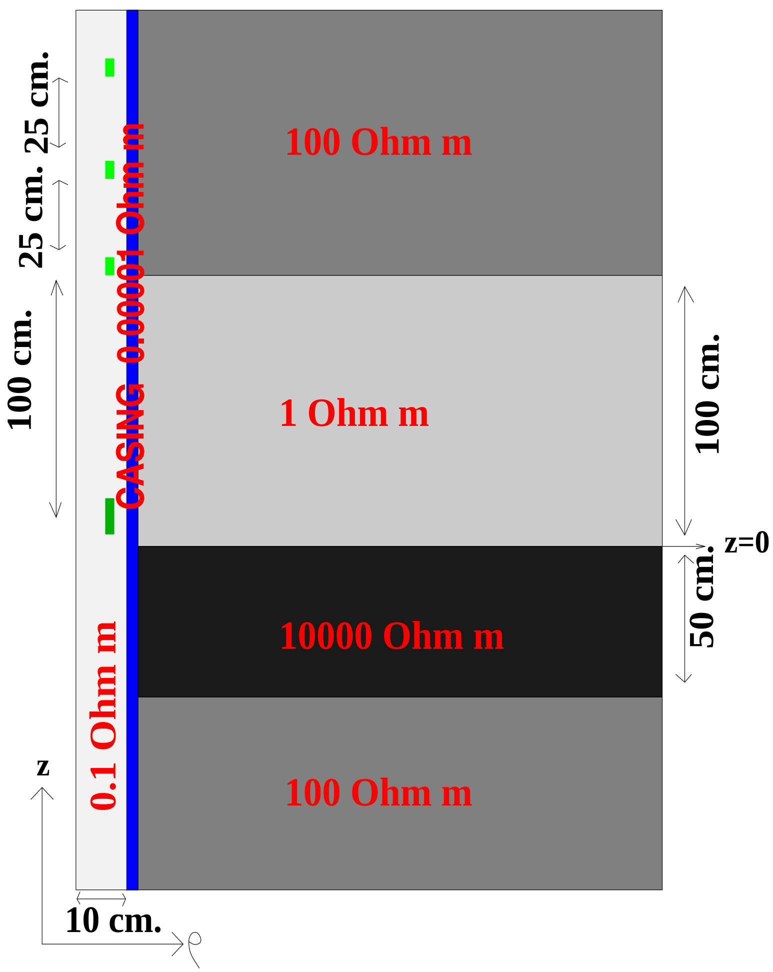
Deviated Cased Wells



Objective: Determine 2nd difference of potential at the receiver antennas.

THROUGH CASING RESISTIVITY INSTRUMENTS

Model Problem with Steel Casing



Frequency: 0 Hz.

Casing resistivity: 10^{-5} Ohm · m.

Casing width: 0.01127 m

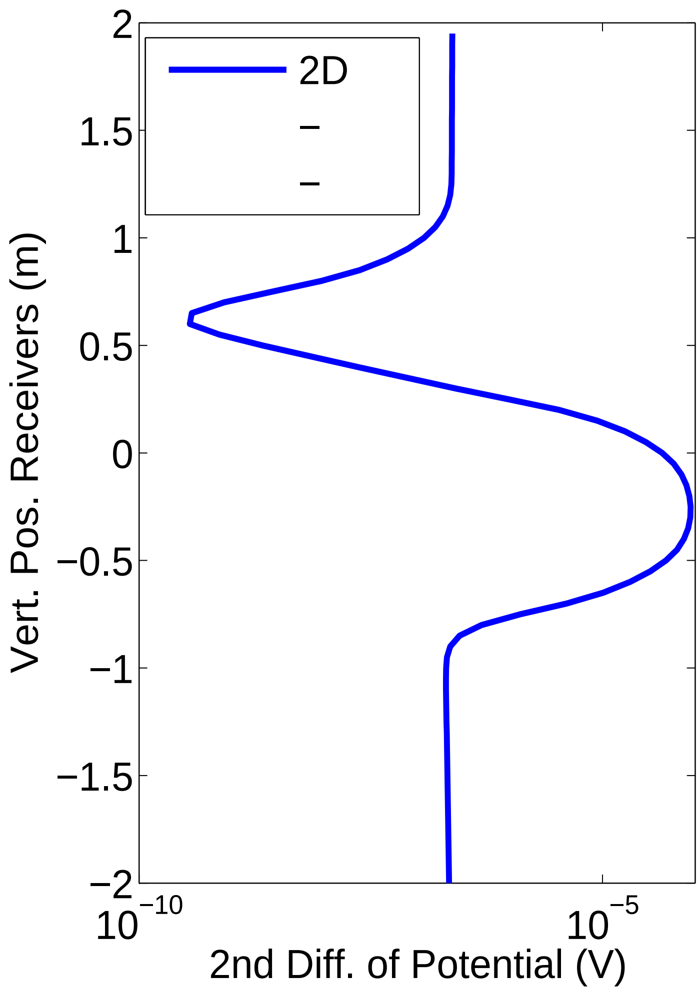
Large constrast in resistivity

Electrodes

Size (domain): 1000m x 2000m

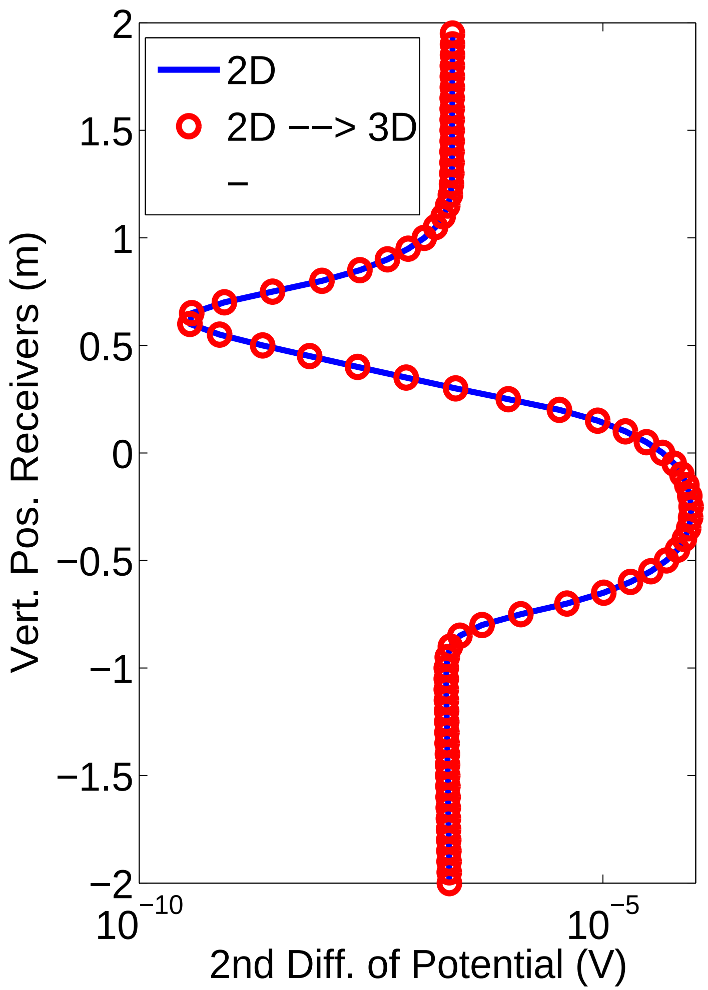
THROUGH CASING RESISTIVITY INSTRUMENTS

Axisymmetric problem



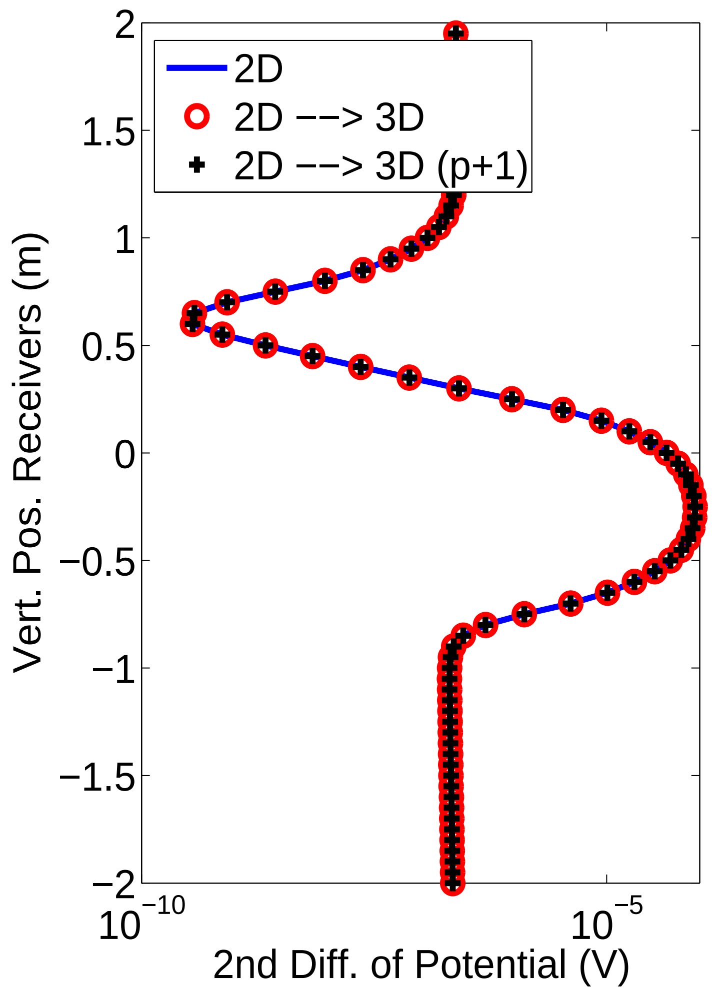
THROUGH CASING RESISTIVITY INSTRUMENTS

Axisymmetric problem



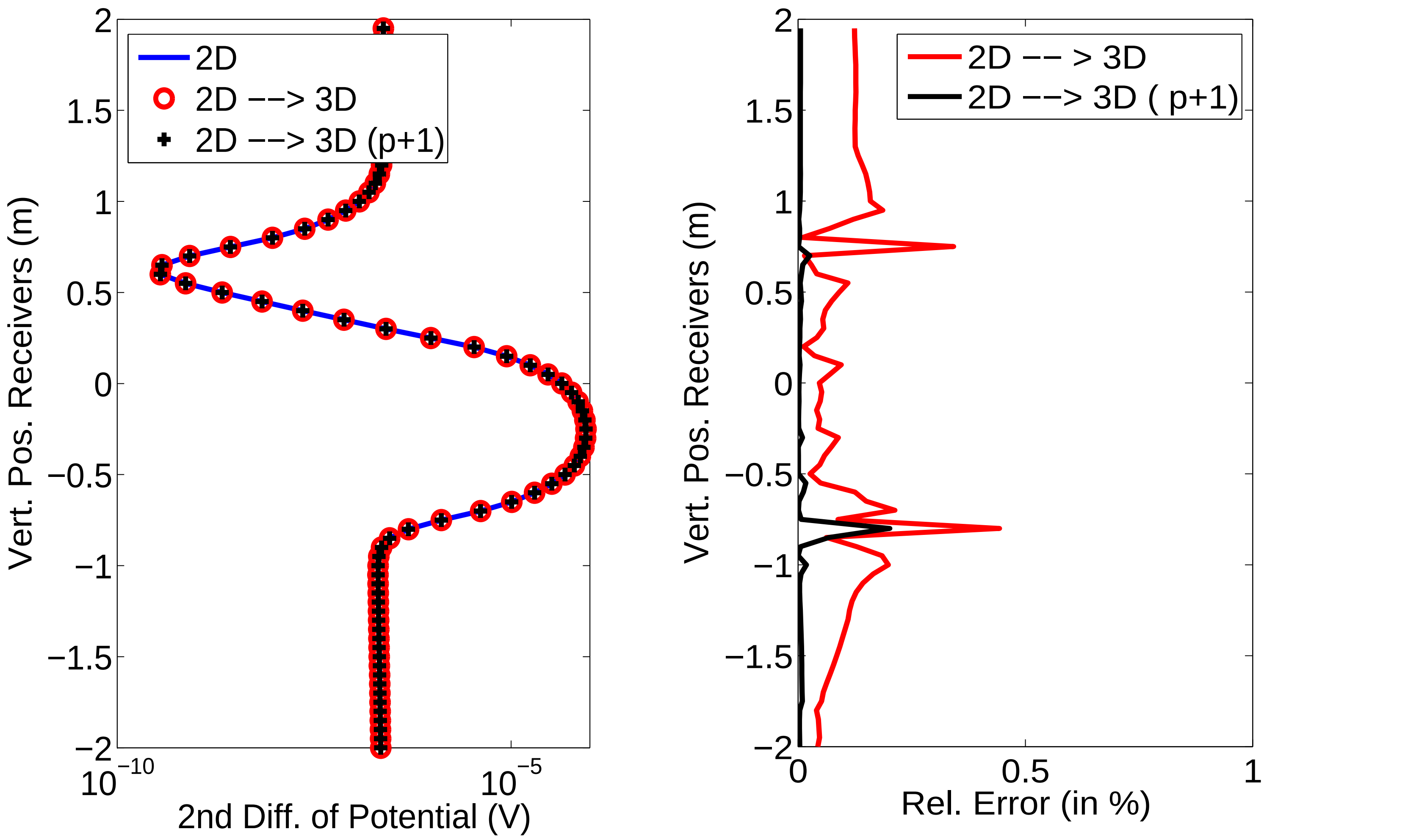
THROUGH CASING RESISTIVITY INSTRUMENTS

Axisymmetric problem



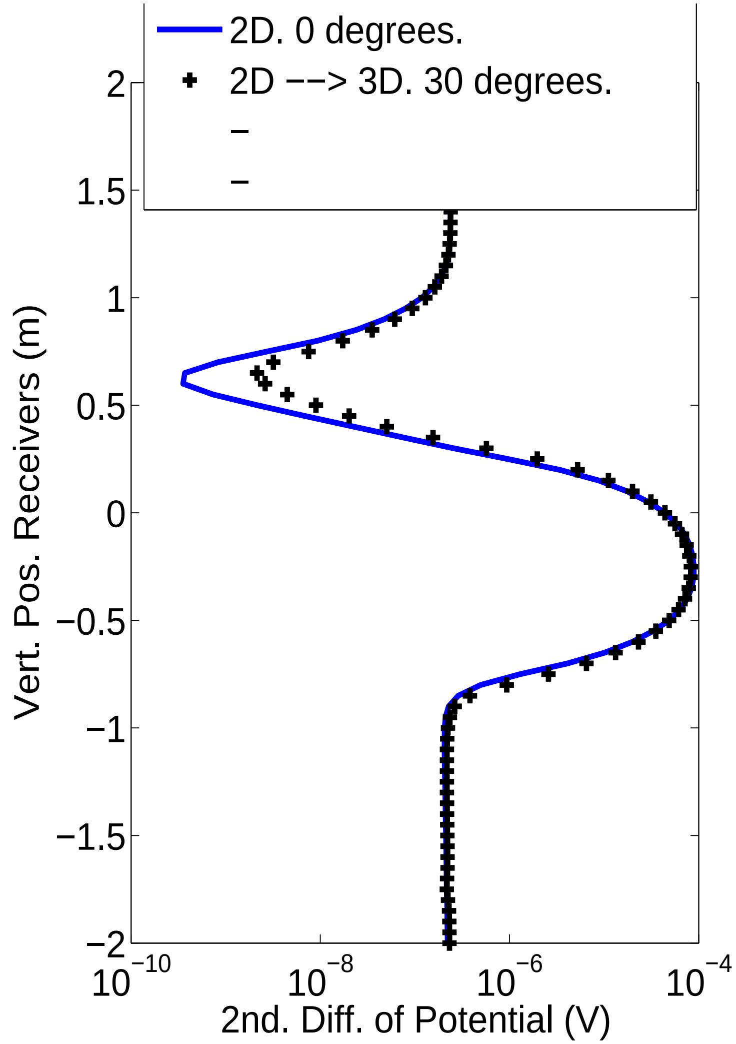
THROUGH CASING RESISTIVITY INSTRUMENTS

Axisymmetric problem



THROUGH CASING RESISTIVITY INSTRUMENTS

30 degrees deviated well



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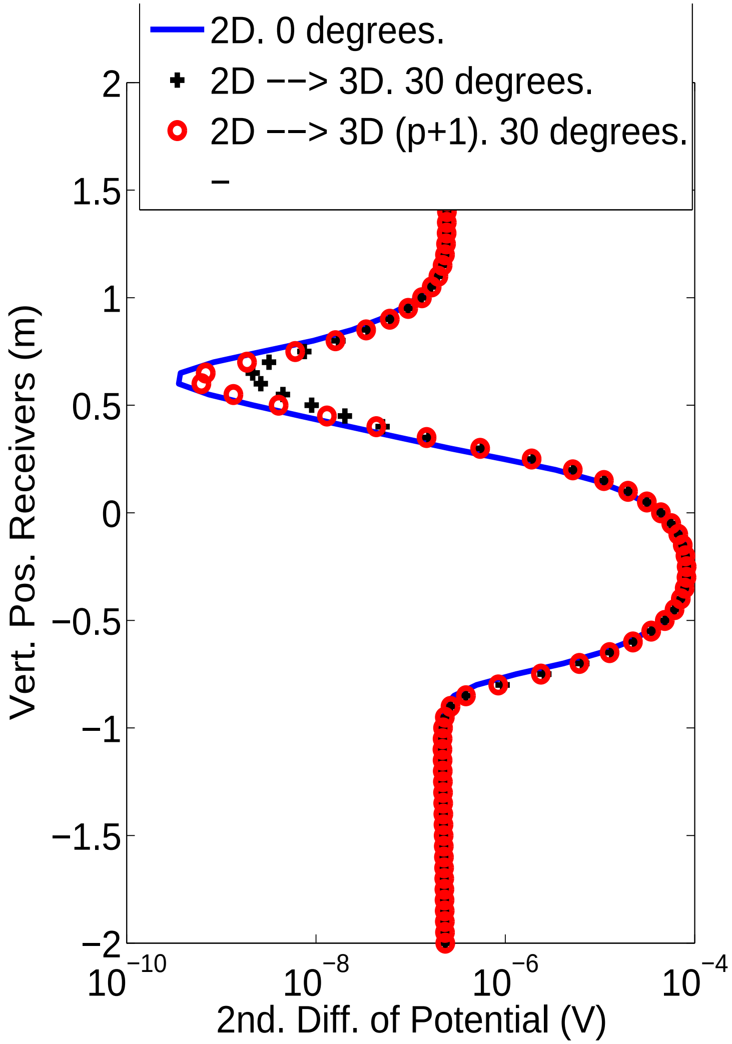

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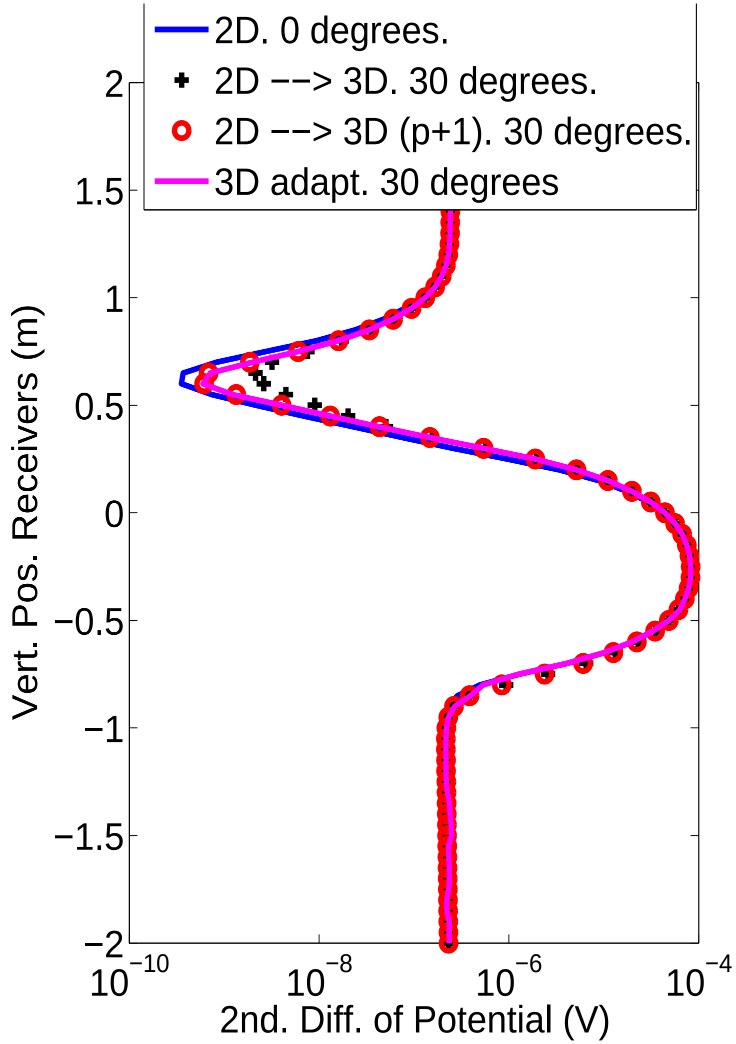

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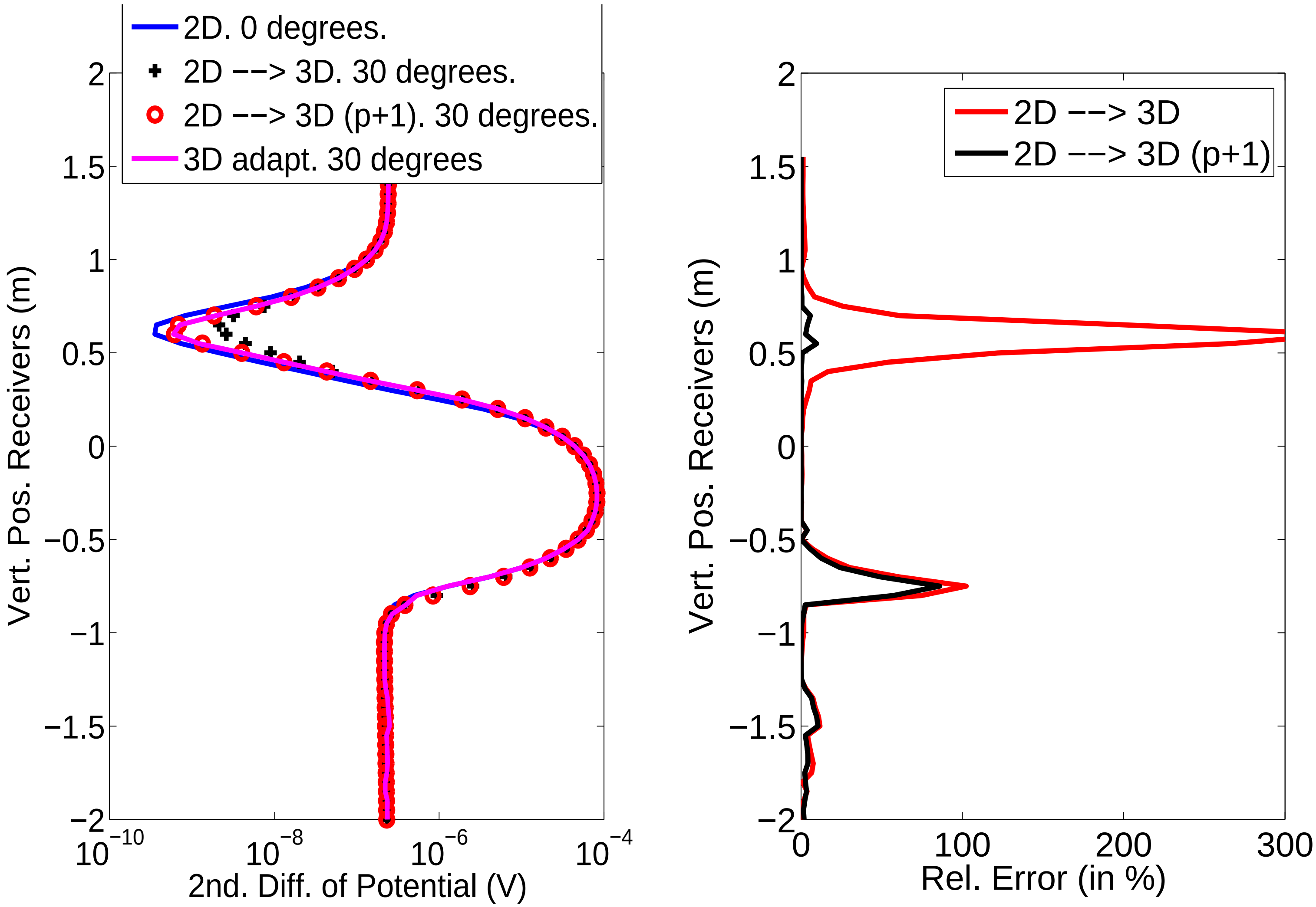
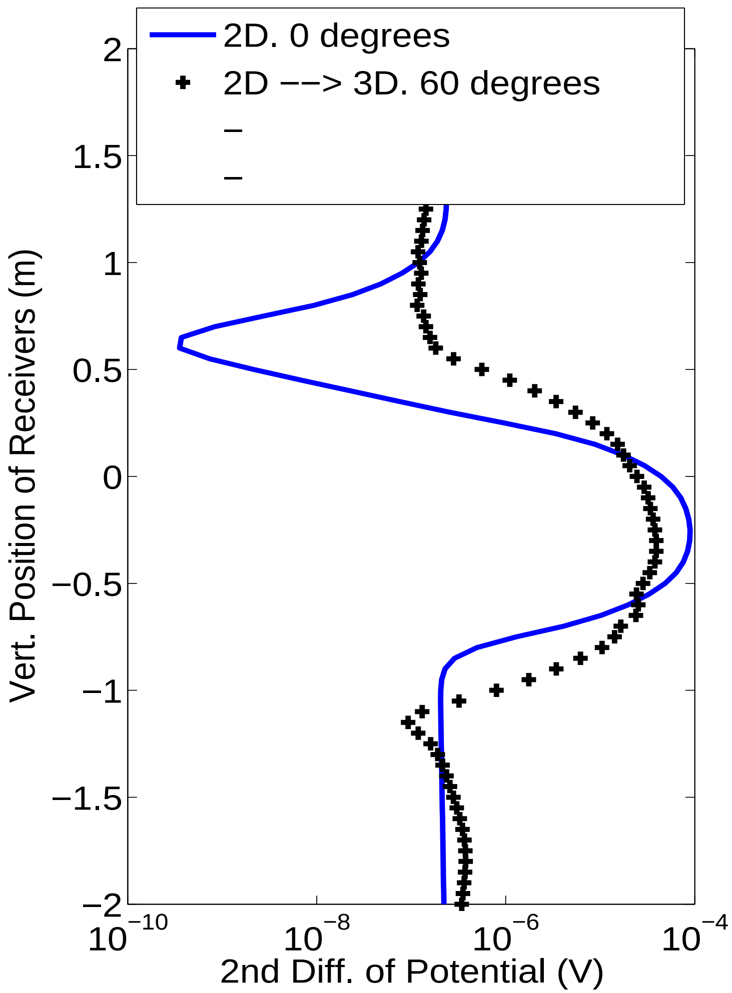


THROUGH CASING RESISTIVITY INSTRUMENTS

60 degrees deviated well



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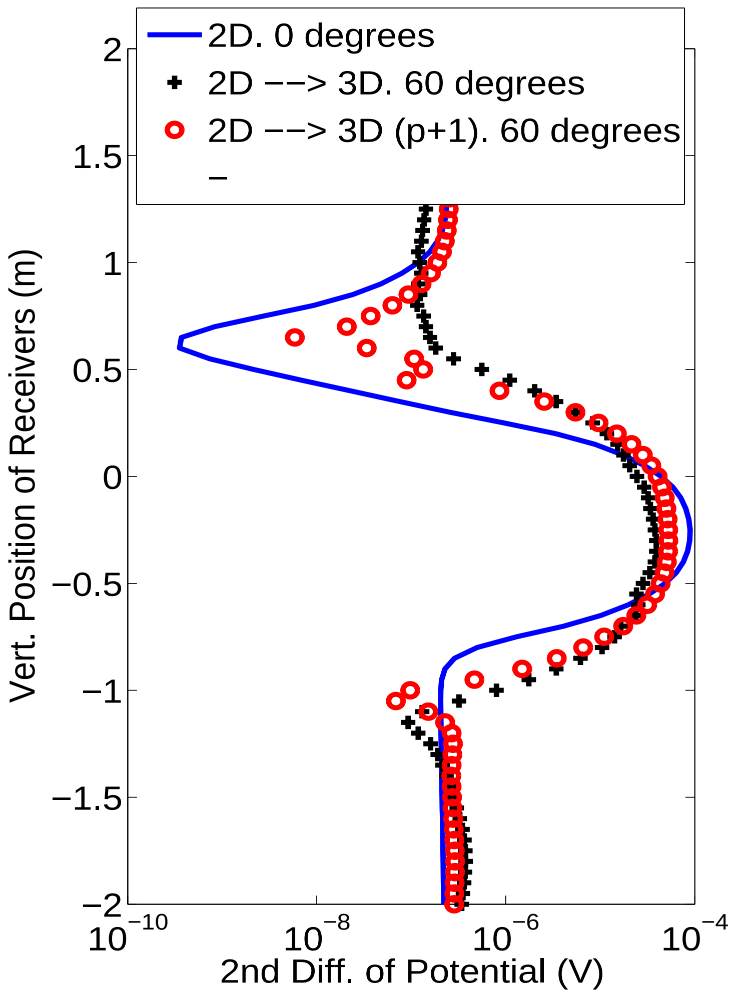

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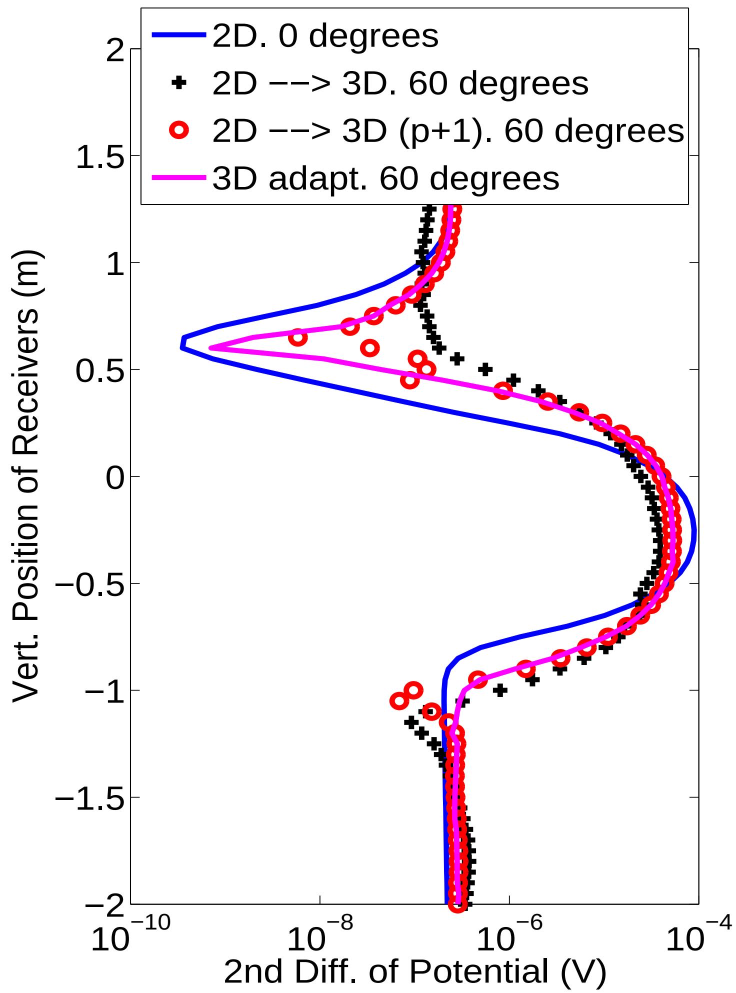

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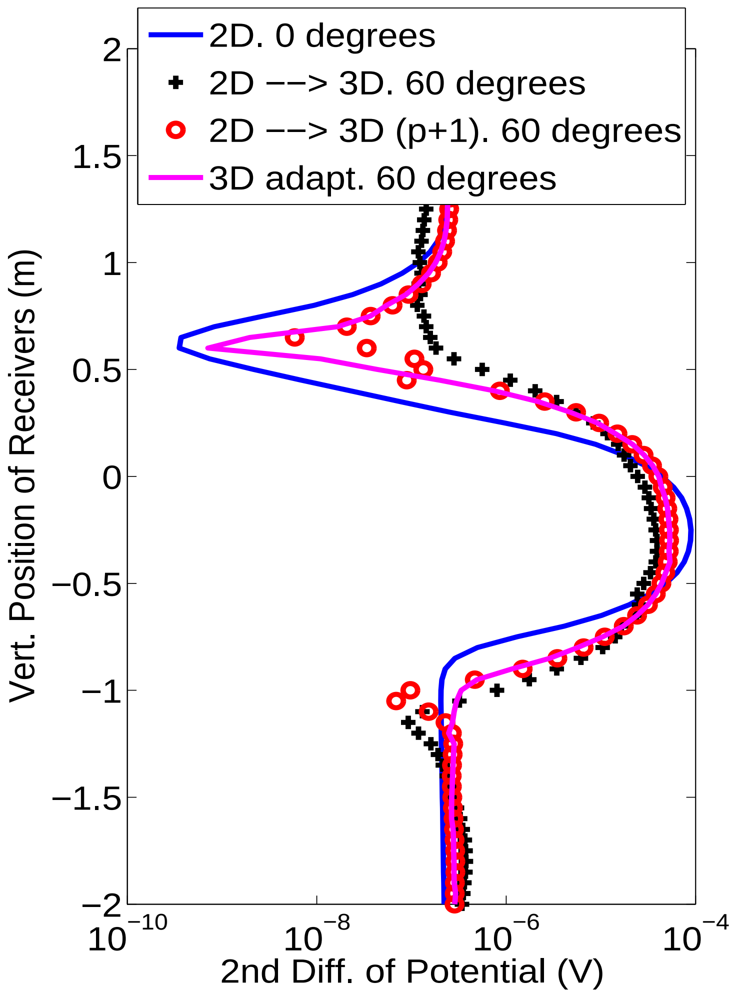
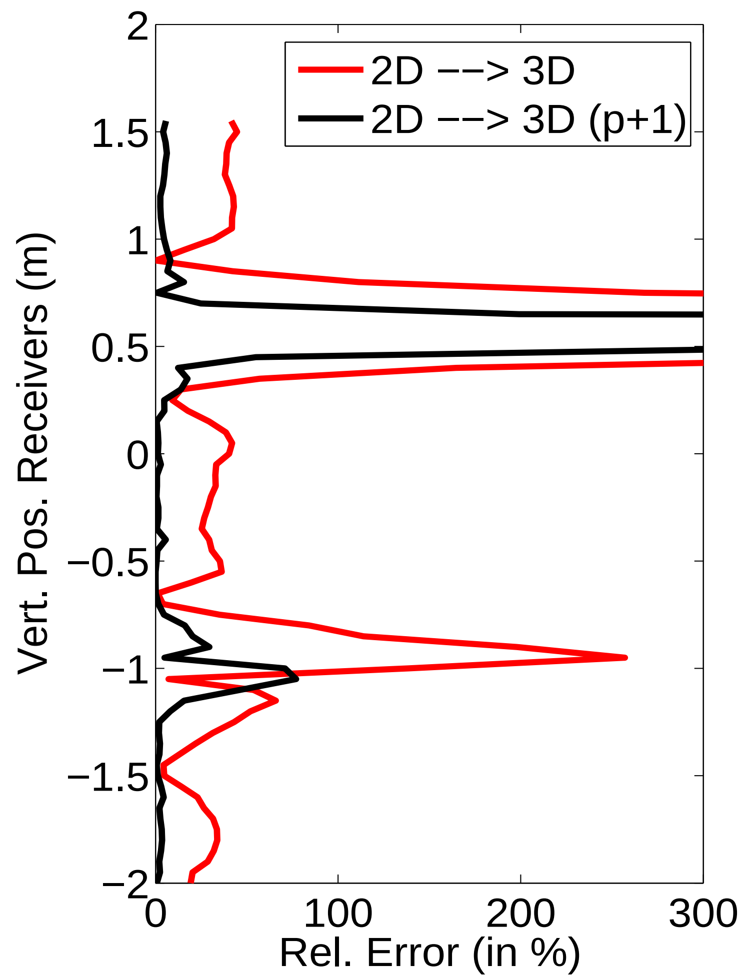
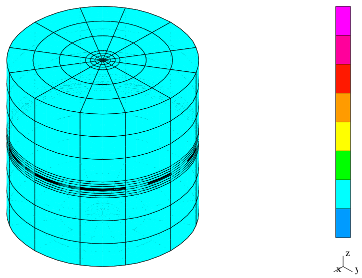




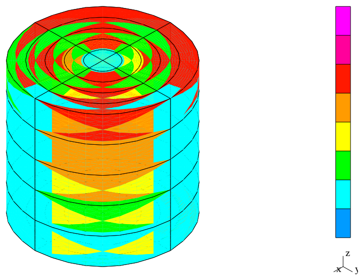
THE hp-FINITE ELEMENT METHOD (FEM)

The h-Finite Element Method



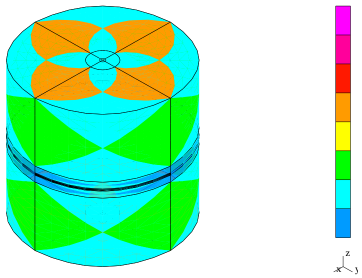
1. Convergence limited by the polynomial degree, and large material contrasts.
2. Optimal h-grids do NOT converge exponentially in real applications.
3. They may "lock" (100$\%$ error).

The p-Finite Element Method



1. Exponential convergence feasible for analytical ("nice") solutions.
2. Optimal p-grids do NOT converge exponentially in real applications.
3. If initial h-grid is not adequate, the p-method will fail miserably.

The hp-Finite Element Method

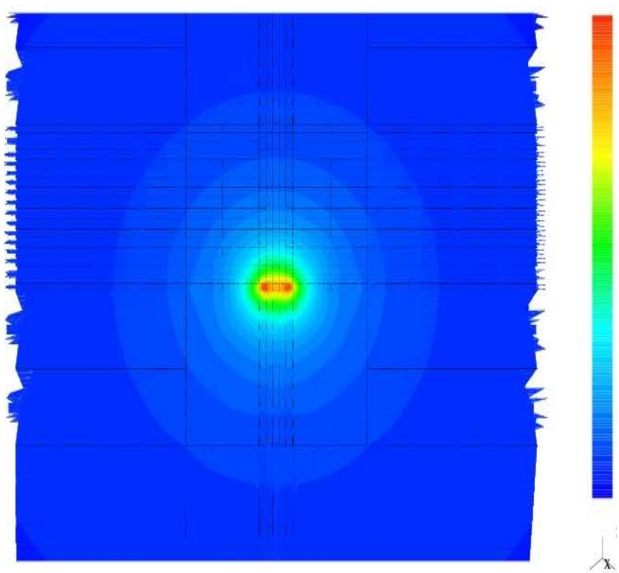


1. Exponential convergence feasible for ALL solutions.
2. Optimal hp-grids DO converge exponentially in real applications.
3. If initial hp-grid is not adequate, results will still be great.

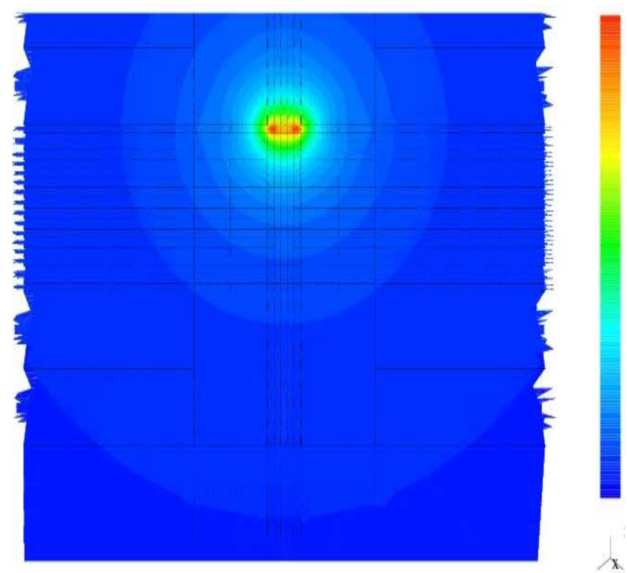
GOAL-ORIENTED ADAPTIVITY

Mathematical Formulation (Goal-Oriented Adaptivity)

DIRECT PROBLEM - Ψ -
2D Cross-Section



DUAL PROBLEM - G -
2D Cross-Section



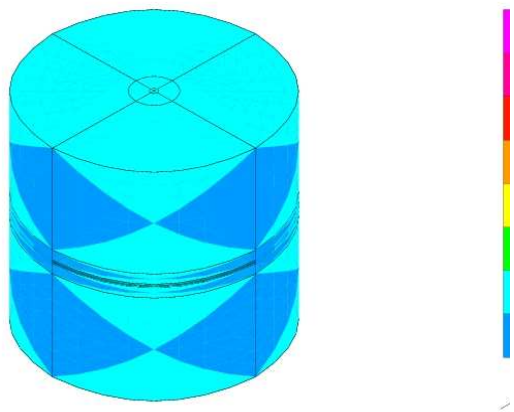
Representation Formula for the Error in the Quantity of Interest:

$$L(\Psi)=b(\Psi,G) = \int_\Omega \sigma \, \nabla\Psi \, \nabla G \, dV$$ (electrostatics)

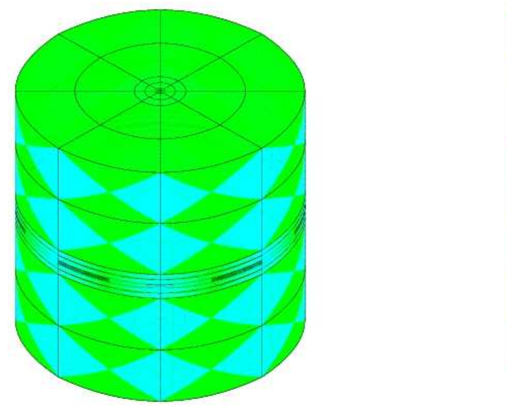
SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Algorithm for Goal-Oriented Adaptivity - STEP I -

Solve Direct and Dual Problems on Grid hp

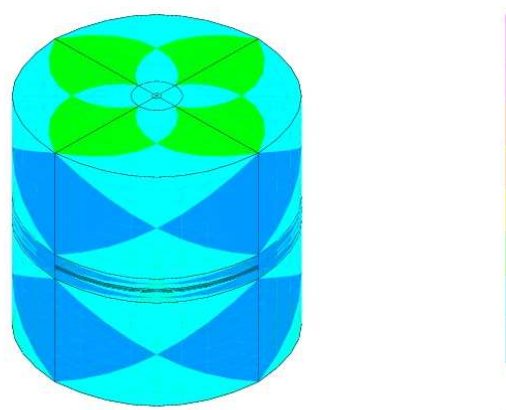


$\longrightarrow$ **Solve Direct and Dual Problems on Grid $h/2, p+1$**



Use the fine grid solution to estimate the coarse grid error function.
Apply the fully automatic goal-oriented hp-adaptive algorithm.

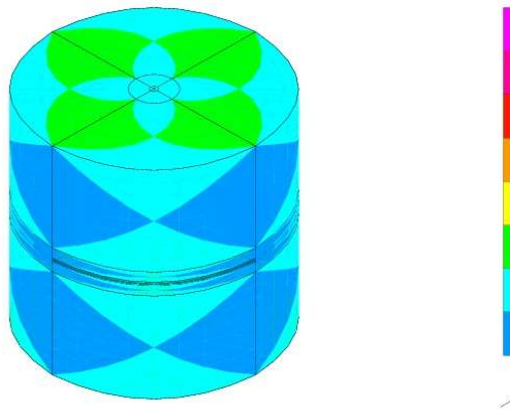
Next optimal hp-grid:



SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

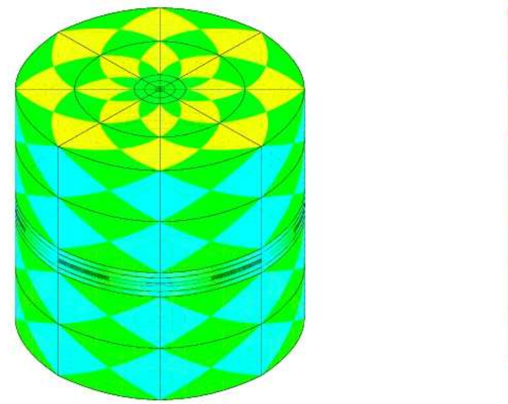
Algorithm for Goal-Oriented Adaptivity - STEP II -

Solve Direct and Dual Problems on Grid hp



→

Solve Direct and Dual Problems on Grid $h/2, p+1$



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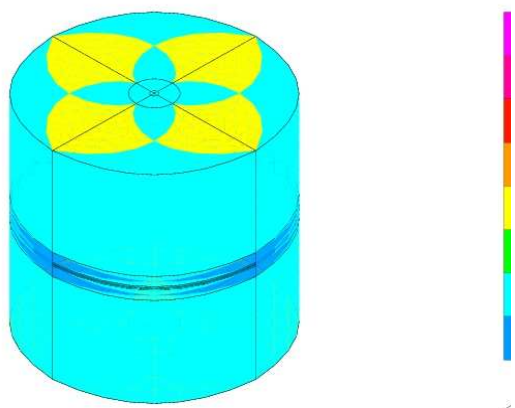
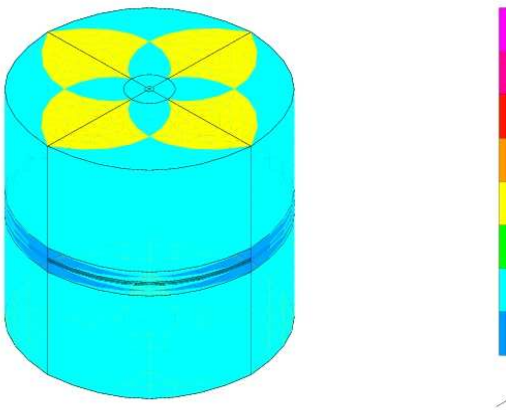


SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

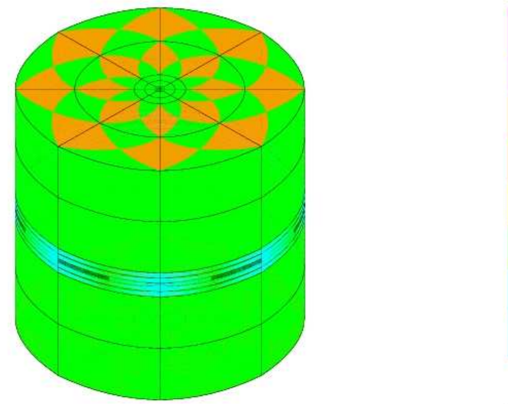
Algorithm for Goal-Oriented Adaptivity - STEP III -

Solve Direct and Dual Problems on Grid hp



$\longrightarrow$

Solve Direct and Dual Problems on Grid $h/2, p+1$



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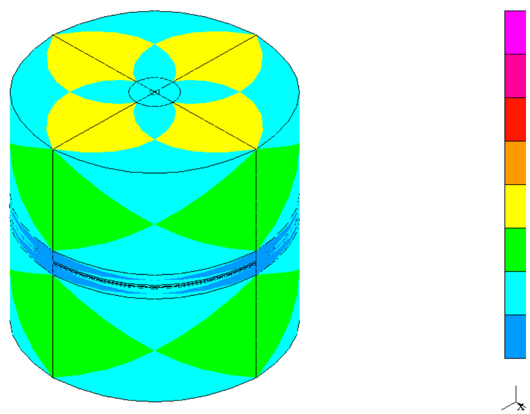
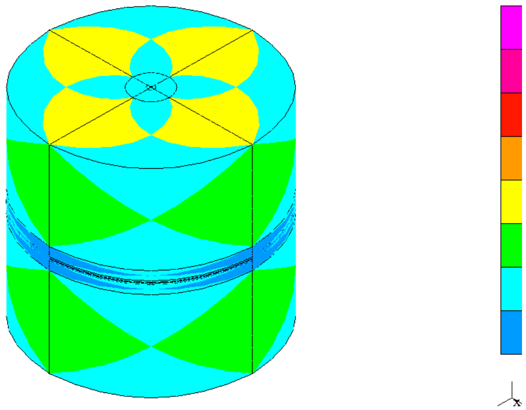
Next optimal hp-grid:



SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

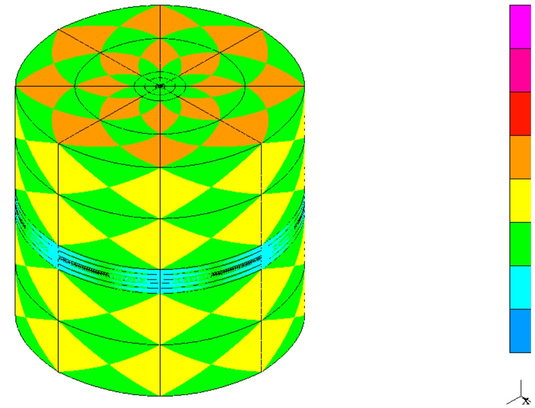
Algorithm for Goal-Oriented Adaptivity - STEP IV -

Solve Direct and Dual Problems on Grid hp



$\longrightarrow$

Solve Direct and Dual Problems on Grid $h/2, p+1$



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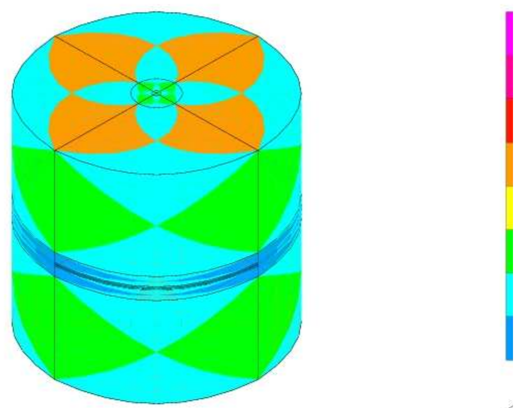
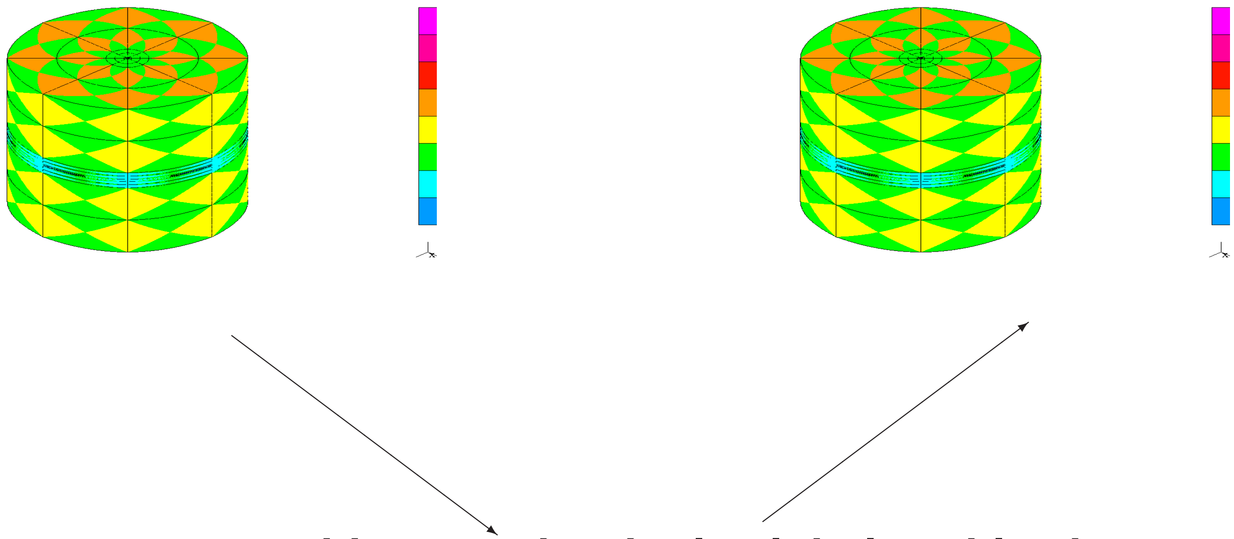


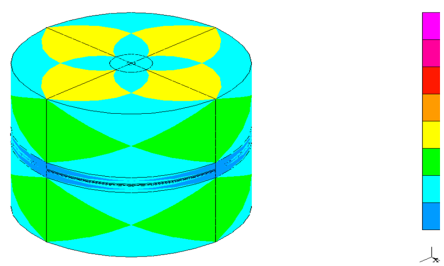
GOAL-ORIENTED TWO-GRID SOLVER

Multigrid (two-grid) Solver (Ax=b)

Fine Grid Smoothing (Sol. Local Problems) **Fine Grid Smoothing (Sol. Local Problems)**



Coarse Grid Correction (Sol. Global Problem)



V-cycle

GOAL-ORIENTED TWO-GRID SOLVER

Challenges (Iterative Solver)

The use of goal-oriented adaptivity. A new strategy for selecting the optimal relaxation parameter has been implemented. This strategy minimizes the error in the quantity of interest rather than in the energy-norm.

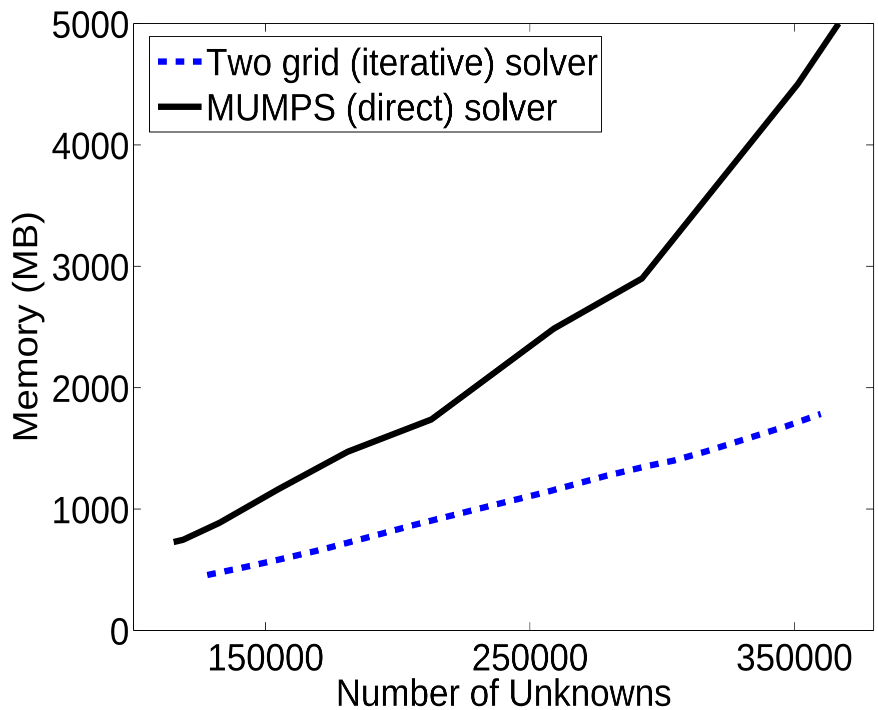
The presence of (arbitrary) elongated elements. An additional edge-based (global) smoother has been implemented. This additional smoother makes the convergence of the iterative solver independent of the aspect ratio of the elements.

Convergence theory for elongated elements. Under development.

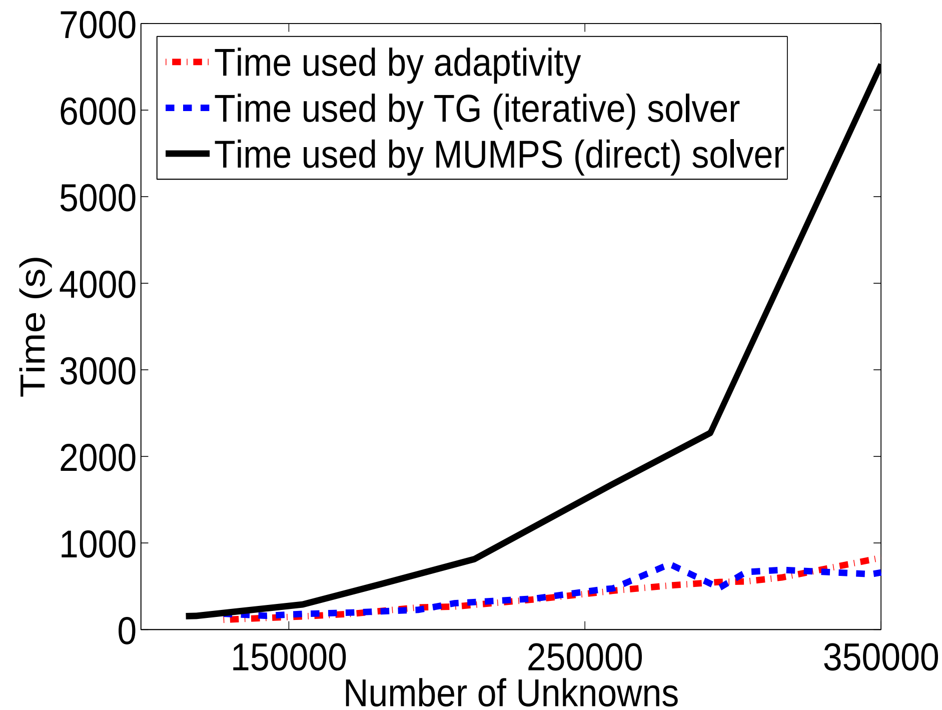
GOAL-ORIENTED TWO-GRID SOLVER

Axisymmetric Model Problem (solved in 3D)

MEMORY



TIME

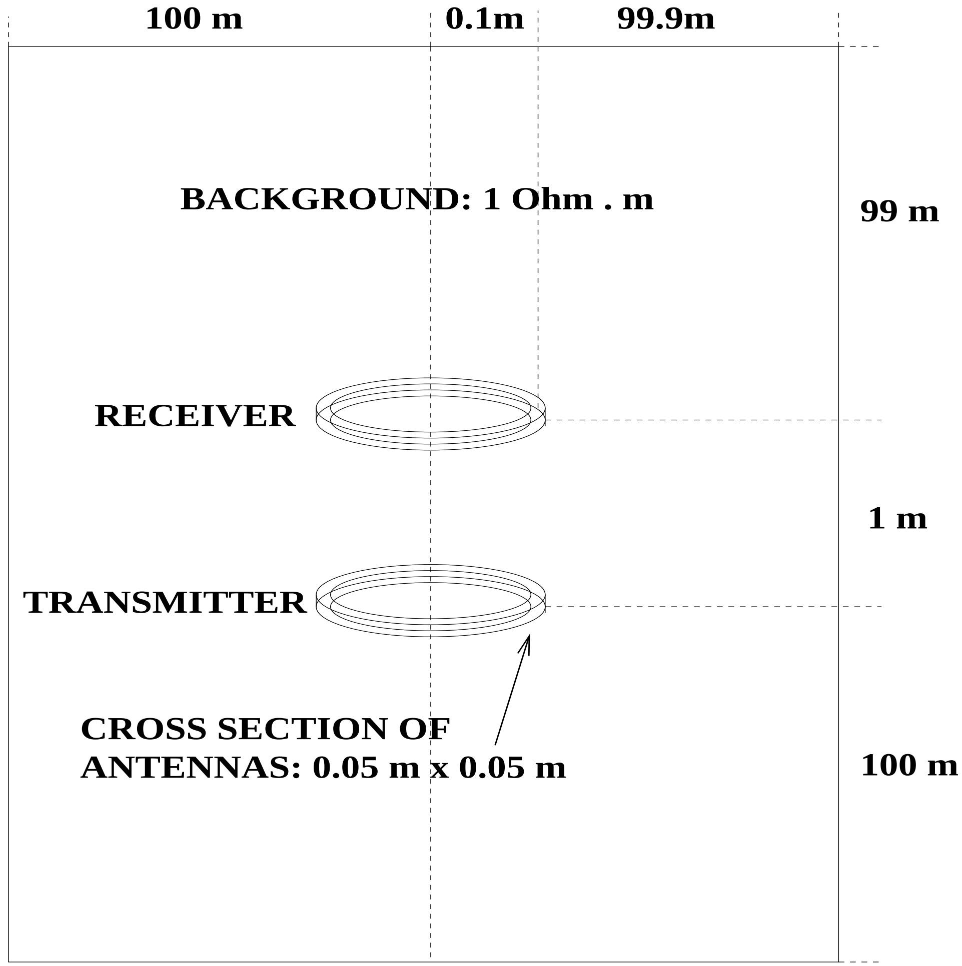


1.2 Ghz processor

Iterative solvers are needed for simulation of 3D resistivity logging applications

LATEROLOG INSTRUMENTS

Electrode Problem



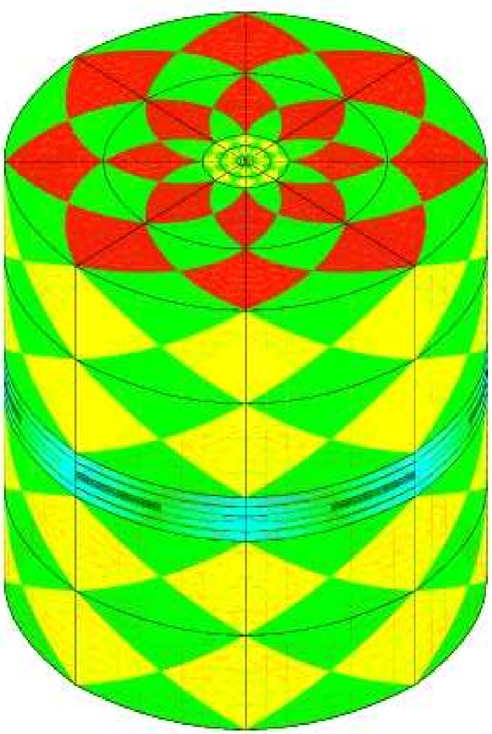
Loop antenna in a homogeneous media at DC.

Computational domain:
100 m x 100 m.

LATEROLOG INSTRUMENTS

Electrode Problem

Final hp-grid



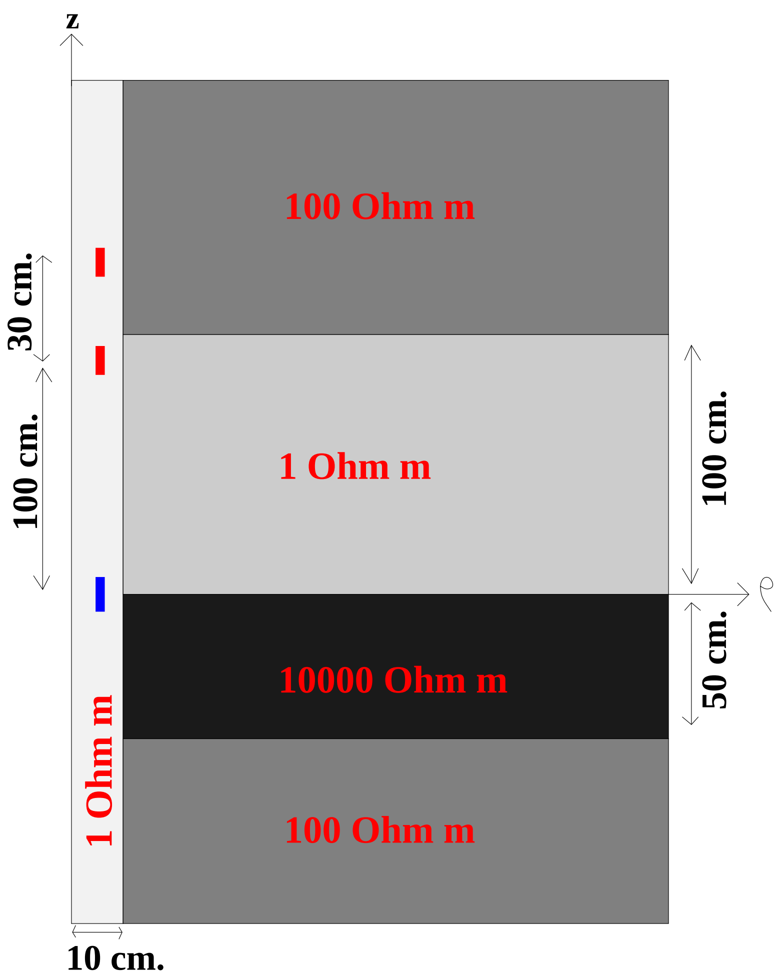
Final solution

2D Solution: 0.078131

3D Solution: 0.078121

LATEROLOG INSTRUMENTS

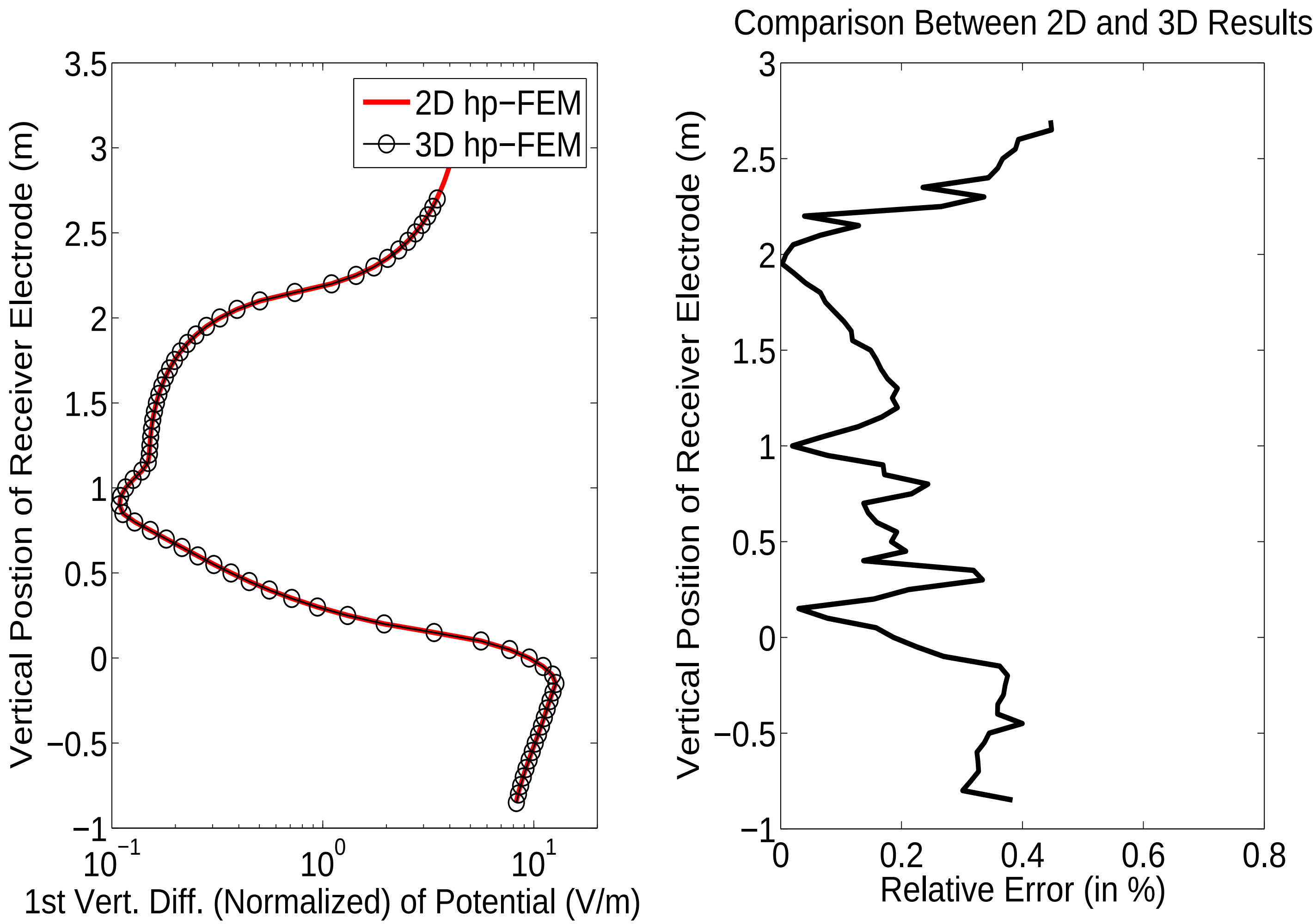
Axisymmetric Model Problem



- **Borehole and four materials on the formation.**
- **Size of computational domain: 100 m $\times$ 100 m.**
- **Size of electrode: 0.05 m $\times$ 0.05 m.**
- **Objective: Compute First Vertical Difference of Potential.**

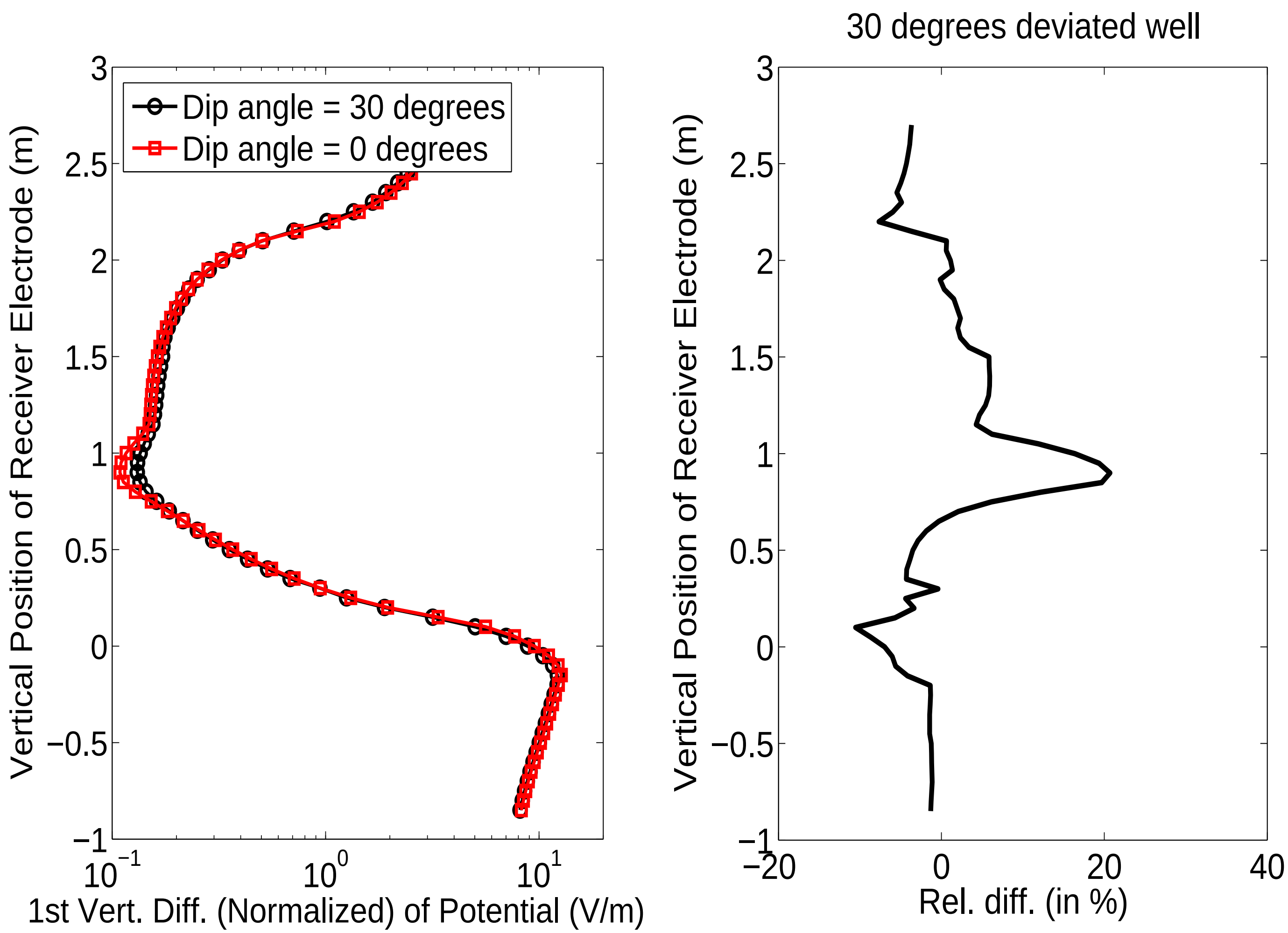
LATEROLOG INSTRUMENTS

Axisymmetric Model Problem



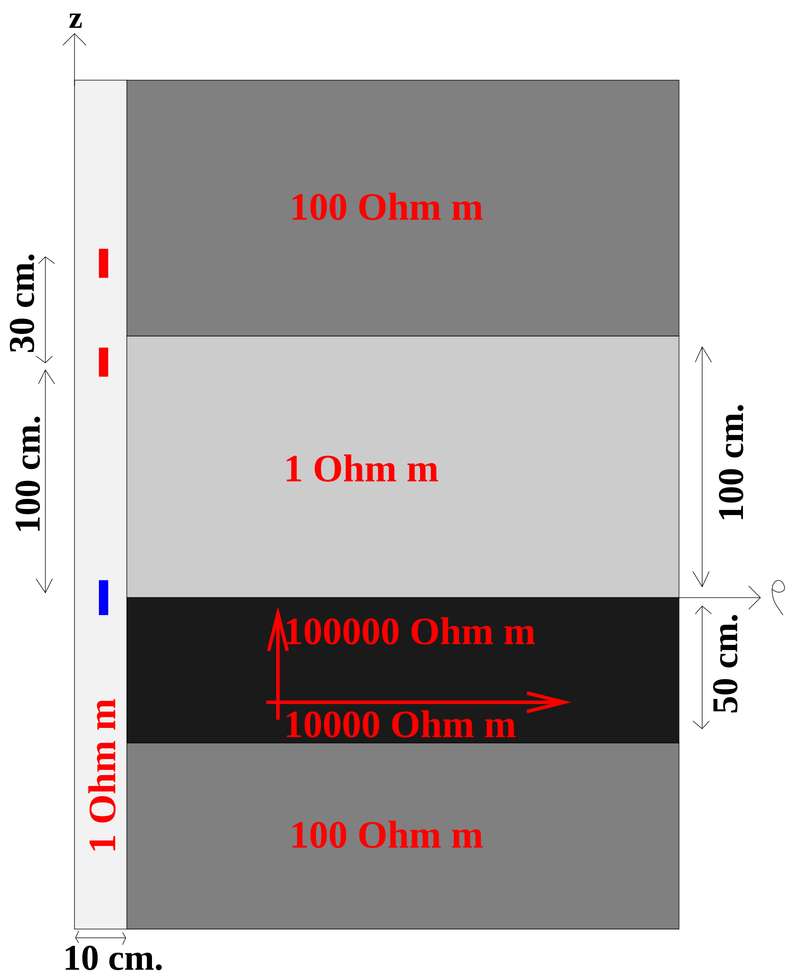
LATEROLOG INSTRUMENTS

Model Problem: Deviated Well



LATEROLOG INSTRUMENTS

Axisymmetric Model Problem



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LATEROLOG INSTRUMENTS

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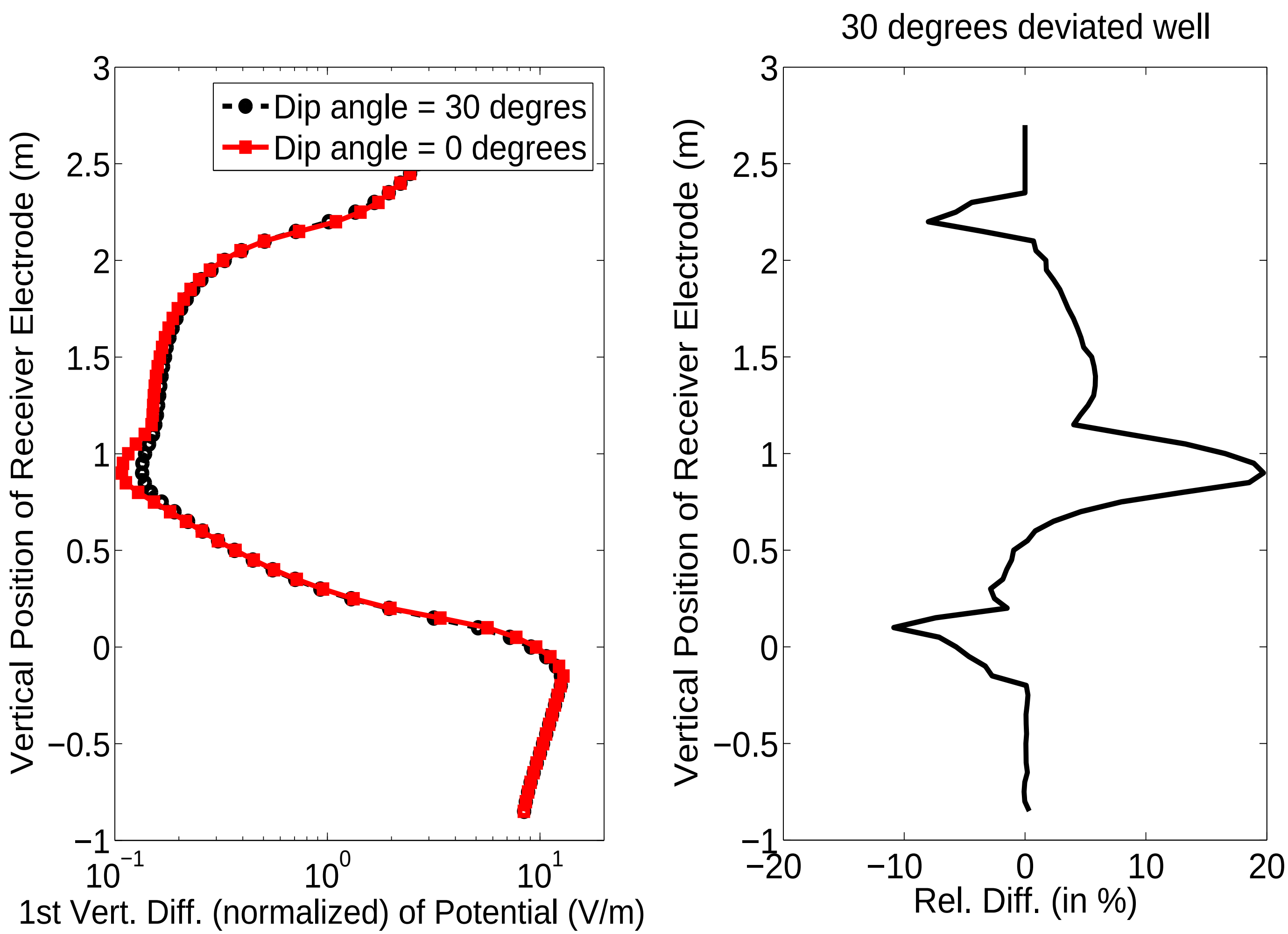
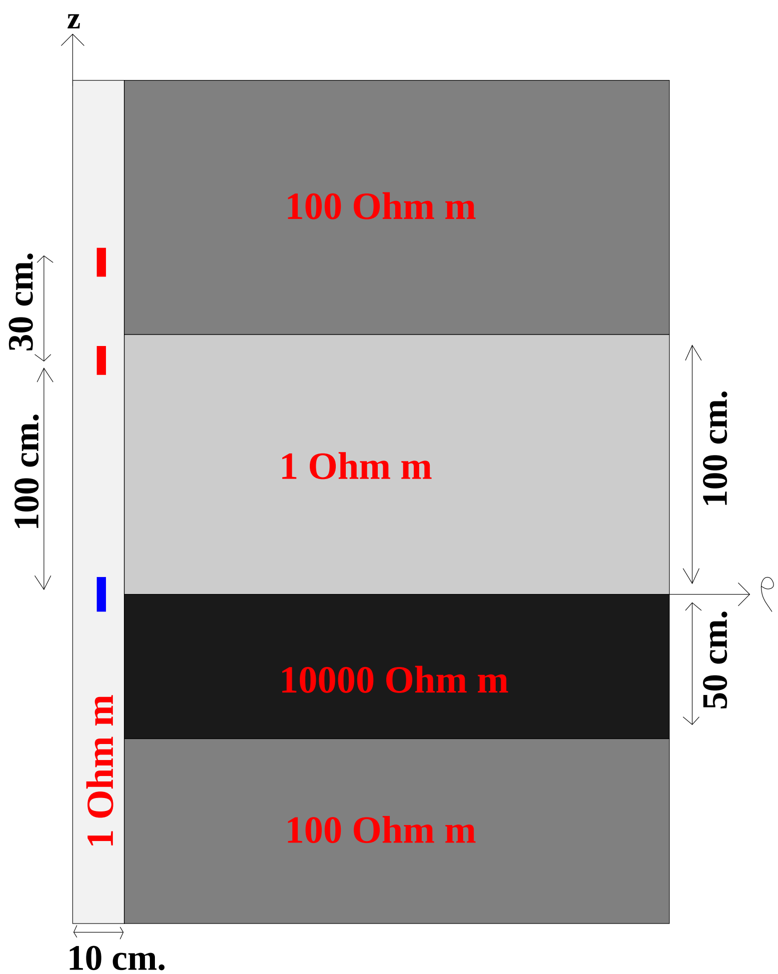
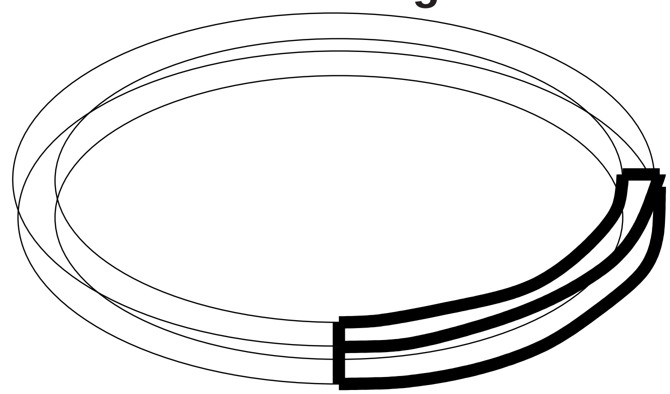


LATEROLOG INSTRUMENTS

Axisymmetric Model Problem

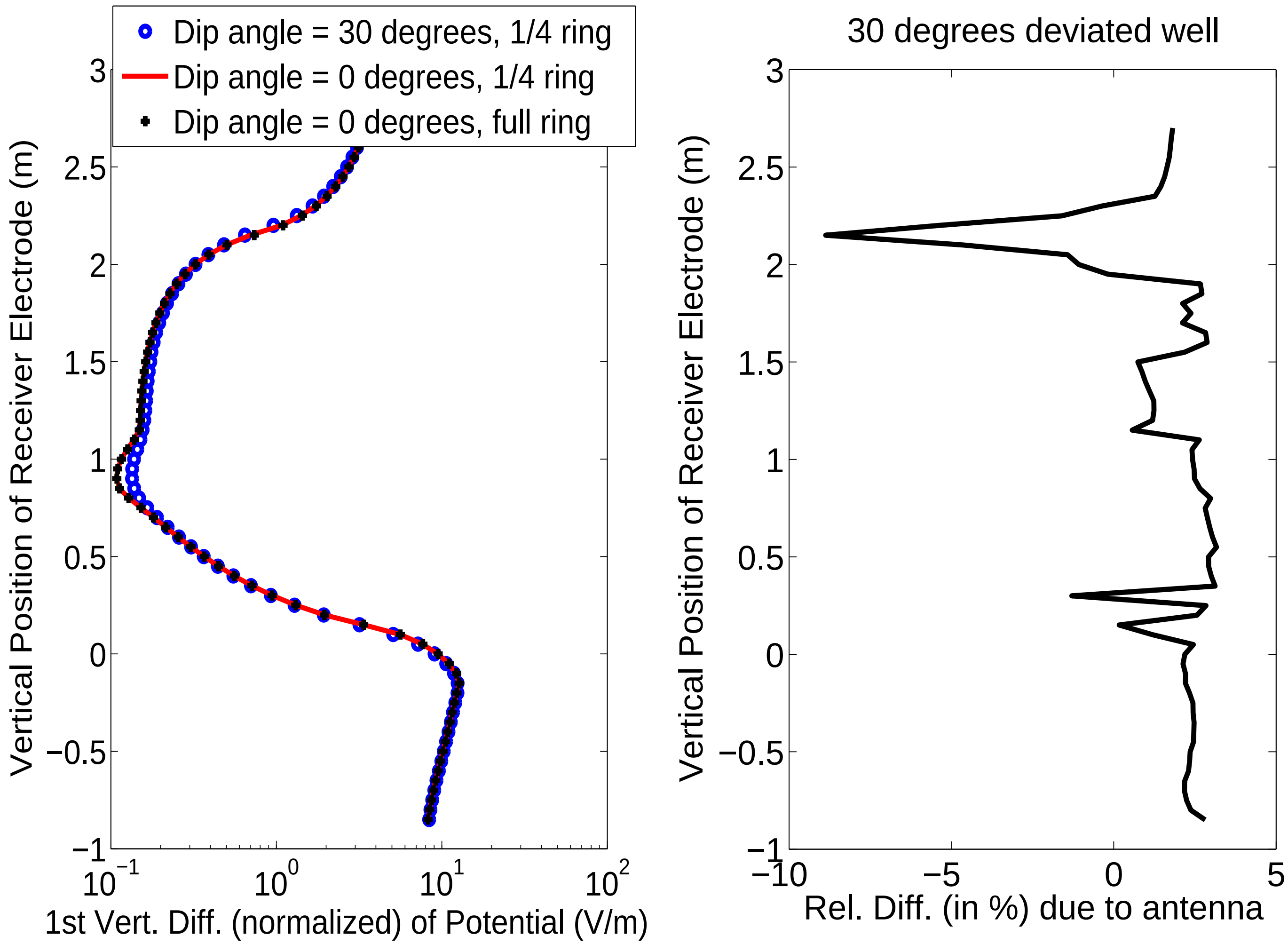


Transmitter Electrode: One fourth of a ring.



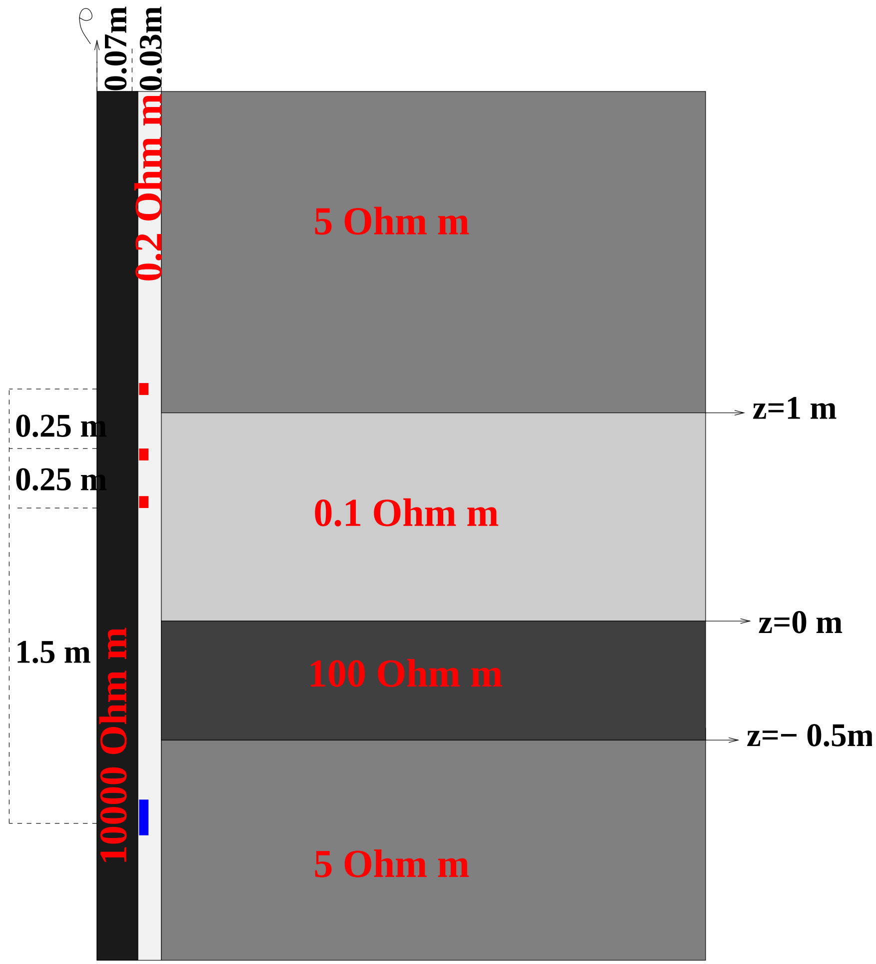
LATEROLOG INSTRUMENTS

Model Problem: Deviated Well



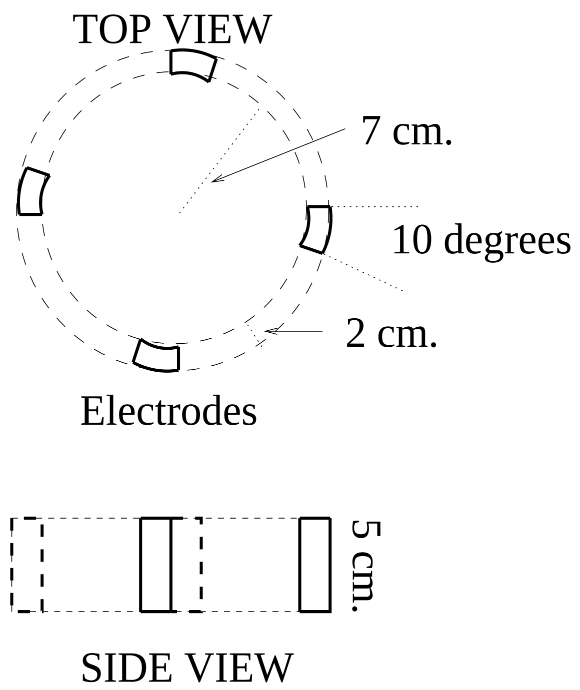
LATEROLOG INSTRUMENTS

Objective: Compute 2nd Diff. of Potential

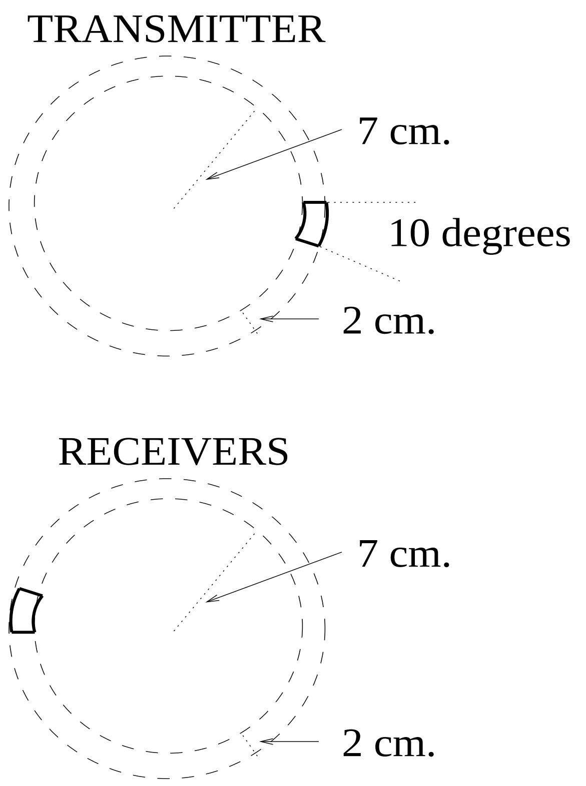


LATEROLOG INSTRUMENTS

PATCH ELECTRODE

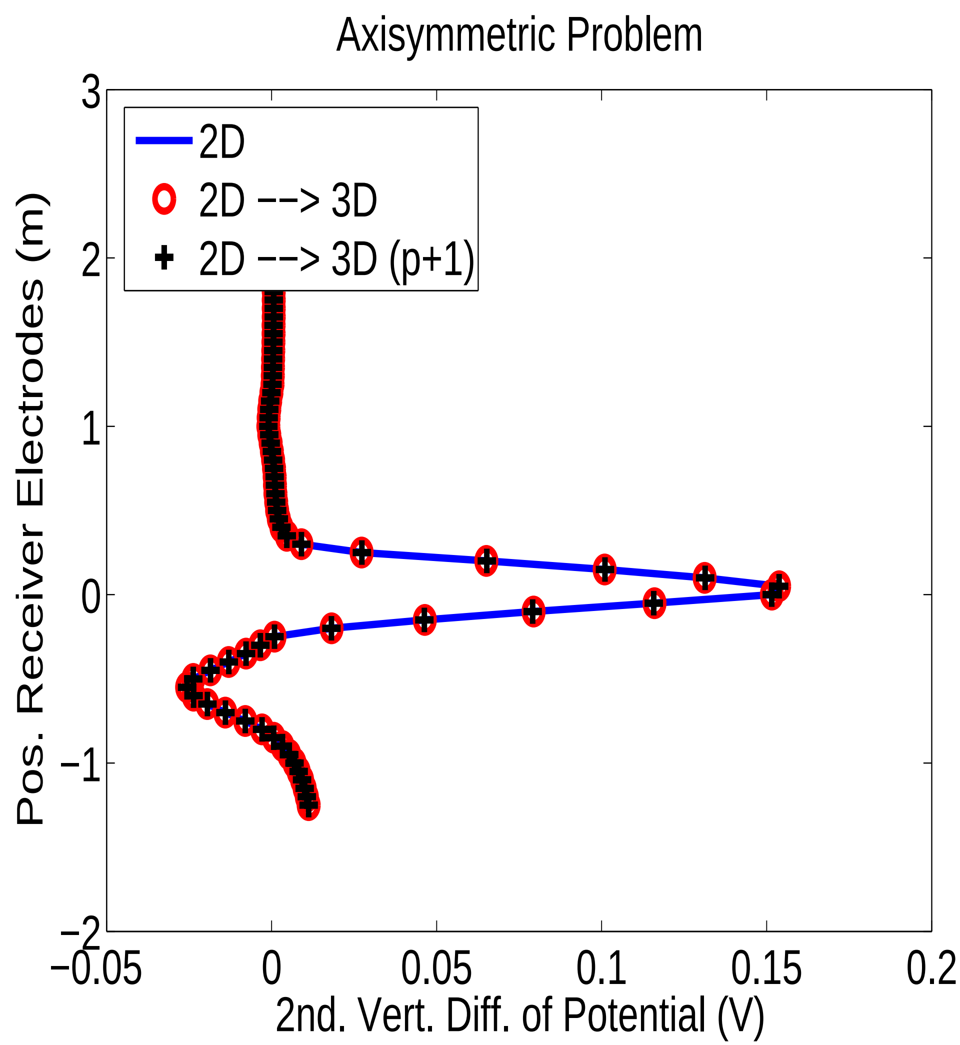


ONE PATCH



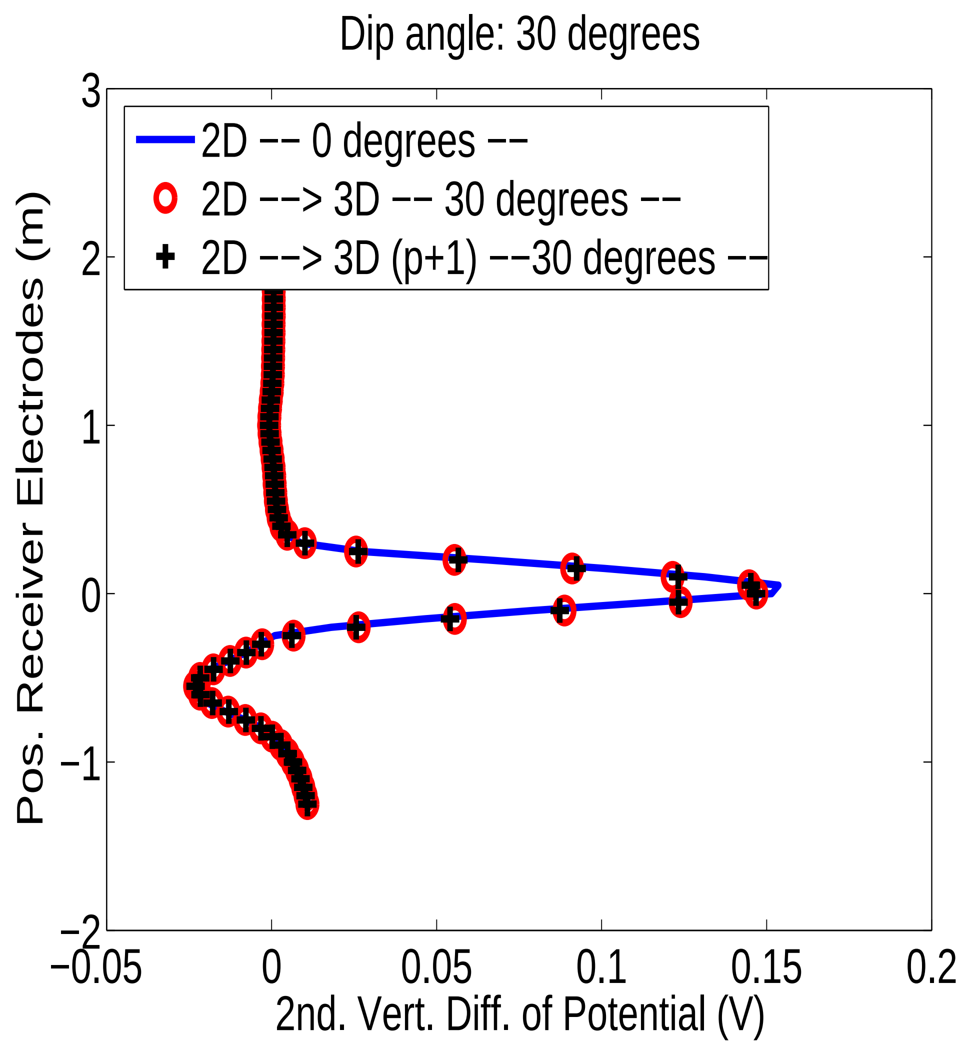
LATEROLOG INSTRUMENTS

Axisymmetric problem



LATEROLOG INSTRUMENTS

Deviated Well



LATEROLOG INSTRUMENTS

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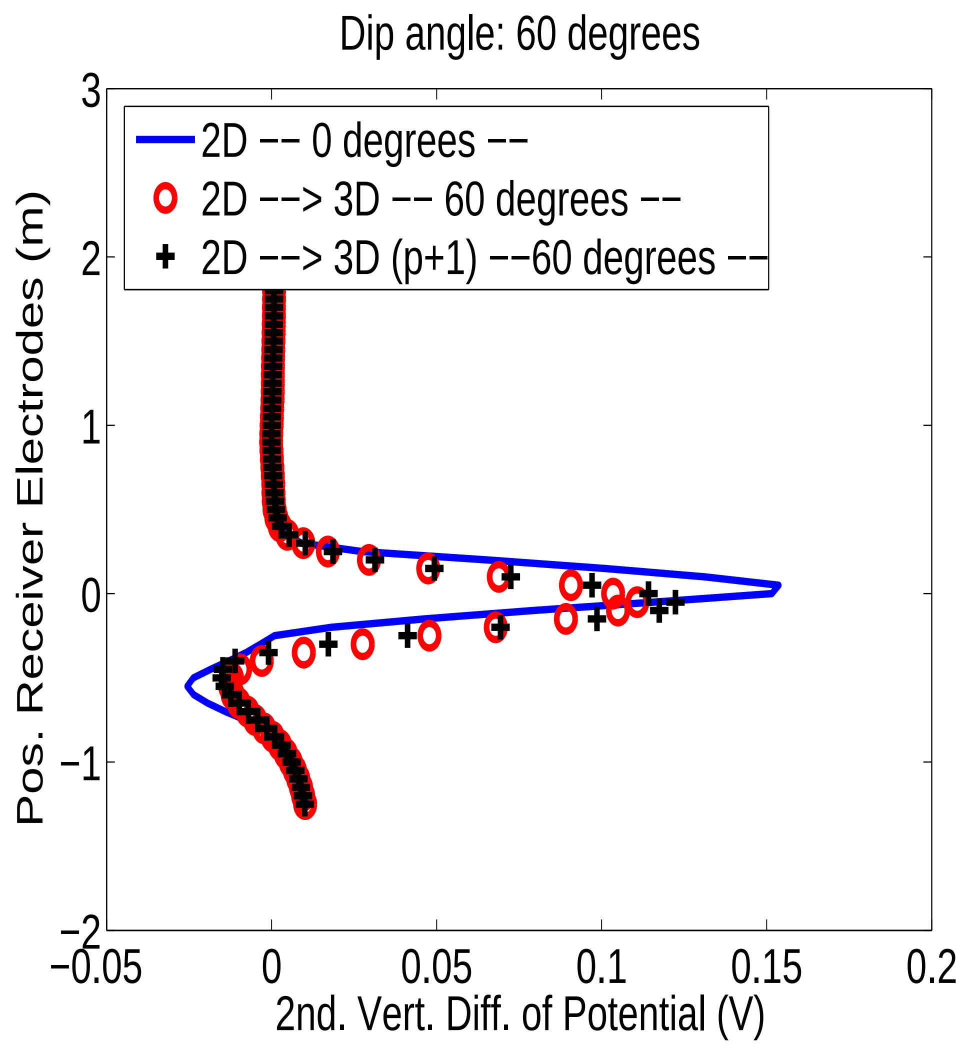
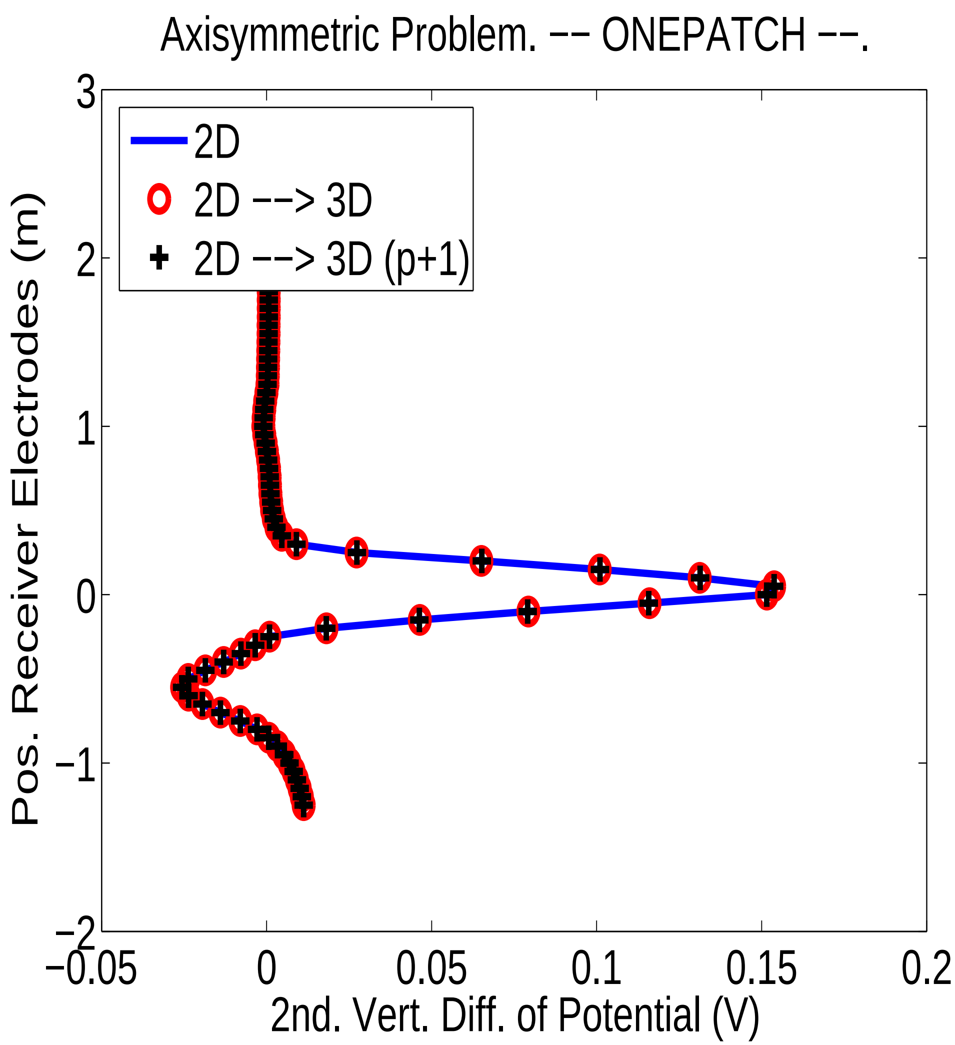


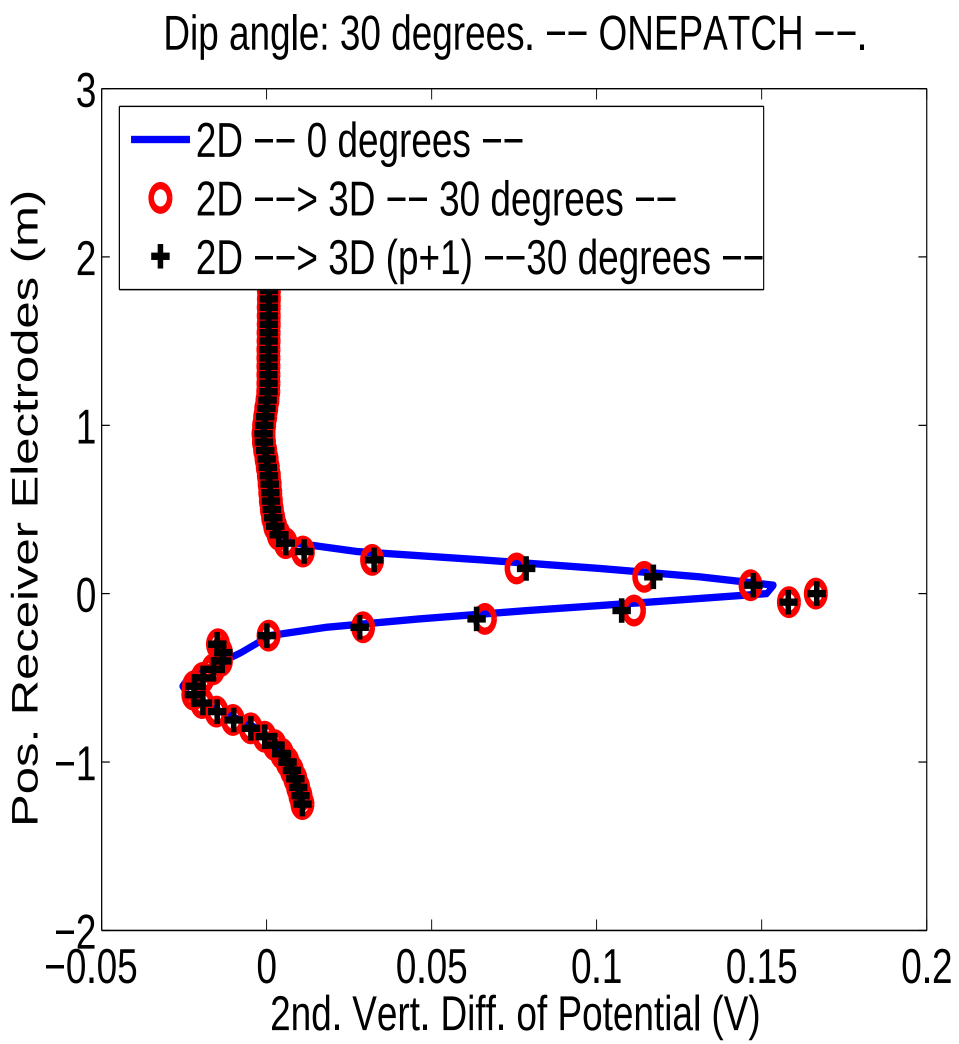
LATEROLOG INSTRUMENTS

Axisymmetric Problem



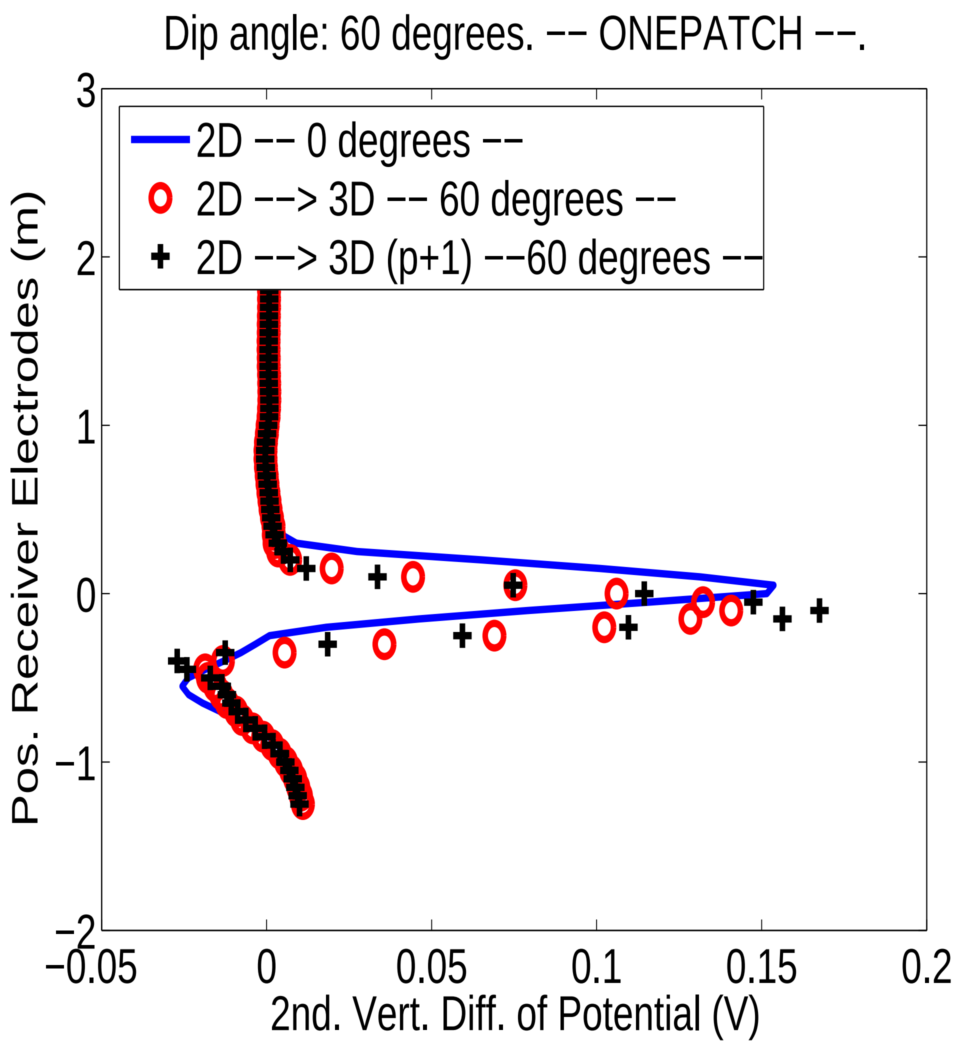
LATEROLOG INSTRUMENTS

Deviated Well



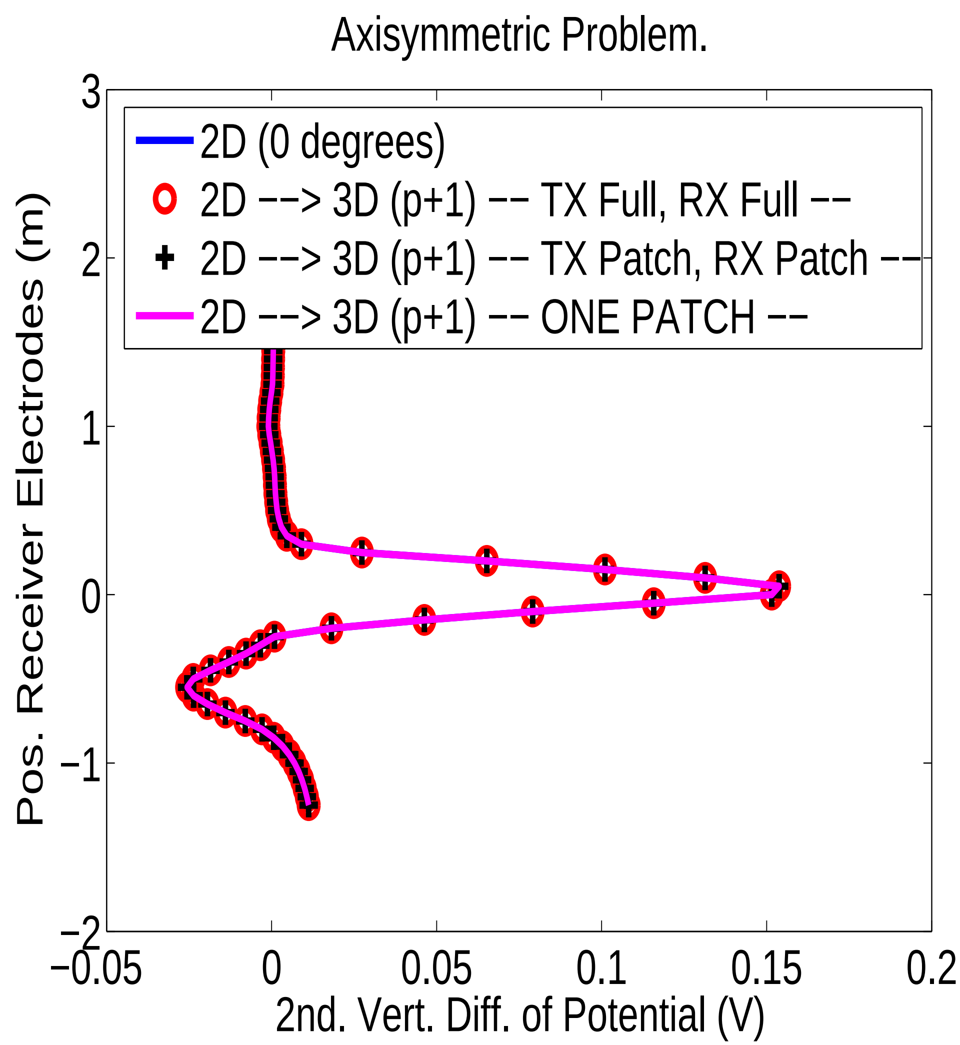
LATEROLOG INSTRUMENTS

Deviated Well



LATEROLOG INSTRUMENTS

Axisymmetric Problem



LATEROLOG INSTRUMENTS

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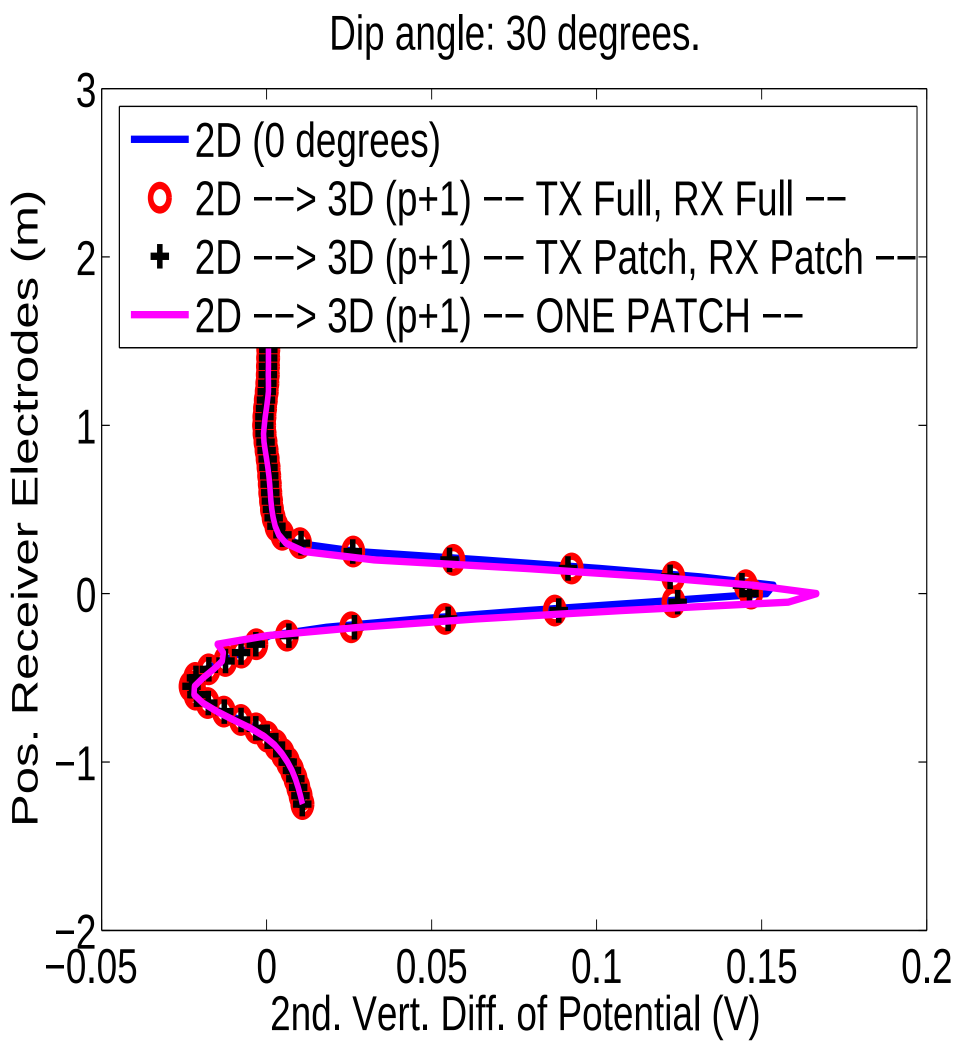

LATEROLOG INSTRUMENTS

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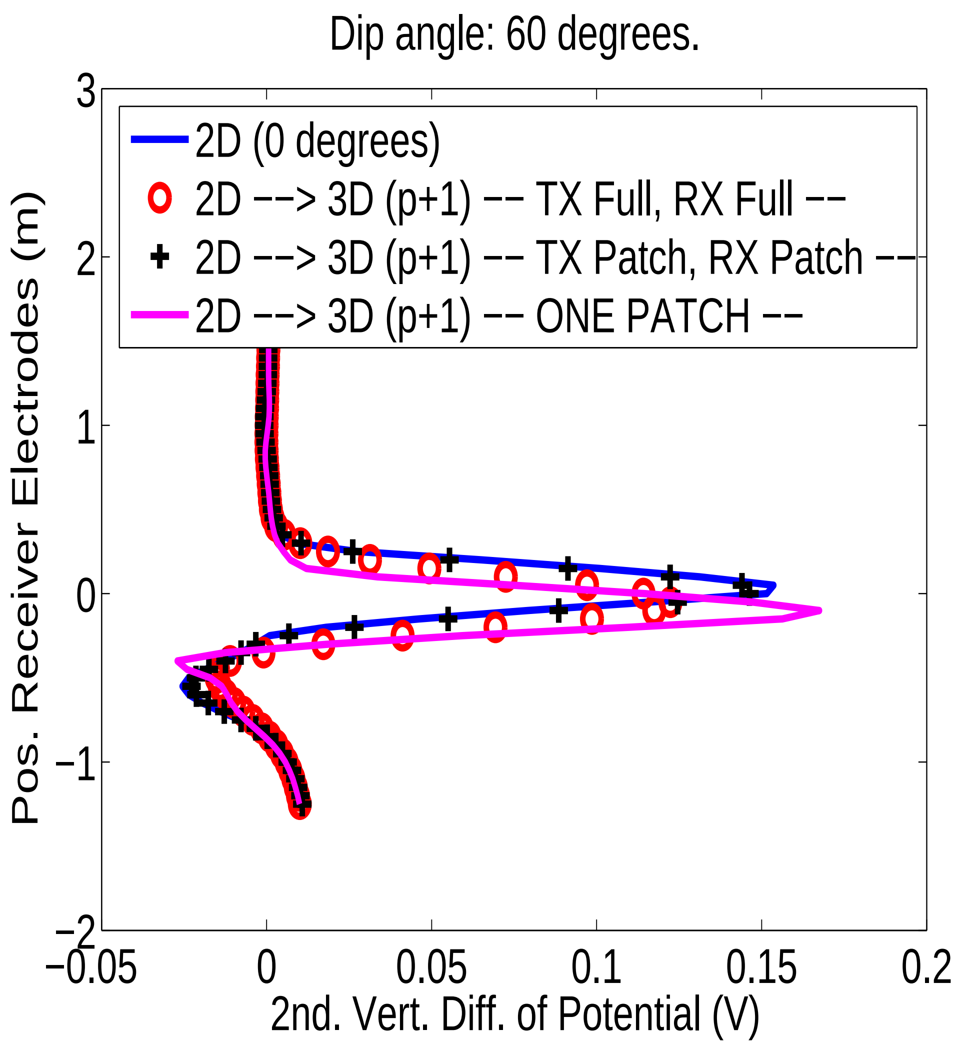
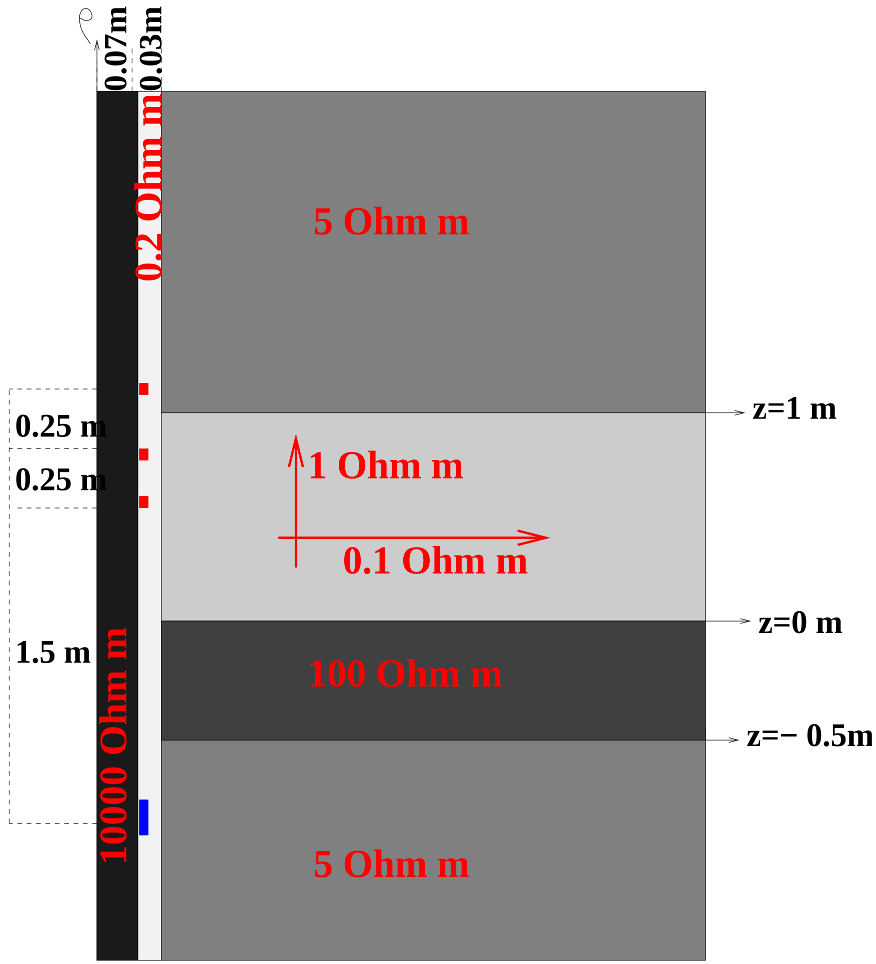


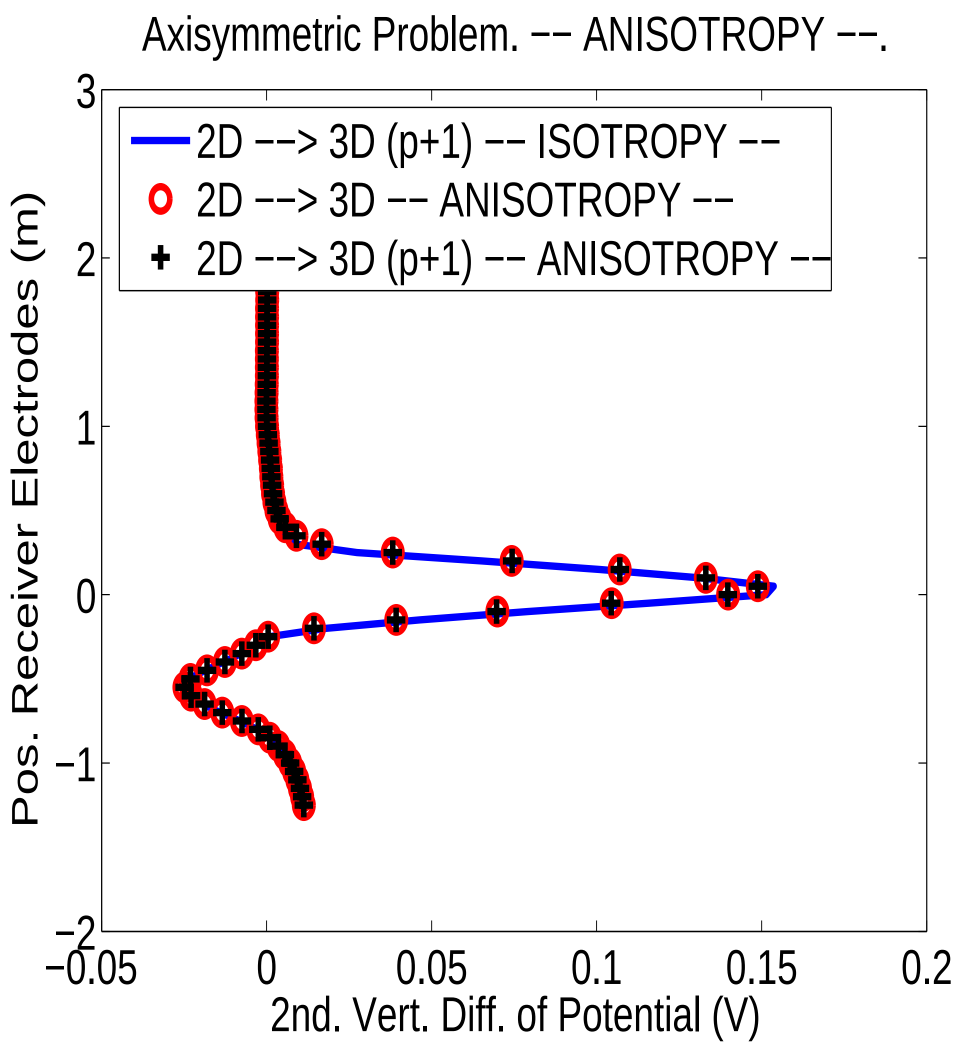
LATEROLOG INSTRUMENTS

Anisotropy in Deviated Wells



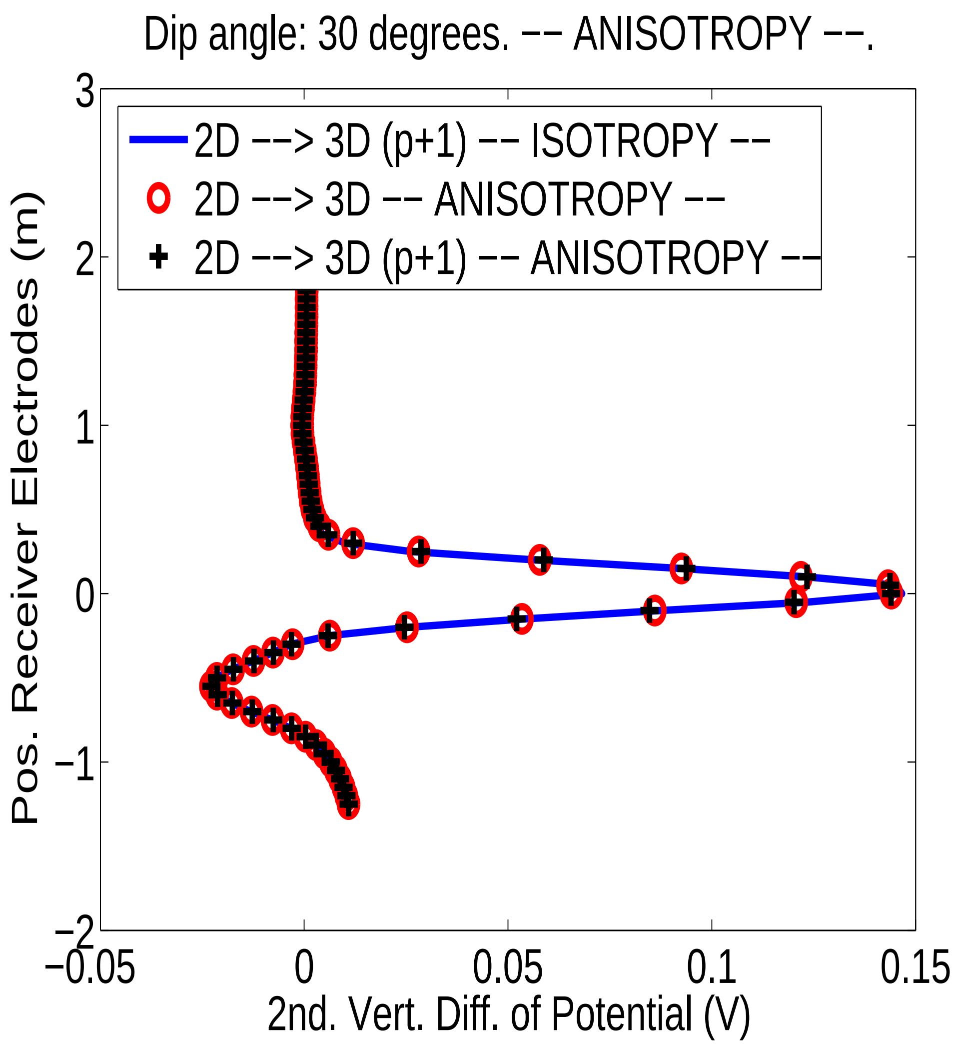
LATEROLOG INSTRUMENTS

Axisymmetric problem



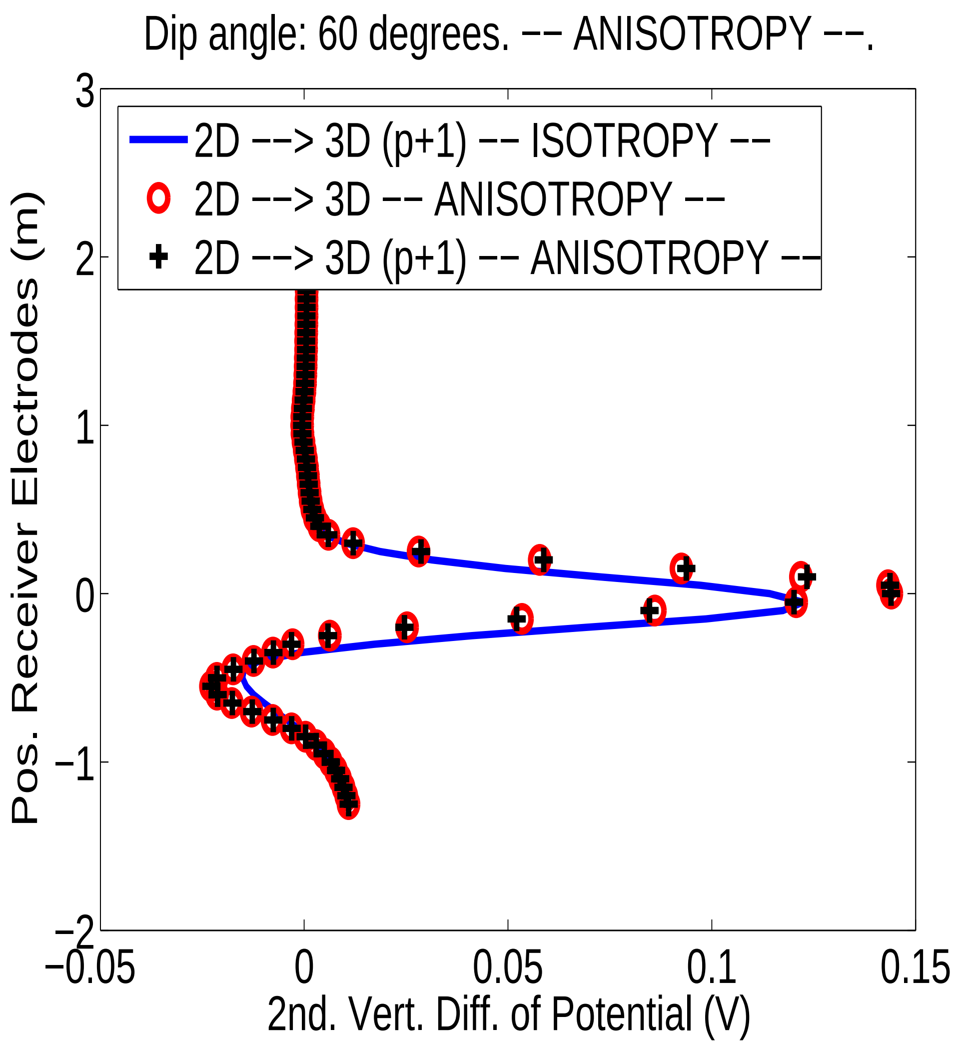
LATEROLOG INSTRUMENTS

Deviated Well



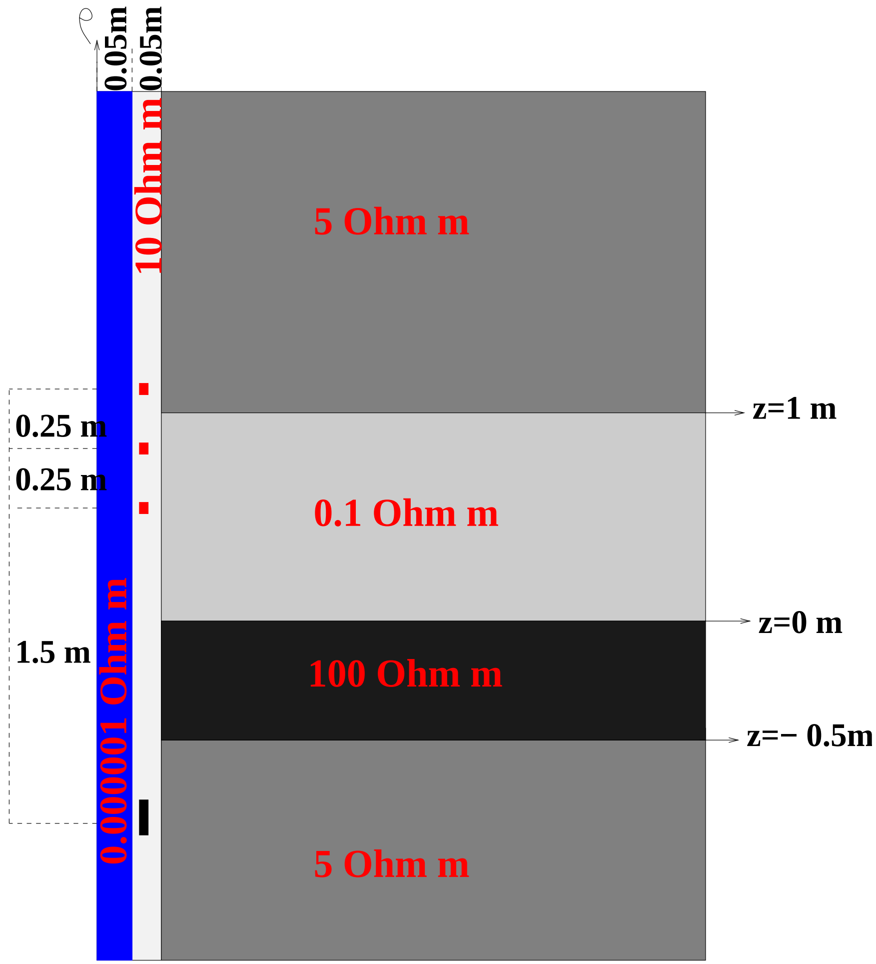
LATEROLOG INSTRUMENTS

Deviated Well



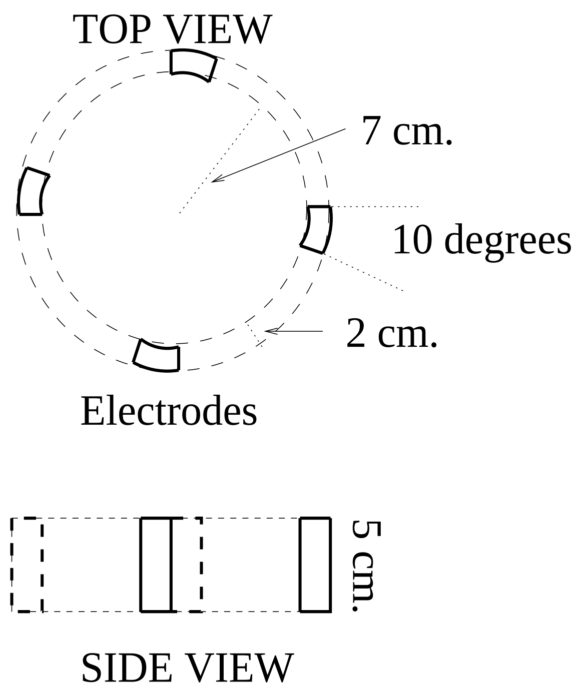
LWD (at DC)

Objective: Compute 2nd Diff. of Potential

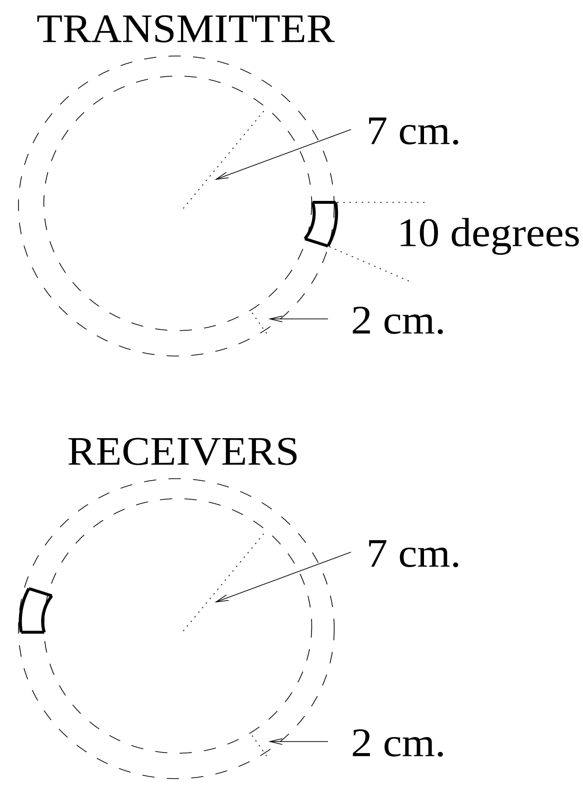


LWD (at DC)

PATCH ELECTRODE

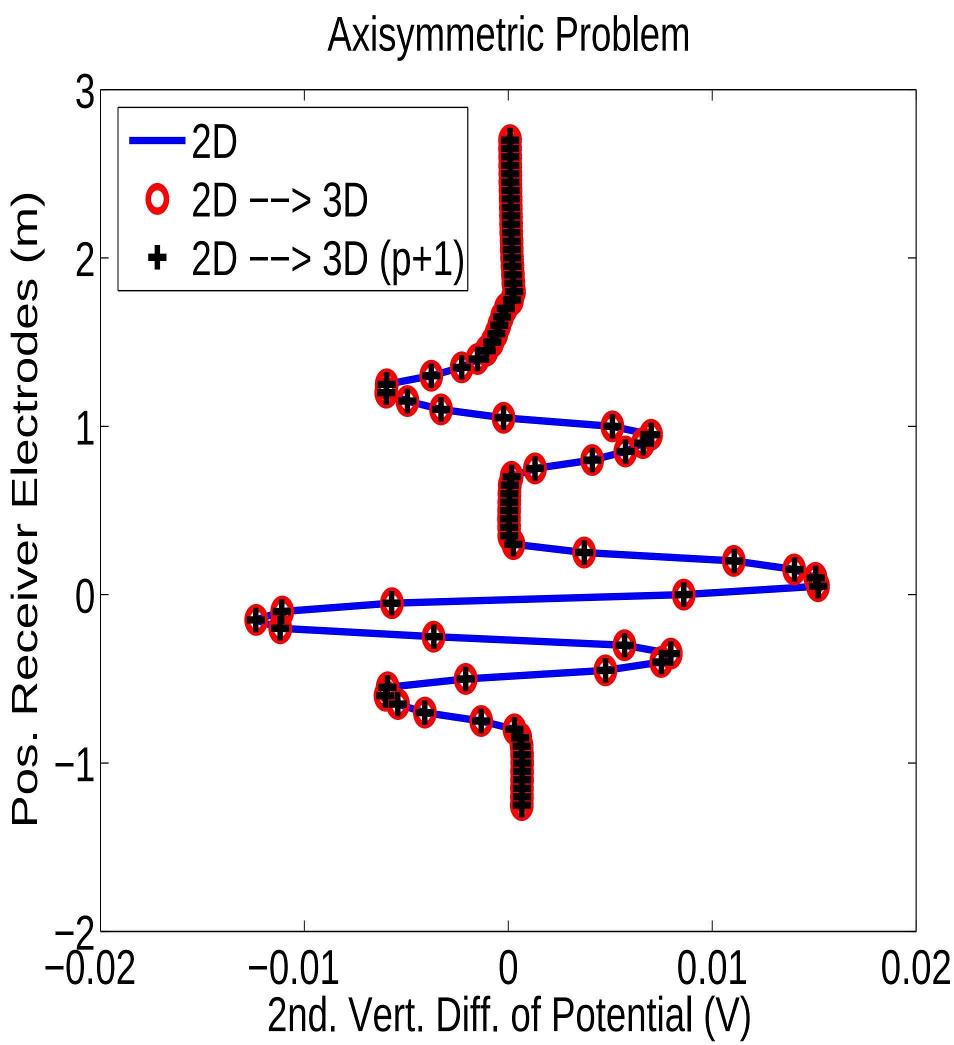


ONE PATCH



LWD (at DC)

Axisymmetric problem



LWD (at DC)

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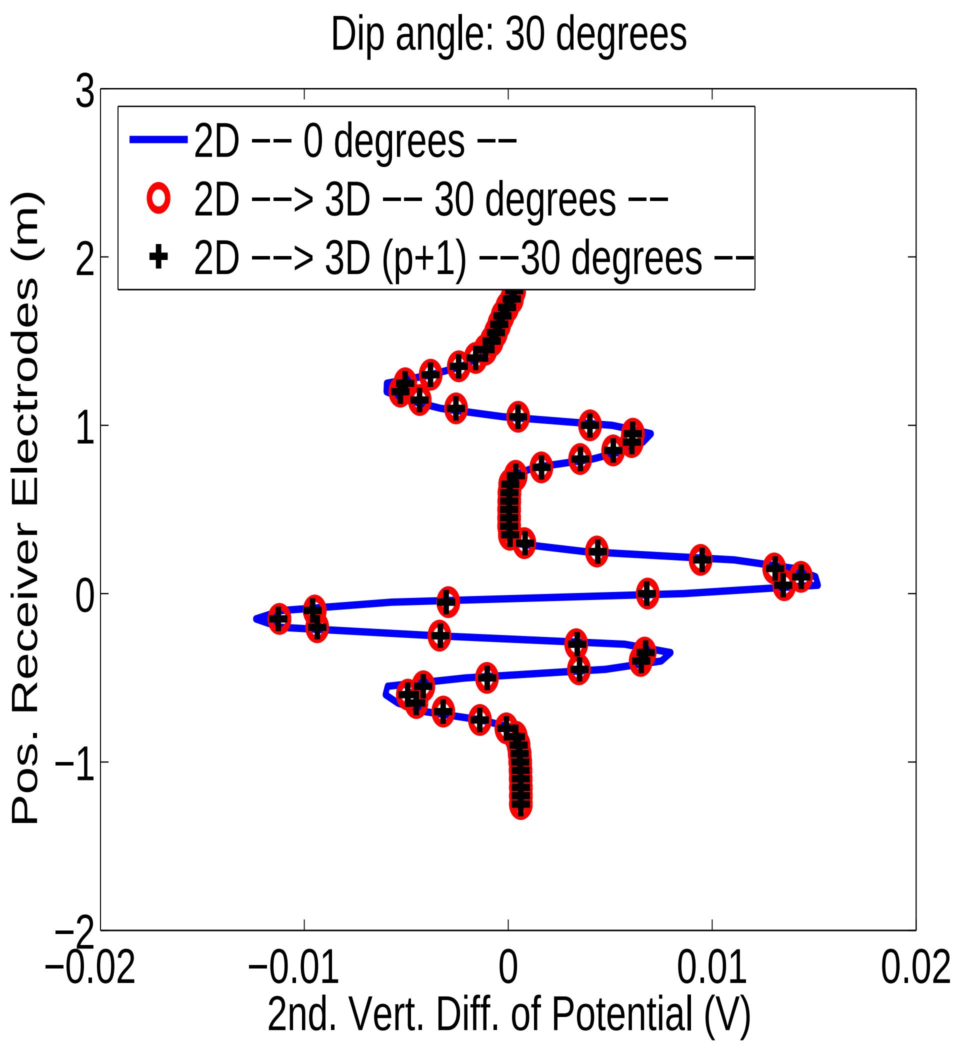

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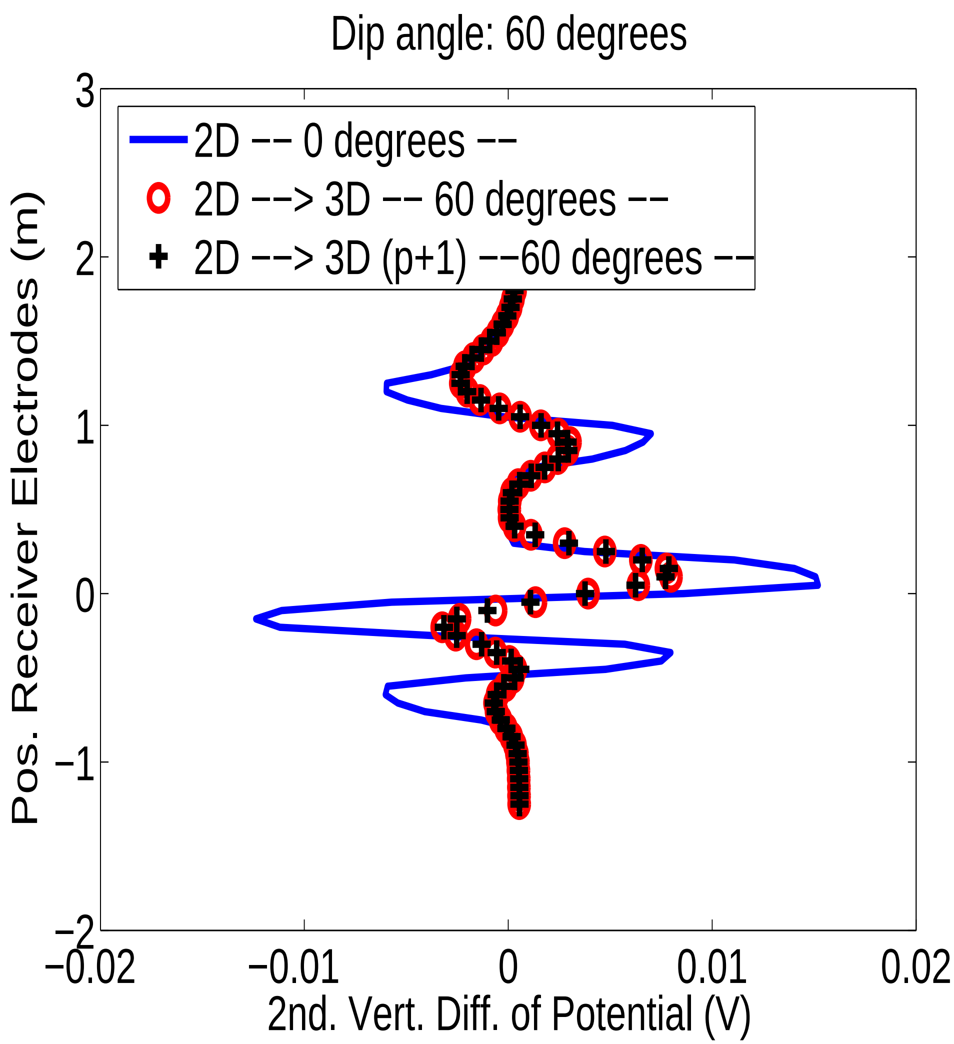

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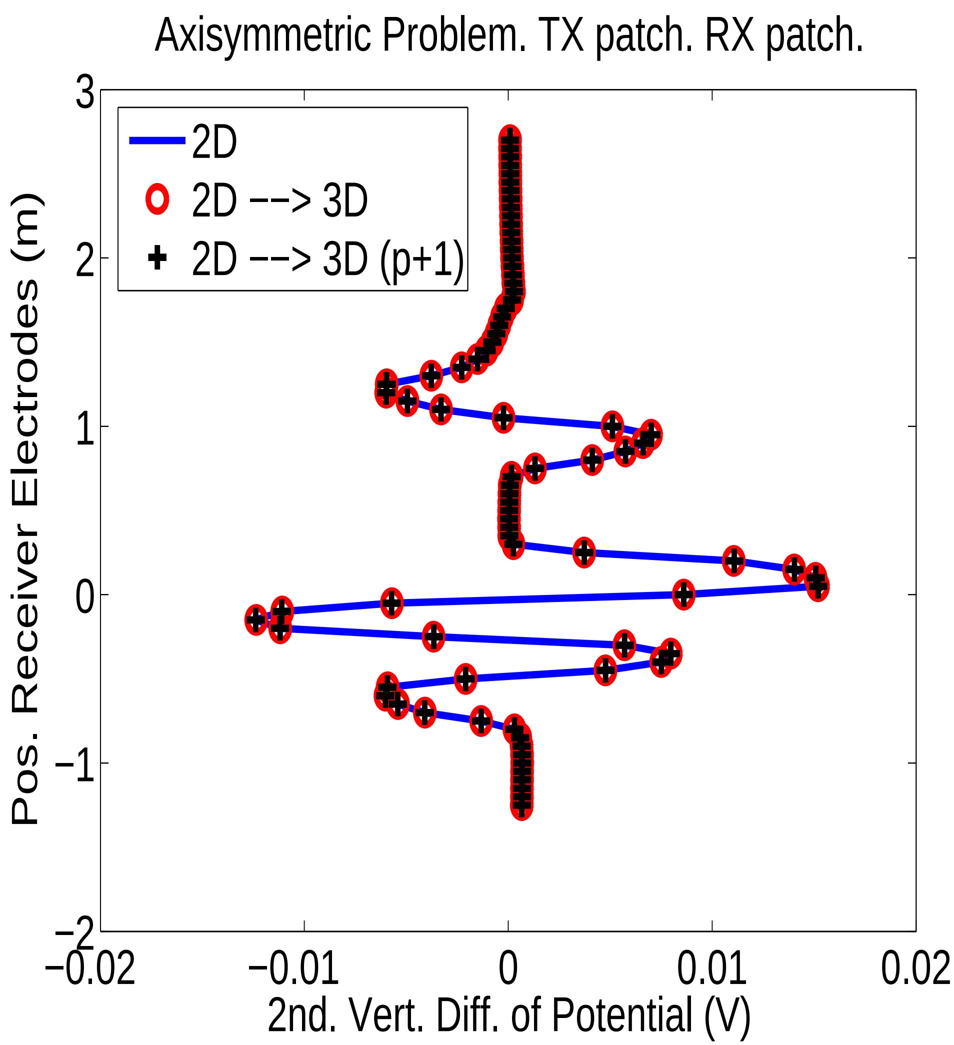

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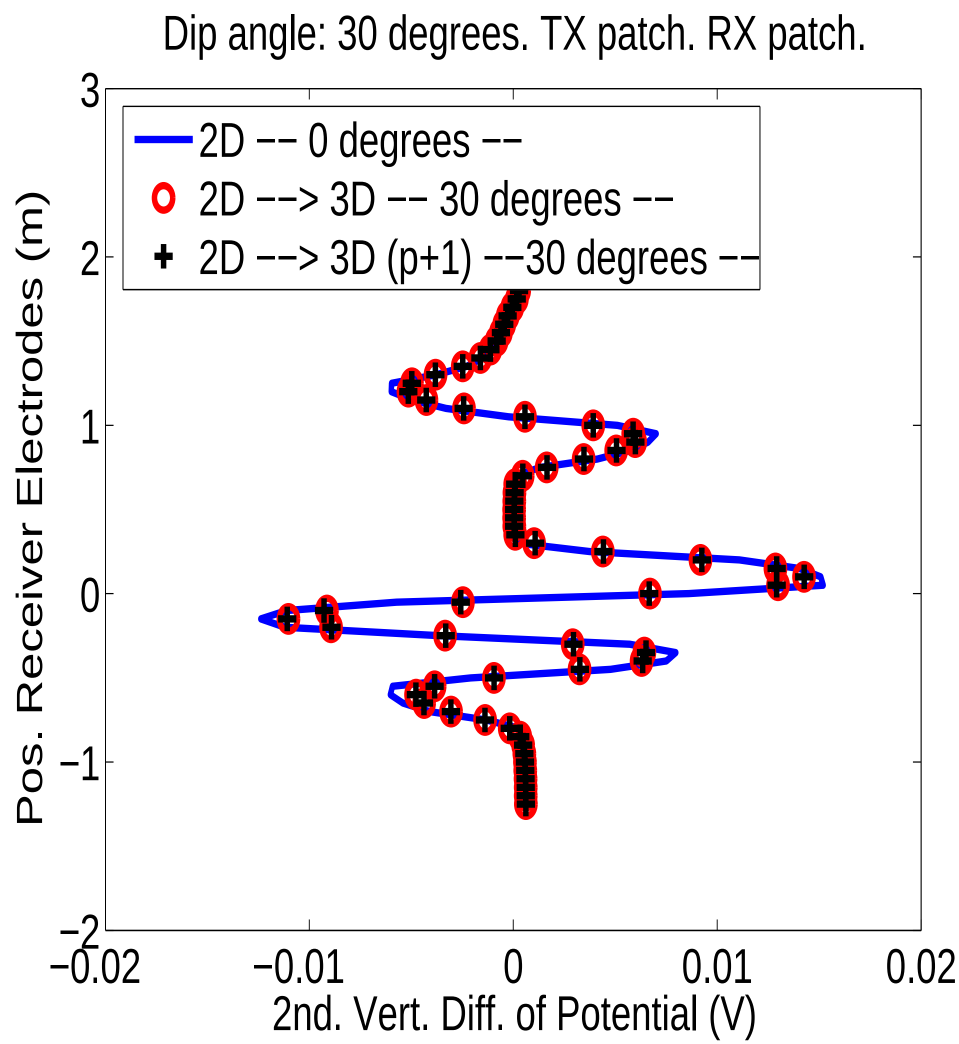

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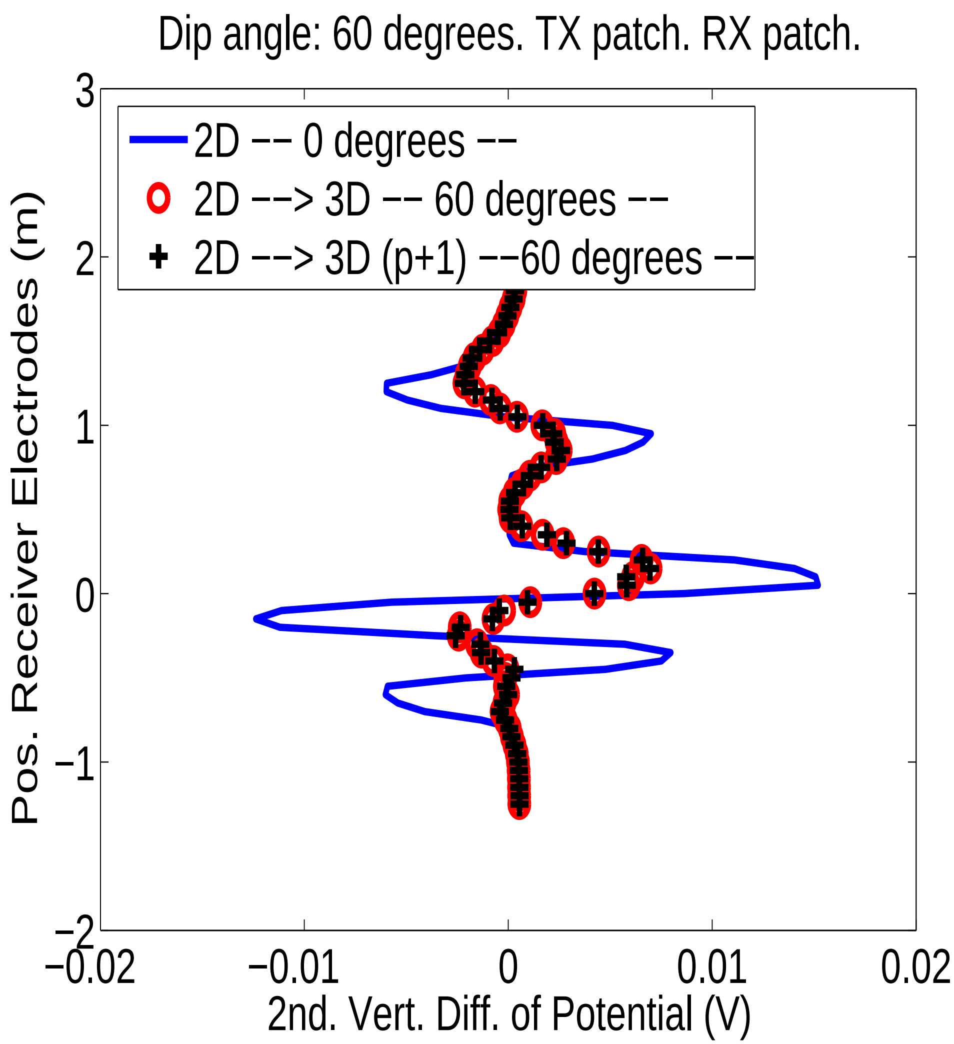

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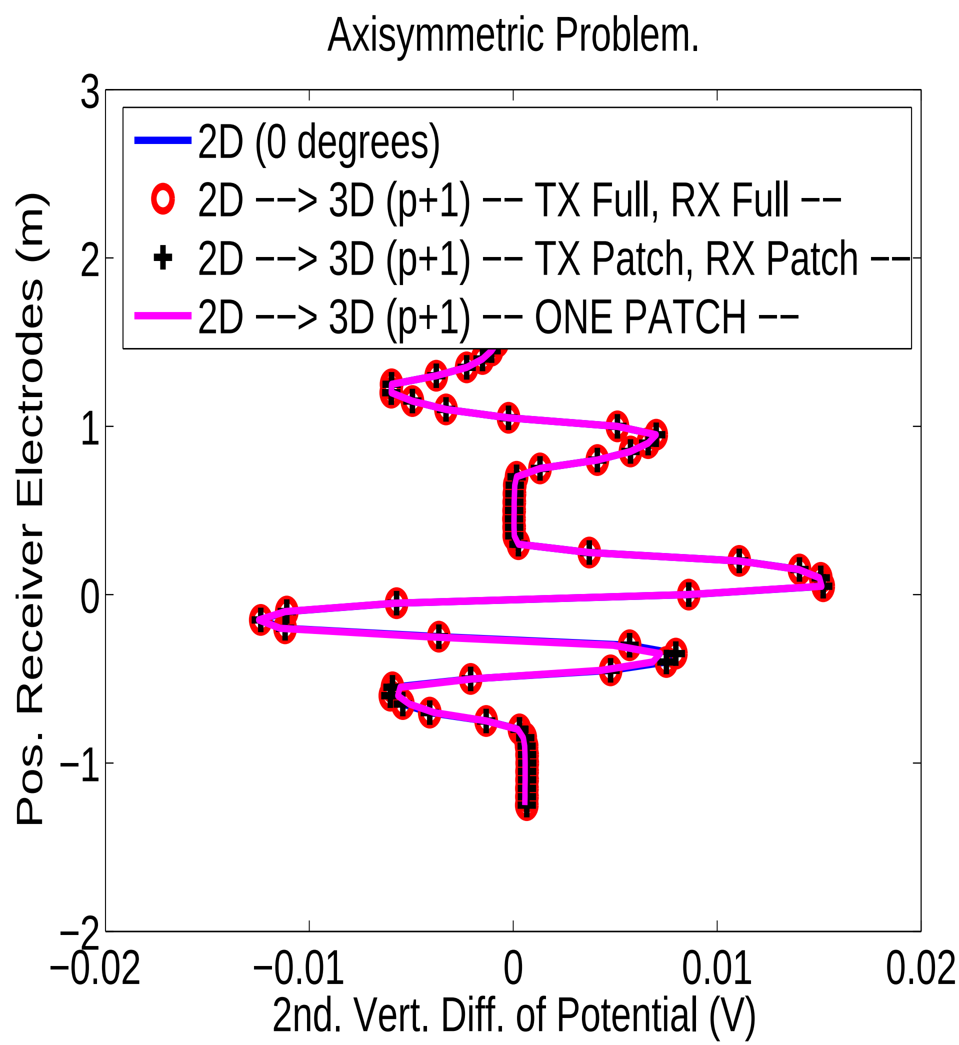

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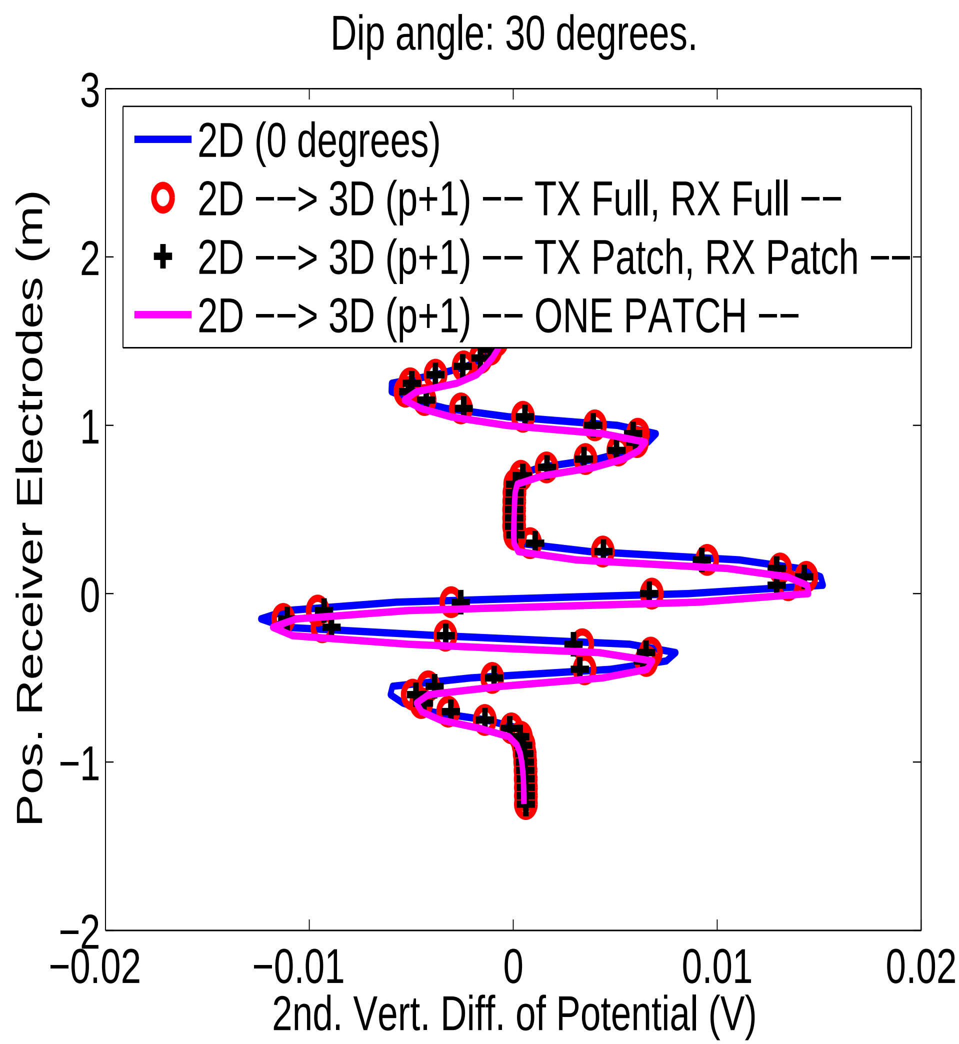
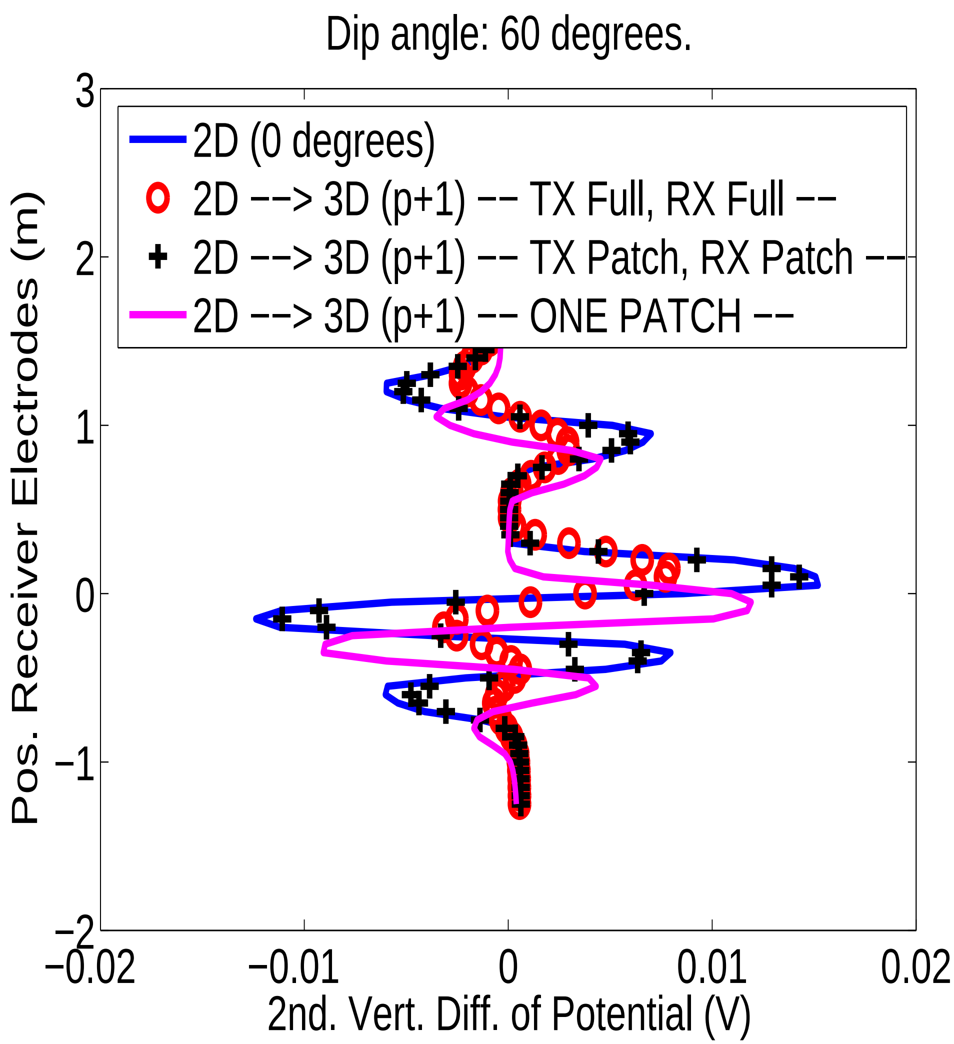


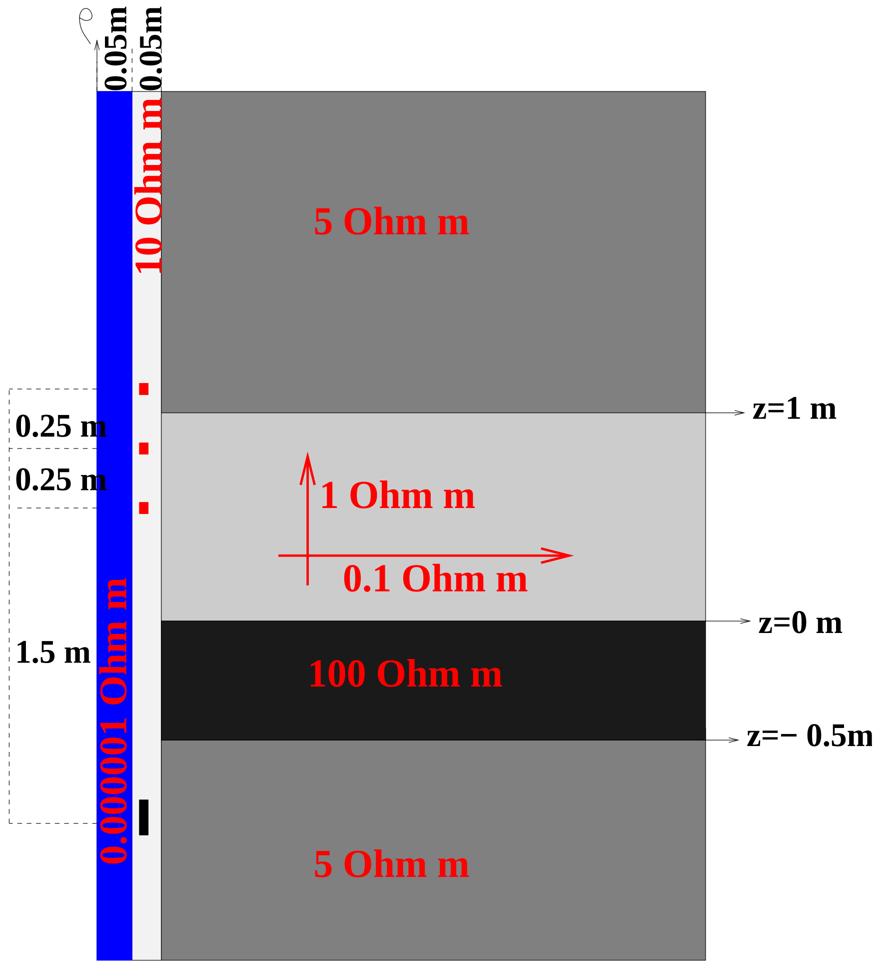
LWD (at DC)

Deviated Well



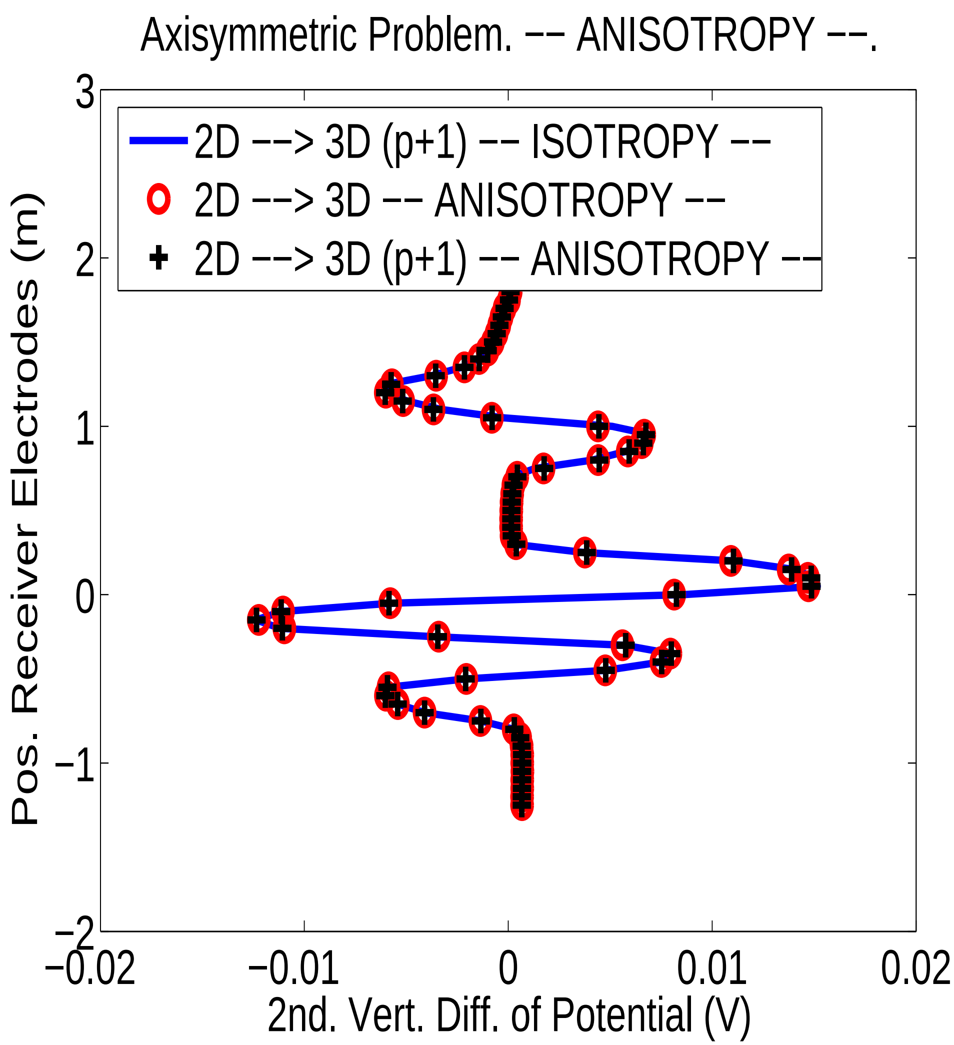
LWD (at DC)

Anisotropy in Deviated Wells



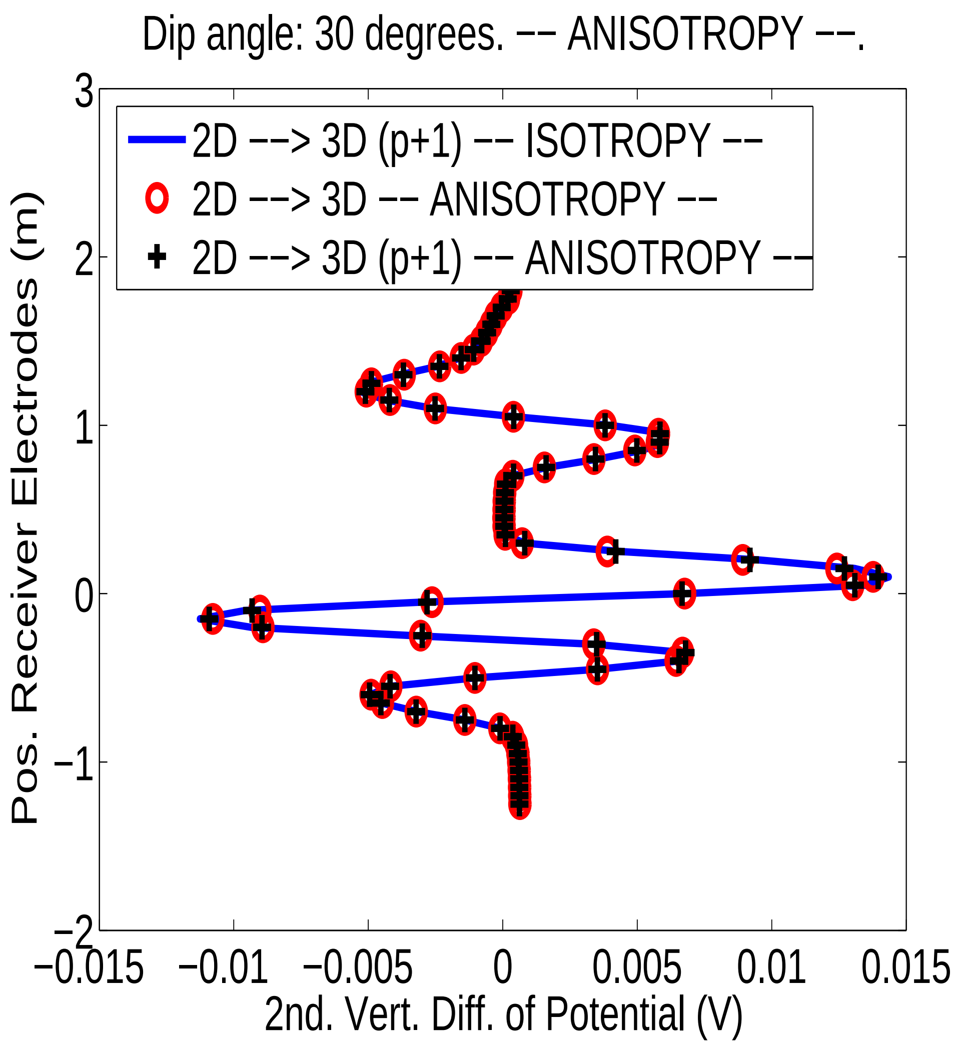
LWD (at DC)

Axisymmetric problem



LWD (at DC)

Deviated Well



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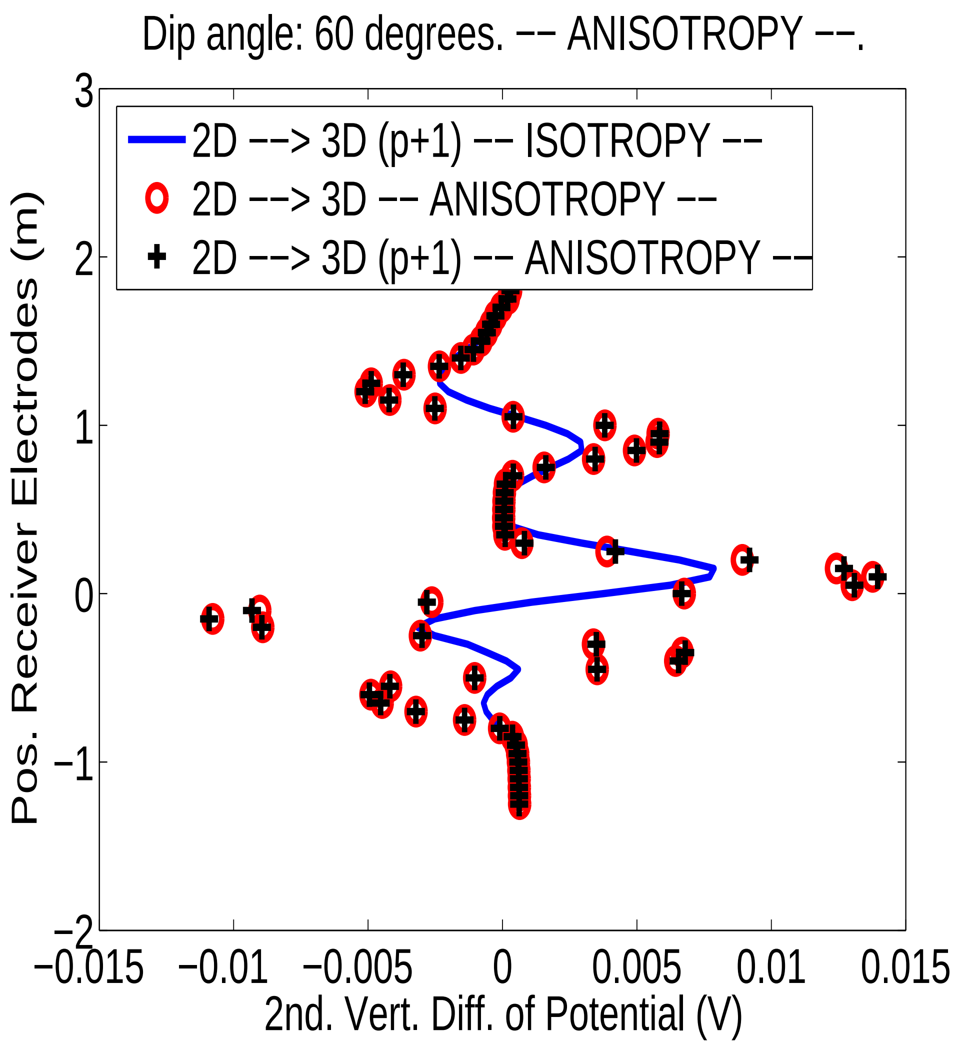

CONCLUSIONS AND FUTURE WORK

Conclusions

- **The 3D goal-oriented self-adaptive hp-Finite Element software provides reliable solutions (with guaranteed errors) for a variety of resistivity logging instruments, including laterolog and through casing resistivity tools.**
- **Within the framework of hp-Finite Elements, we are able to simulate multi-physics and multi-dimensional problems.**

Future Work

- **3D AC simulator.**
- **Parallel iterative solver.**

ACKNOWLEDGMENTS

ارامكو السعودية
Saudi Aramco
BAKER HUGHES
Baker Atlas
bp
Chevron
ConocoPhillips
Agip
Eni
ExxonMobil
HALLIBURTON®
IMP Innovar
INSTITUTO MEXICANO DEL PETRÓLEO
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