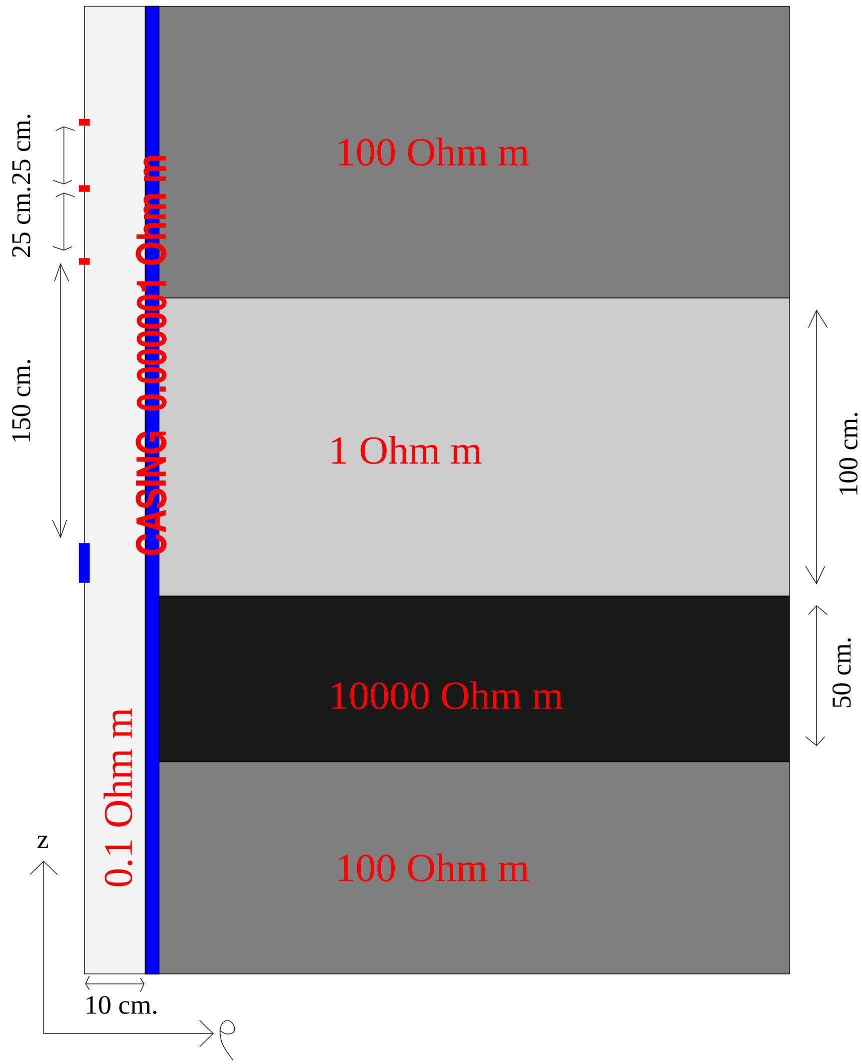
Through Casing Resistivity Logging Problem (DC)



Axisymmetric 3D problem.

Five different materials.

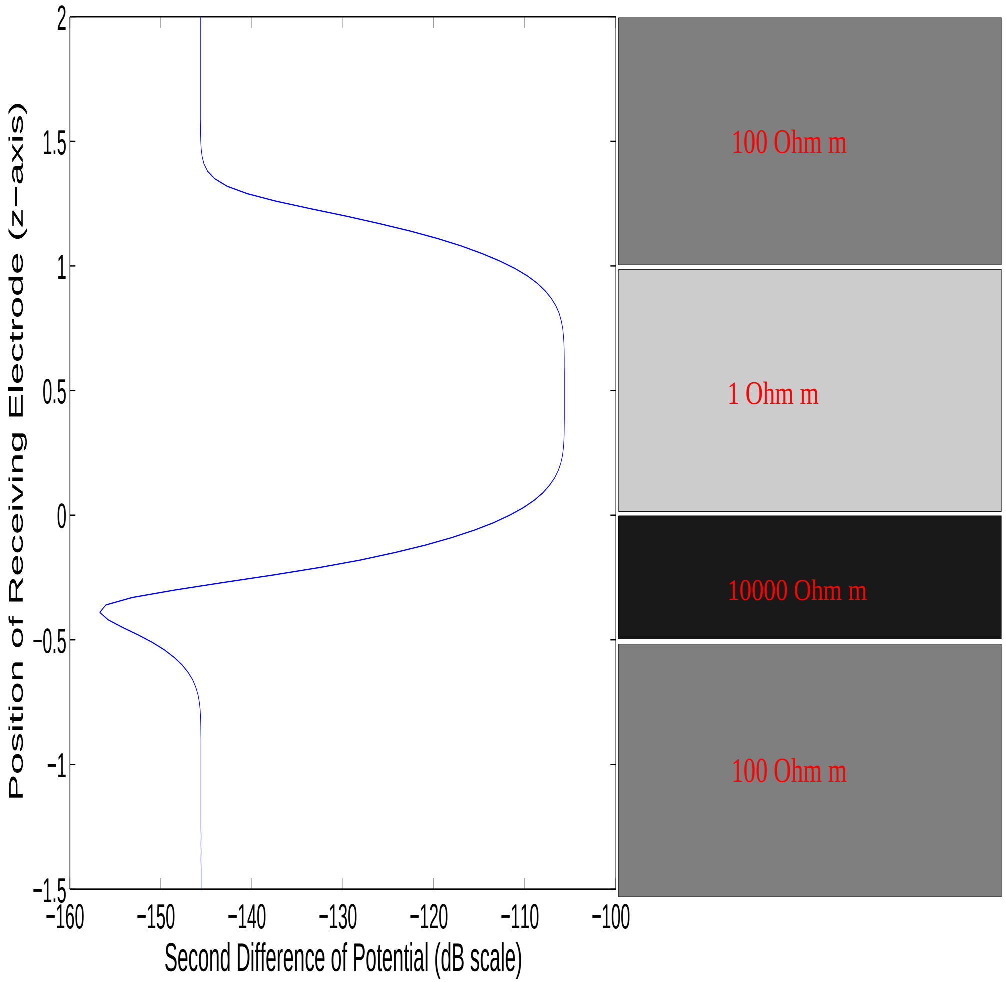
Size of computational domain: SEVERAL MILES.

Material properties varying by up to TEN orders of magnitude (10000000000!!!).

Objective: Determine Second Difference of Potential Receiving Electrodes.

Through Casing Resistivity Logging Problem (DC)

Final Log Obtained by Our Finite Element Software



Second Difference of Potential (in the decibel scale) vs Position of the receiving electrodes (with respect z).

Through Casing Resistivity Tool (TCRT).

Five different Materials with varying conductivity by up to 10 orders of magnitude.

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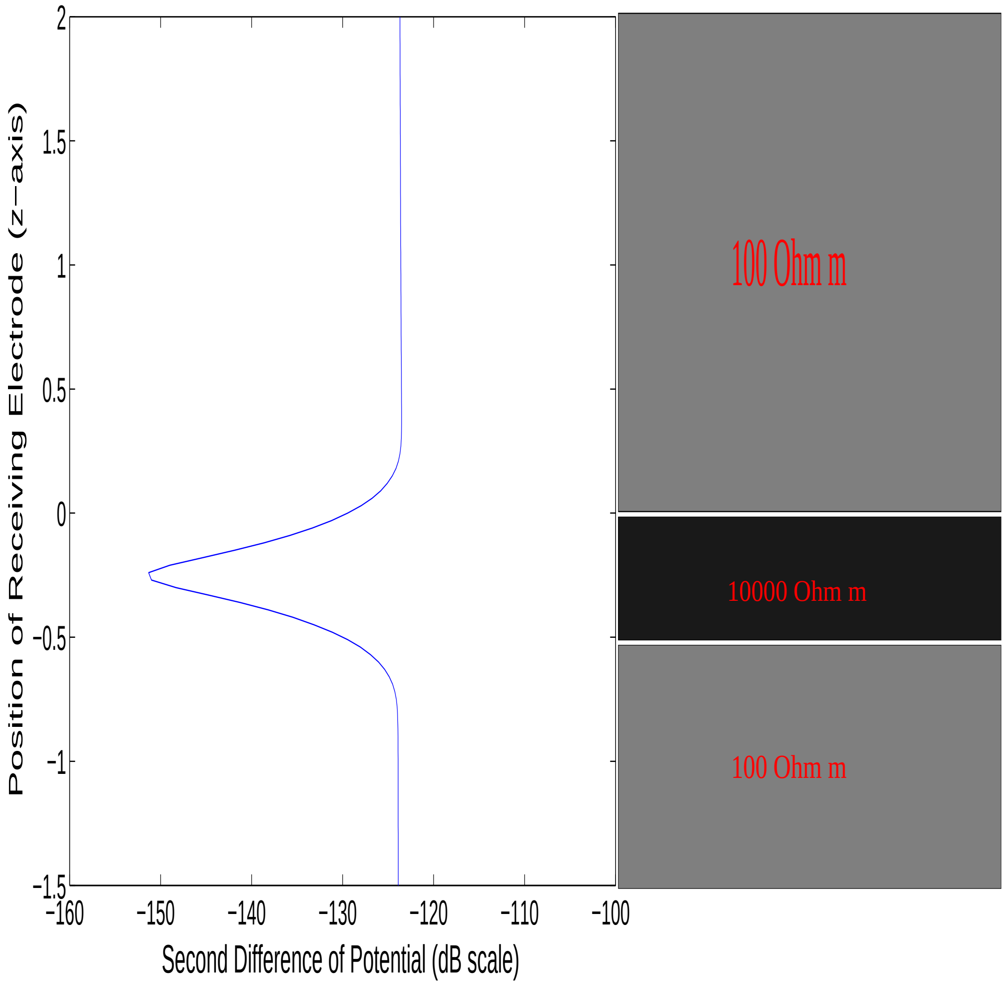

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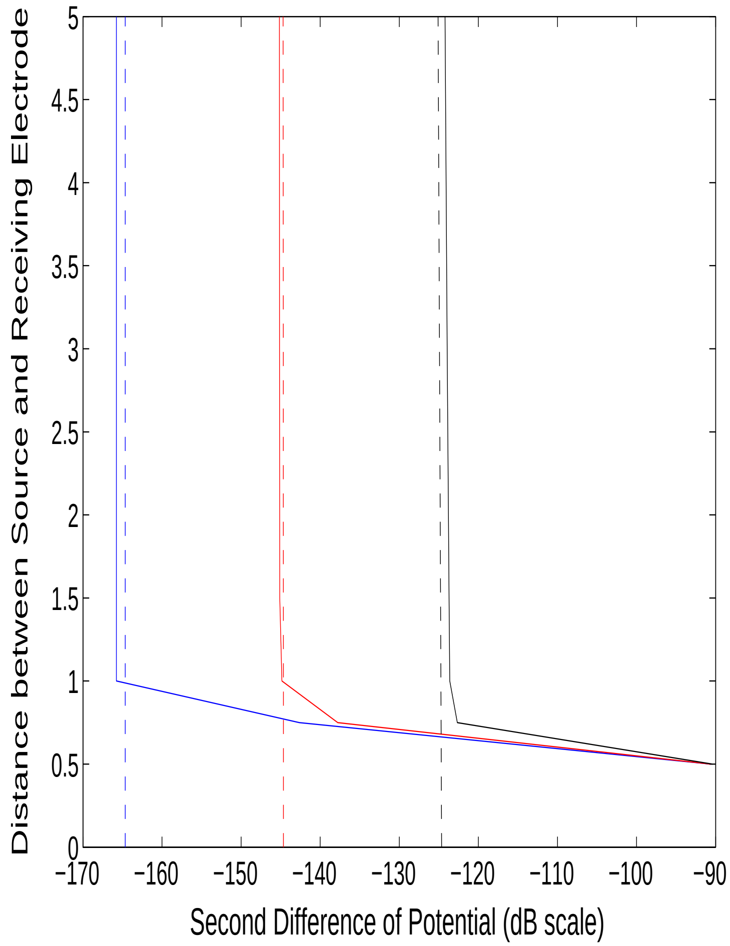
Through Casing Resistivity Logging Problem (DC)

Logging Through Casing (Kaufman's Approx. Formula)



Formation Resistivity= 10000

Formation Resistivity = 100

Formation Resistivity = 1

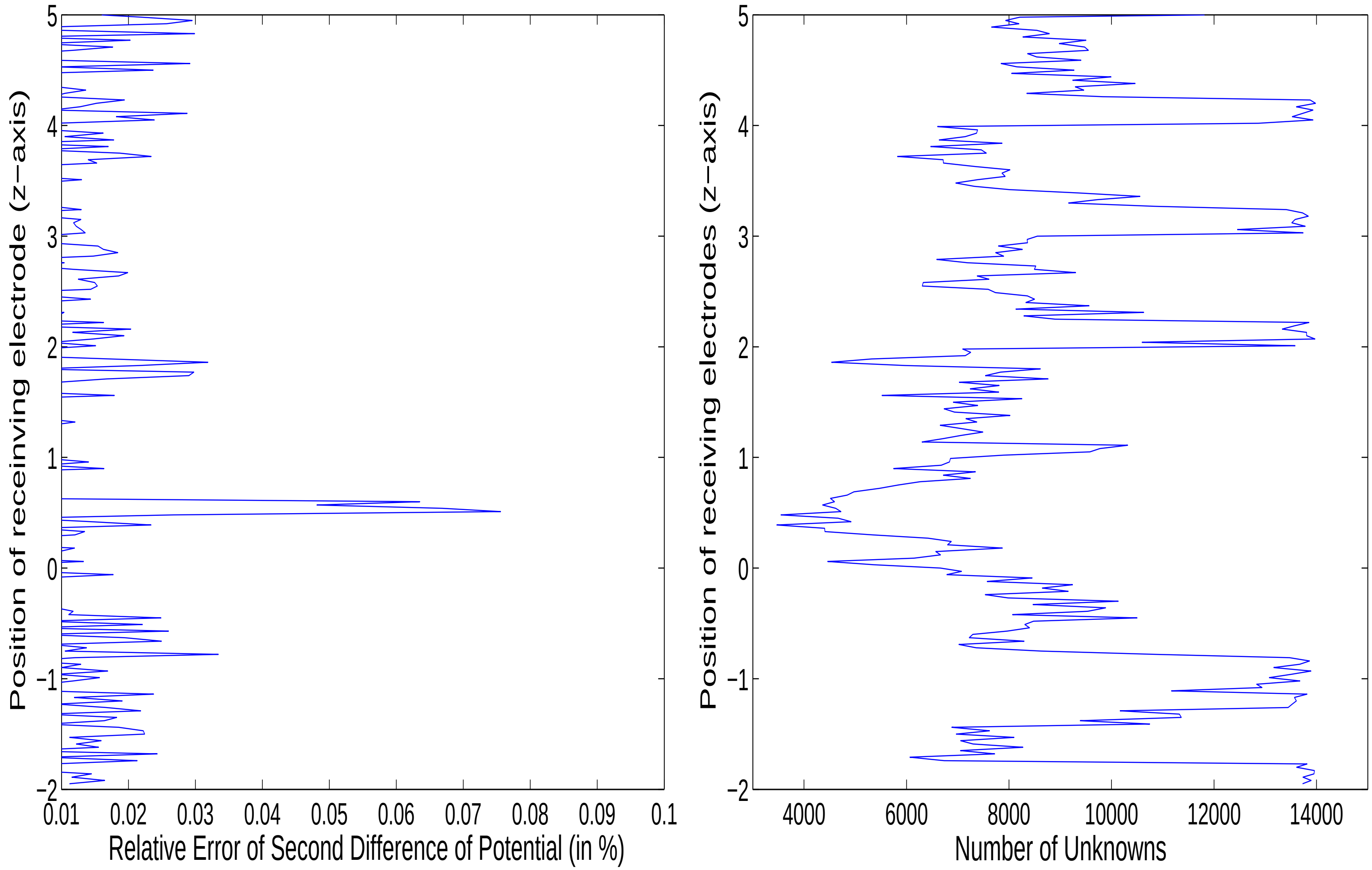
Continuous Lines:
Obtained from our Finite Element Software.

Dashed Lines:
Obtained from Kaufman's Approx. Formula.

Kaufman's Approximation Formula has 20 % relative error!!!

Through Casing Resistivity Logging Problem (DC)

Approximation Error



Joint Industry Research Consortium on Formation Evaluation (Fourth Annual Meeting)

A New Fully Automatic Goal-Oriented hp-Adaptive Finite Element Strategy for Simulations of Resistivity Logging Instruments.

David Pardo (dzubiaur@yahoo.es),
L. Demkowicz, C. Torres-Verdin, L. Tabarovsky, A. Bespalov

Collaborators: D. Xue, J. Kurtz, M. Paszynski, Ch. Larson,
W. Rachowicz, A. Zdunek, L.E. Garcia-Castillo

Acknowledgment: Baker-Atlas

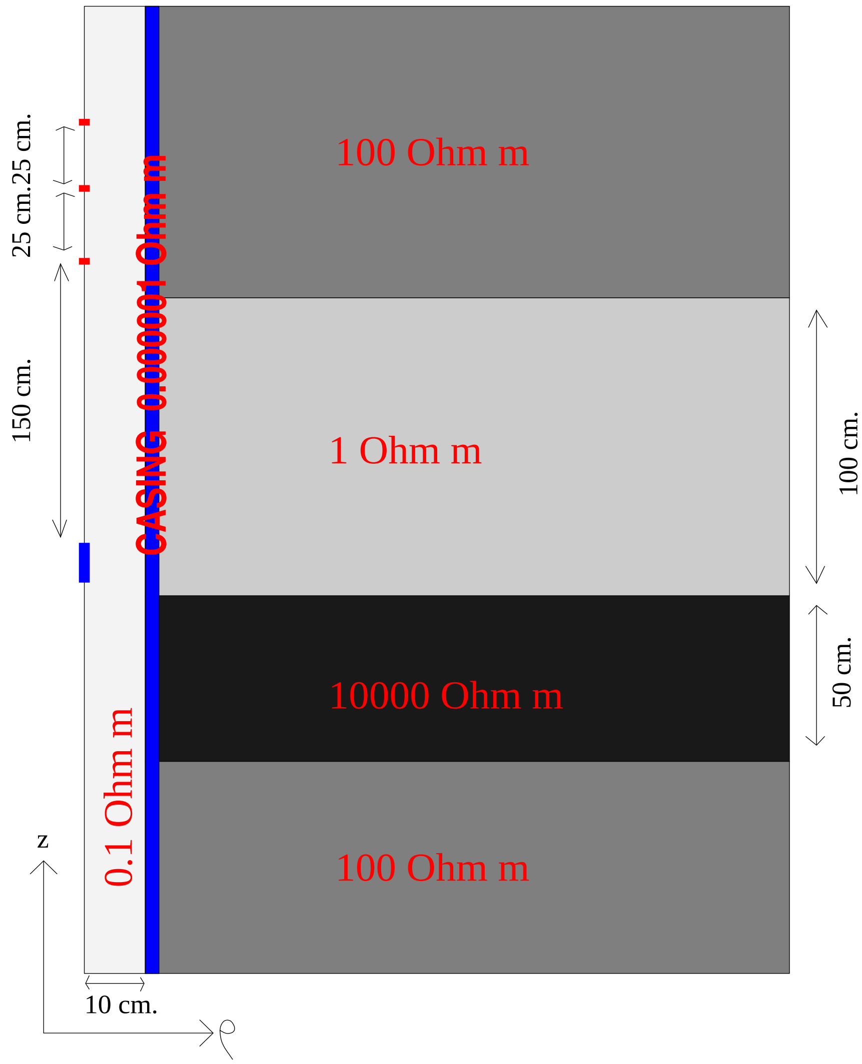
August 19, 2004

Institute for Computational Engineering and Sciences (ICES)
The University of Texas at Austin

OVERVIEW

1. Motivation: A Through Casing Resistivity Tool Problem.
2. Conductive Media Equation.
3. hp-Finite Elements.
4. Fully Automatic Energy Norm hp-Adaptive Strategy.
5. Fully Automatic Goal-Oriented hp-Adaptive Strategy.
6. Numerical Results: DC, AC, 2D, and 3D problems
7. Conclusions and Future Work.

MOTIVATION



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CONDUCTIVE MEDIA EQUATION

Derivation of Conductive Media Equation:

Maxwell's Equations:

$$\begin{cases} \boldsymbol{\nabla} \times \mathrm{H} = (\sigma - j\omega\epsilon)\mathrm{E} + \mathrm{J} \\ \boldsymbol{\nabla} \times \mathrm{E} = (j\omega\mu\epsilon)\mathrm{H} \ , \\ \boldsymbol{\nabla} \cdot \epsilon\mathrm{E} = \rho \ , \\ \boldsymbol{\nabla} \cdot \mu\mathrm{H} = 0 \ , \end{cases}$$

CONDUCTIVE MEDIA EQUATION

Derivation of Conductive Media Equation:

Maxwell's Equations: **Steady state:**

$$\left\{\begin{array}{l}\nabla \times \mathbf{H} = (\sigma - j\omega\epsilon)\mathbf{E} + \mathbf{J} \\ \nabla \times \mathbf{E} = (j\omega\mu\epsilon)\mathbf{H}\ , \\ \nabla \cdot \epsilon\mathbf{E} = \rho\ , \\ \nabla \cdot \mu\mathbf{H} = 0\ ,\end{array}\right. \quad \stackrel{\omega=0}{\Longrightarrow} \quad \left\{\begin{array}{l}\nabla \times \mathbf{H} = \sigma\mathbf{E} + \mathbf{J} \\ \nabla \times \mathbf{E} = 0\ , \\ \nabla \cdot \epsilon\mathbf{E} = \rho\ , \\ \nabla \cdot \mu\mathbf{H} = 0\ .\end{array}\right.$$

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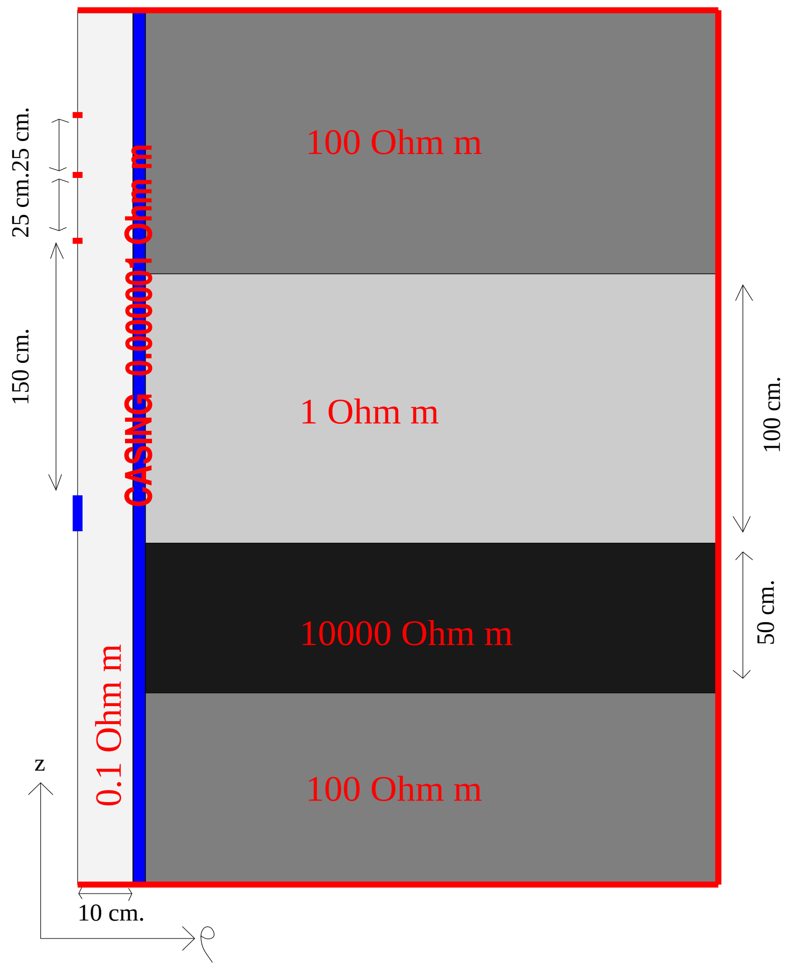
$$\boxed{-\nabla \cdot \sigma \nabla \Psi = \nabla \cdot \mathrm{J}}$$

CONDUCTIVE MEDIA EQUATION

Boundary Conditions



Essential (Dirichlet BC) to make the computational domain finite.

No BC for the center of axisymmetry.

An extra boundary term to model the source electrode.

CONDUCTIVE MEDIA EQUATION

Variational Formulation

3D Variational Form:

$$\begin{cases} \text{Find } \Psi \in \Psi_D + V \text{ such that:} \\ \int_\Omega \sigma \nabla \Psi \nabla \xi \, dV = \int_\Omega \nabla \cdot \mathrm{J} \, \xi \, dV + \int_{\Gamma_N} g \, \xi \, dS \quad \forall \xi \in V \, . \end{cases}$$

Using Cylindrical Coordinates:

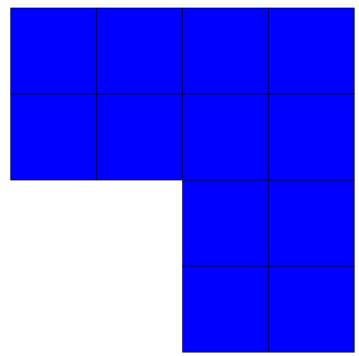
$$\begin{cases} \text{Find } \Psi \in \Psi_D + V \text{ such that:} \\ \int_\Omega \sigma \nabla \Psi \nabla \xi \, \rho \, d\rho d\psi dz = \int_\Omega \nabla \cdot \mathrm{J} \, \xi \, \rho \, d\rho d\psi dz + \int_{\Gamma_N} g \, \xi \, dS \quad \forall \xi \in V \, . \end{cases}$$

Using a Different Notation:

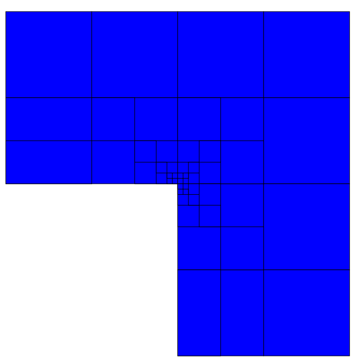
$$\begin{cases} \text{Find } \Psi \in \Psi_D + V \text{ such that:} \\ b(\Psi, \xi) = f(\xi) \quad \forall \xi \in V \, . \end{cases}$$

HP-FINITE ELEMENTS

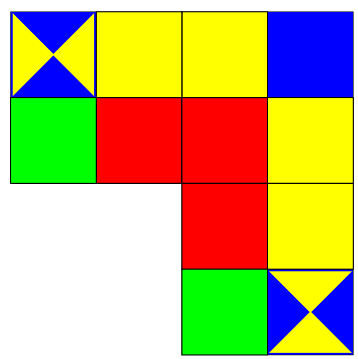
Different refinement strategies for finite elements:



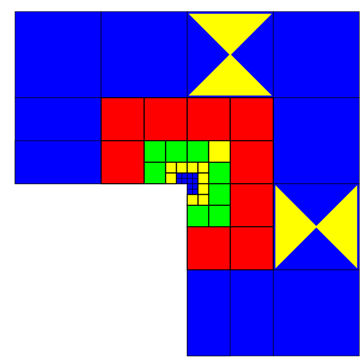
Given initial grid



***h*-refined grid**



***p*-refined grid**



***hp*-refined grid**

HP-FINITE ELEMENTS

EXPONENTIAL CONVERGENCE RATES
EXPONENTIAL CONVERGENCE RATES
EXPONENTIAL CONVERGENCE RATES
for problems WITH and without SINGULARITIES

if we orchestrate an optimal distribution of h and p within the same grid

Smaller dispersion (pollution) error

as *p* increases.

More geometrical details captured

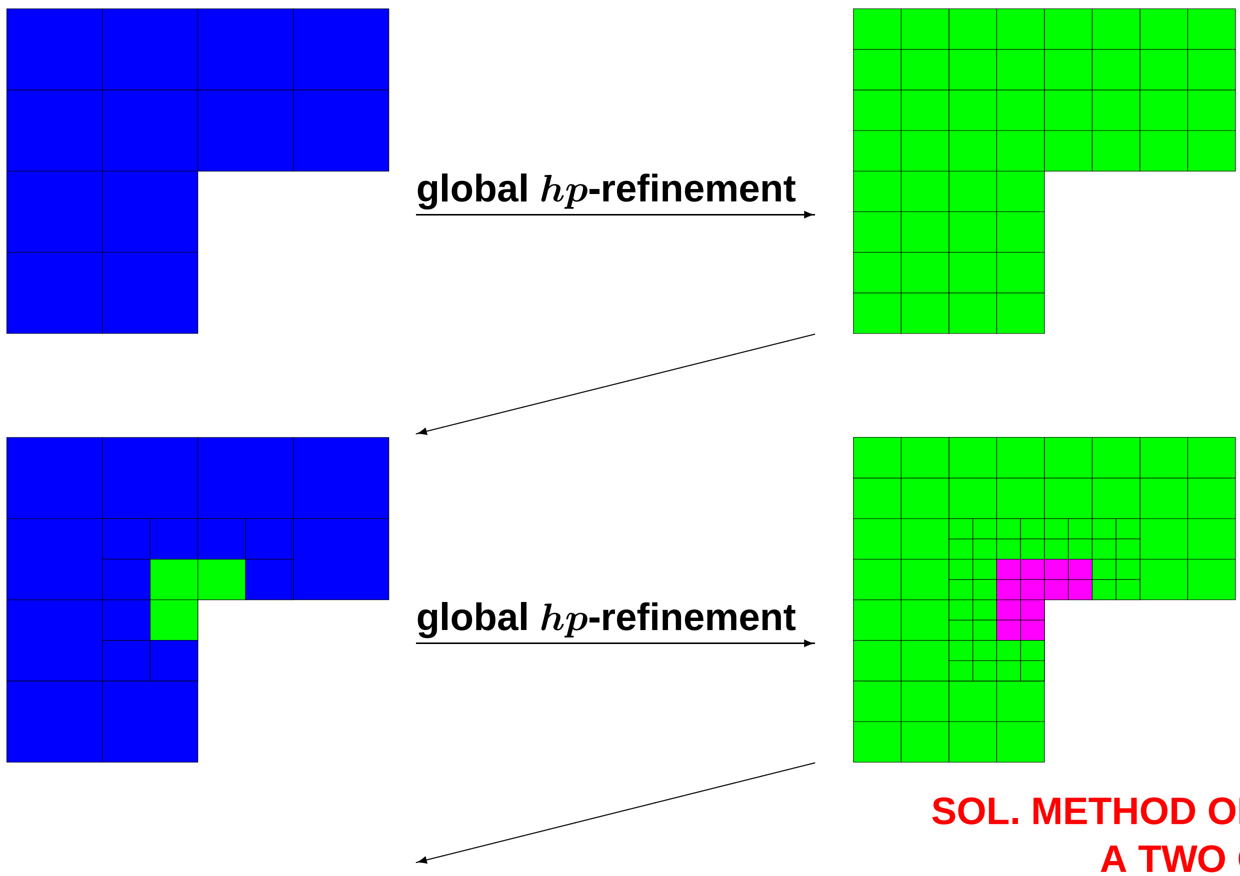
as *h* decreases.

FULLY AUTOMATIC *HP*-ADAPTIVE STRATEGY

Energy norm based fully automatic *hp*-adaptive strategy

Coarse grids
(hp)

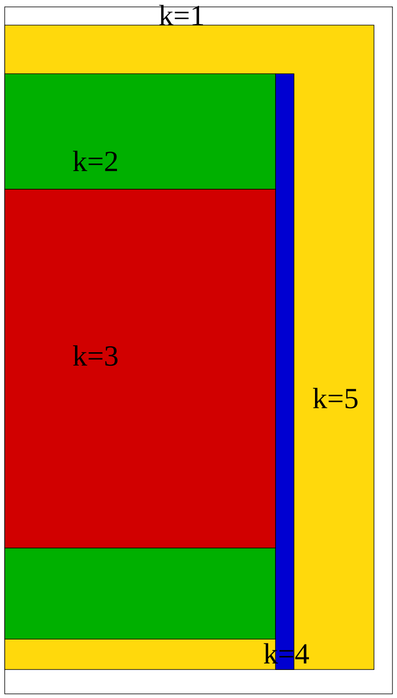
Fine grids
($h/2, p+1$)



SOL. METHOD ON FINE GRIDS:
A TWO GRID SOLVER

FULLY AUTOMATIC *HP*-ADAPTIVE STRATEGY

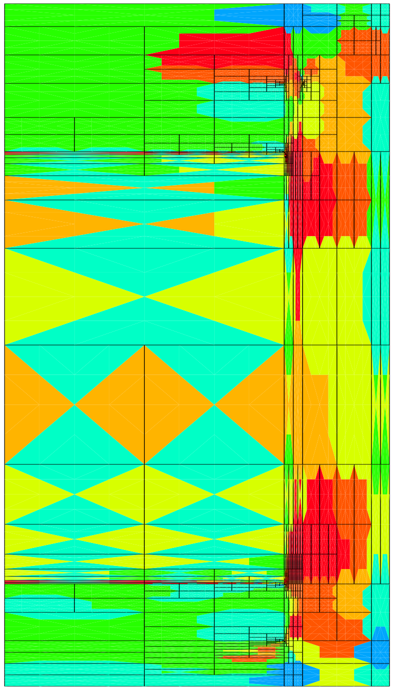
Orthotropic heat conduction example (Sandia National Laboratories)



Equation: $\nabla(\mathrm{K}\nabla u) = f^{(k)}$

$$\mathrm{K} = \mathrm{K}^{(k)} = \begin{bmatrix} K_x^{(k)} & 0 \\ 0 & K_y^{(k)} \end{bmatrix}$$

$$K_x^{(k)} = (25,\ 7,\ 5,\ 0.2,\ 0.05)$$
$$K_y^{(k)} = (25,\ 0.8,\ 0.0001,\ 0.2,\ 0.05)$$

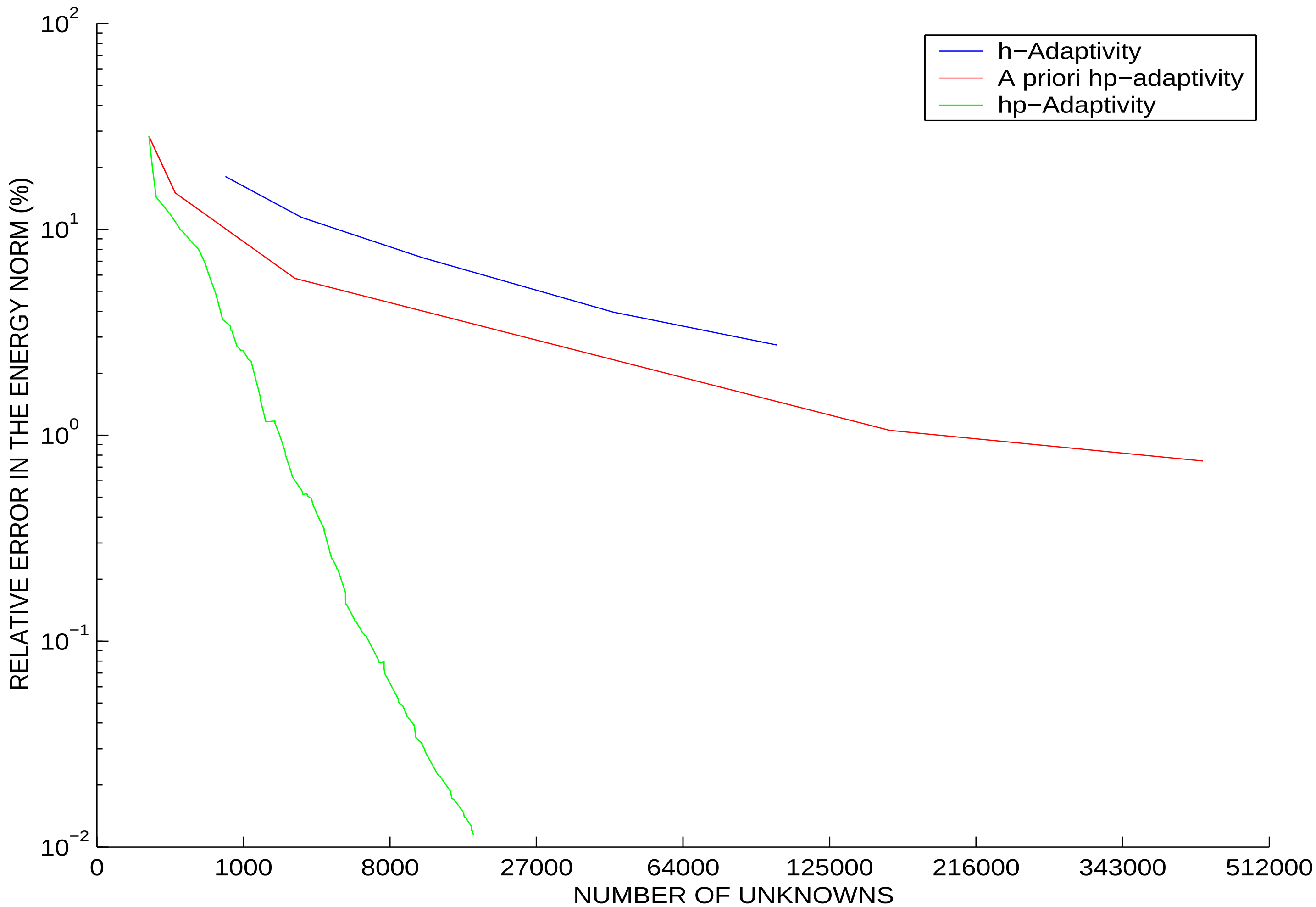


Final hp-Grid

FULLY AUTOMATIC *HP*-ADAPTIVE STRATEGY

Convergence comparison

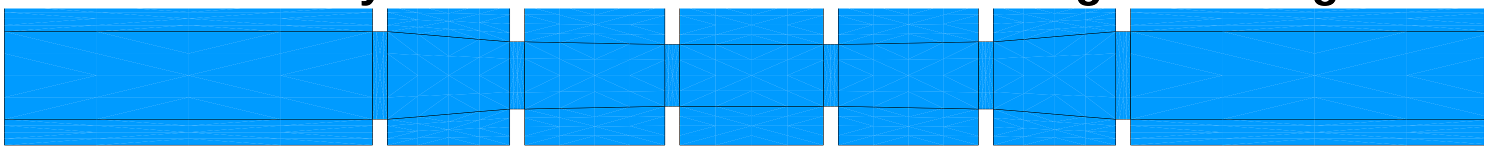
Orthotropic heat conduction example

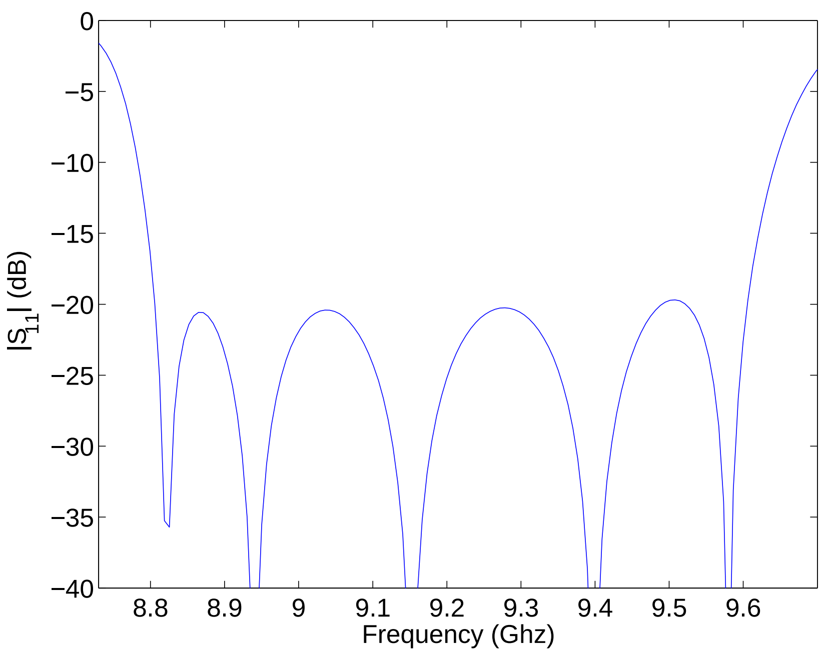


FULLY AUTOMATIC *HP*-ADAPTIVE STRATEGY

Waveguide example with six iris

Geometry of a cross section of the rectangular waveguide





RETURN LOSS OF THE WAVEGUIDE

H-plane six resonant iris filter.

Dominant mode (source): $TE_{10}-$**mode.**

Dimensions $\approx 20 \times 2 \times 1$ **cm.**

Operating Frequency $\approx 8.8 - 9.6$ **Ghz**

Cutoff frequency ≈ 6.56 **Ghz**

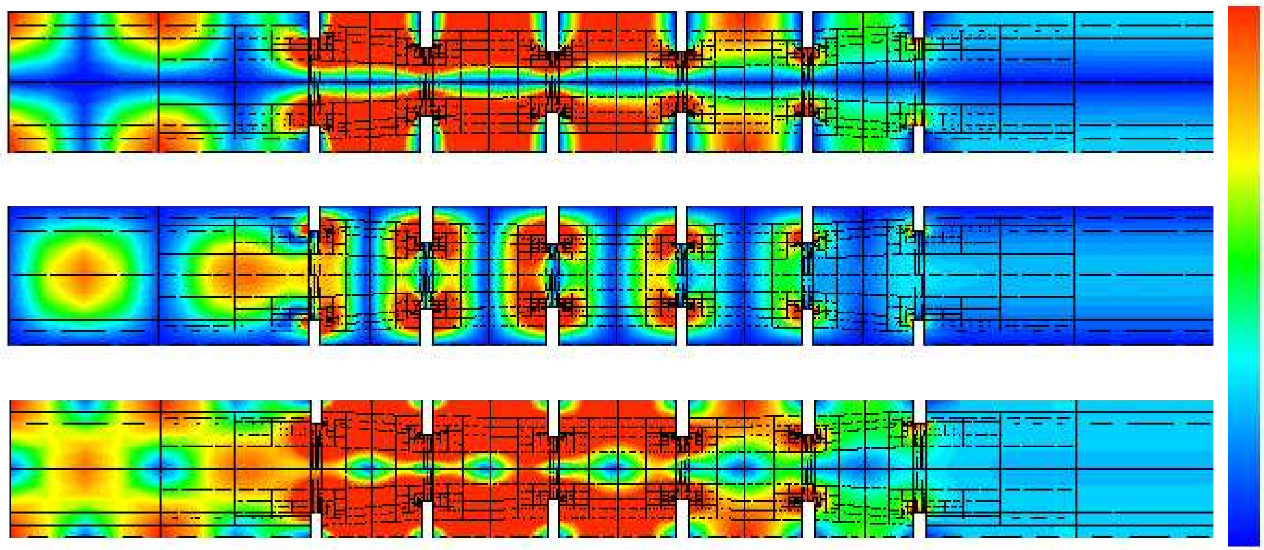
FULLY AUTOMATIC *HP*-ADAPTIVE STRATEGY

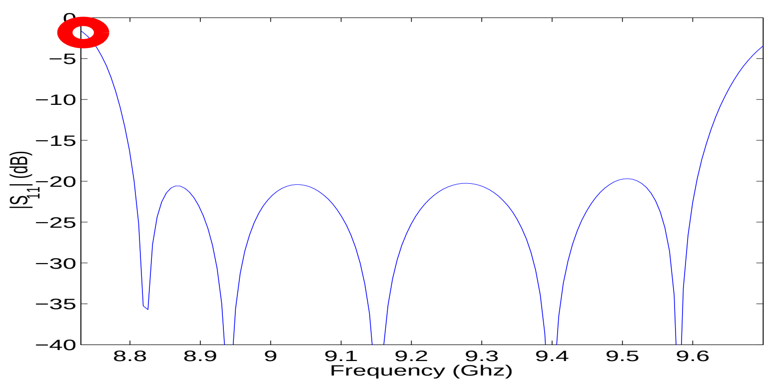
FEM solution for frequency $= 8.72$ Ghz

$|H_x|$

$|H_y|$

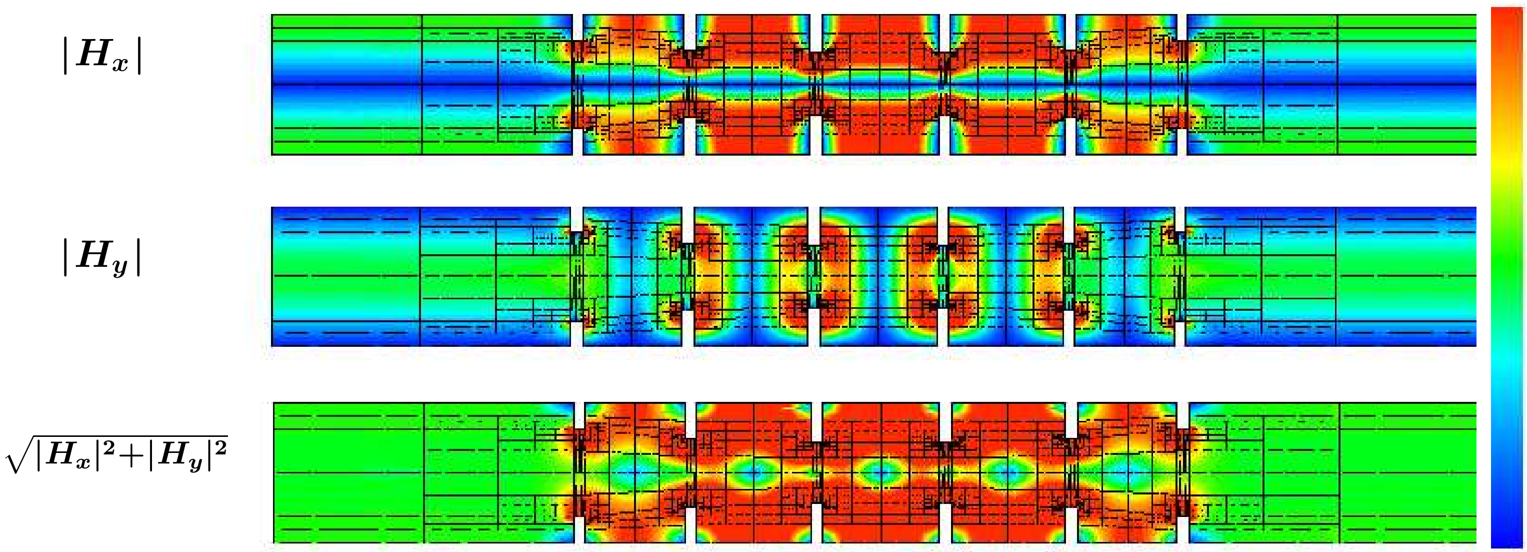
$\sqrt{|H_x|^2+|H_y|^2}$

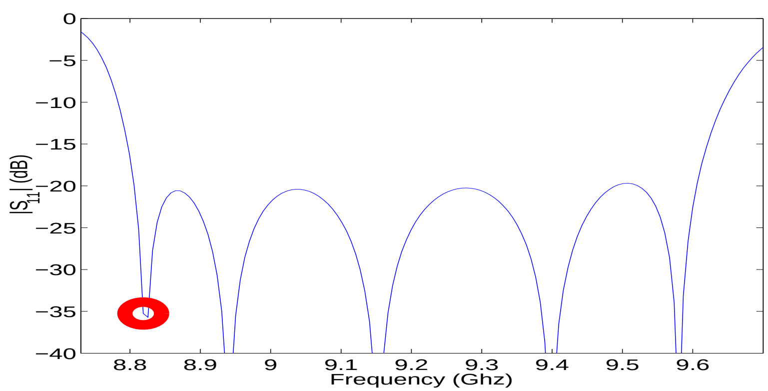




FULLY AUTOMATIC *HP*-ADAPTIVE STRATEGY

FEM solution for frequency $= 8.82$ Ghz





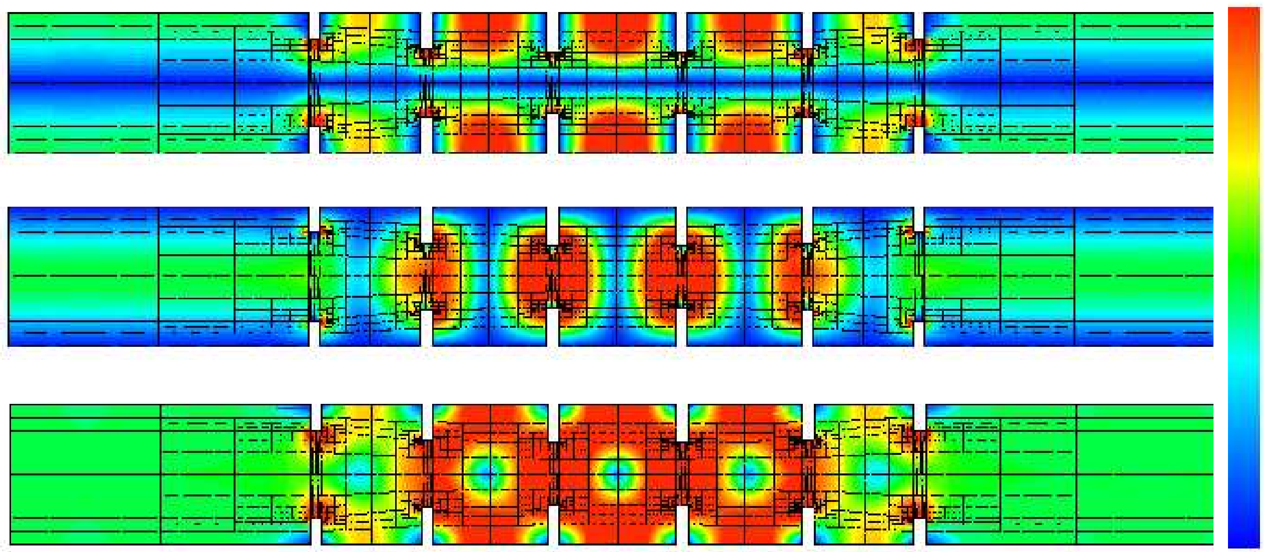
FULLY AUTOMATIC *HP*-ADAPTIVE STRATEGY

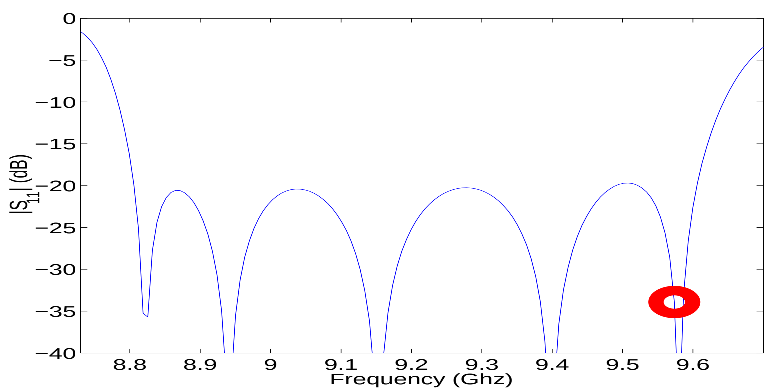
FEM solution for frequency $= 9.58$ Ghz

$|H_x|$

$|H_y|$

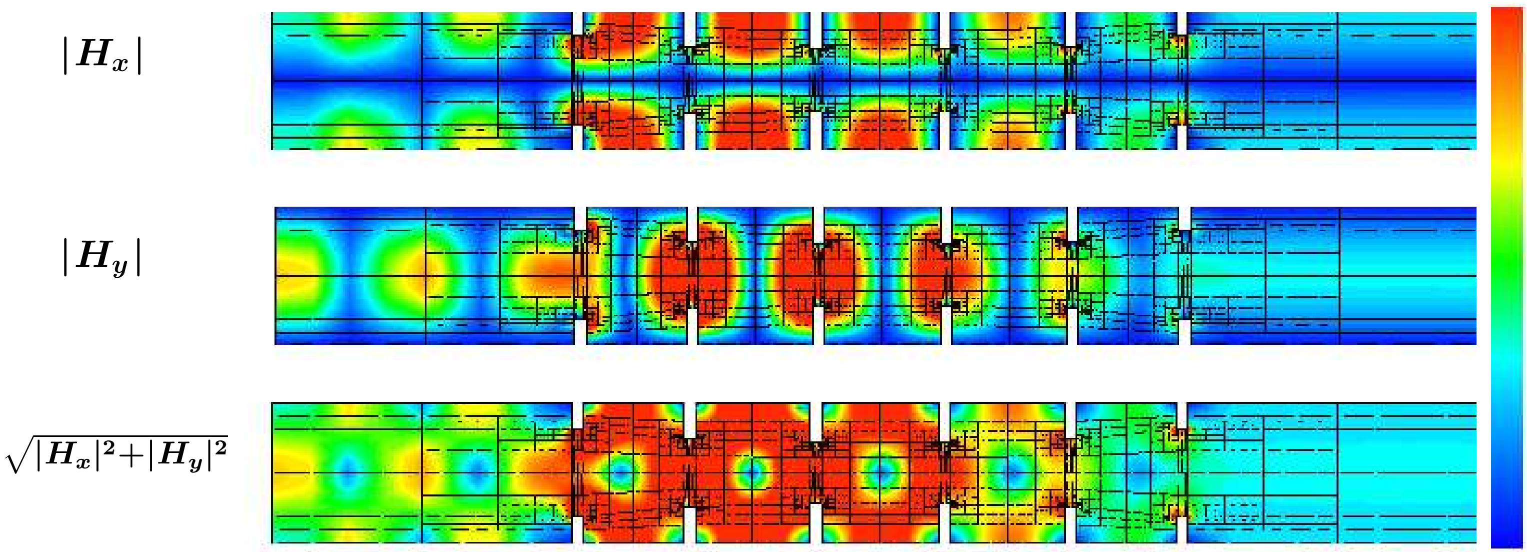
$\sqrt{|H_x|^2+|H_y|^2}$

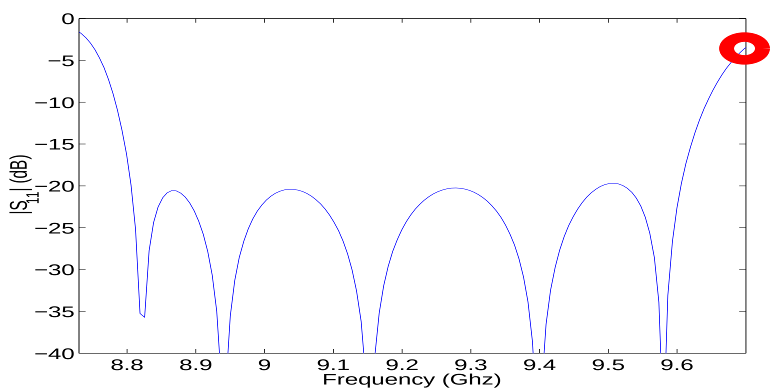




FULLY AUTOMATIC *HP*-ADAPTIVE STRATEGY

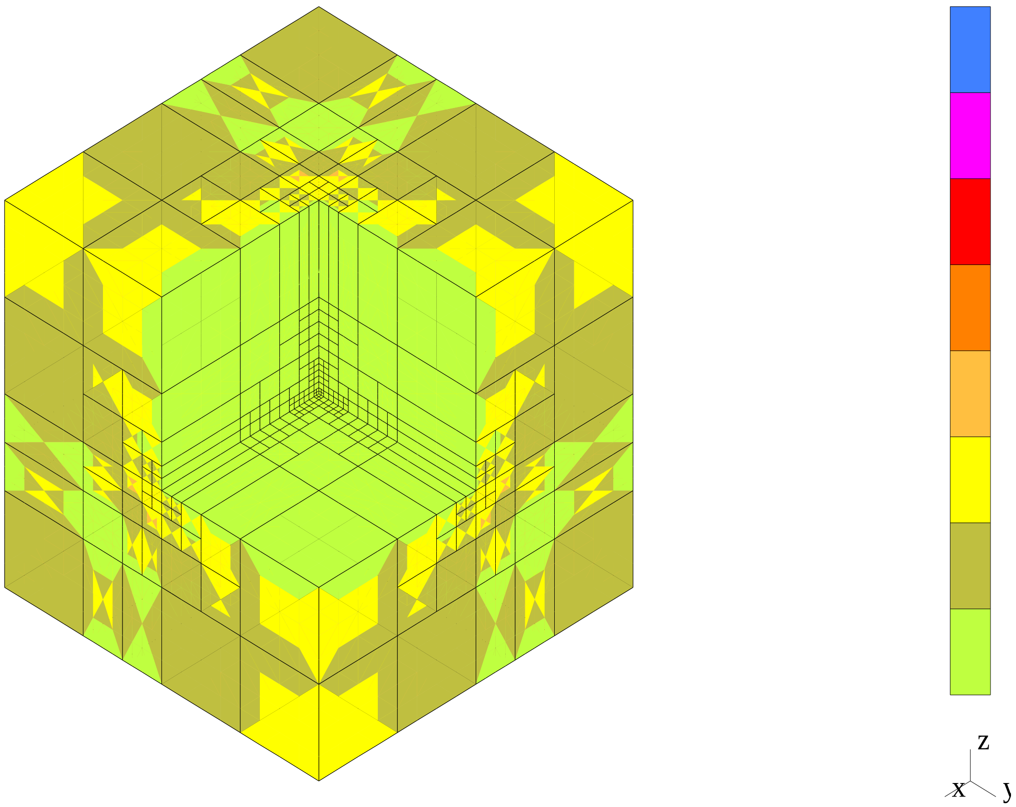
FEM solution for frequency $= 9.71$ Ghz





FULLY AUTOMATIC *HP*-ADAPTIVE STRATEGY

Fichera problem. Final hp-grid.



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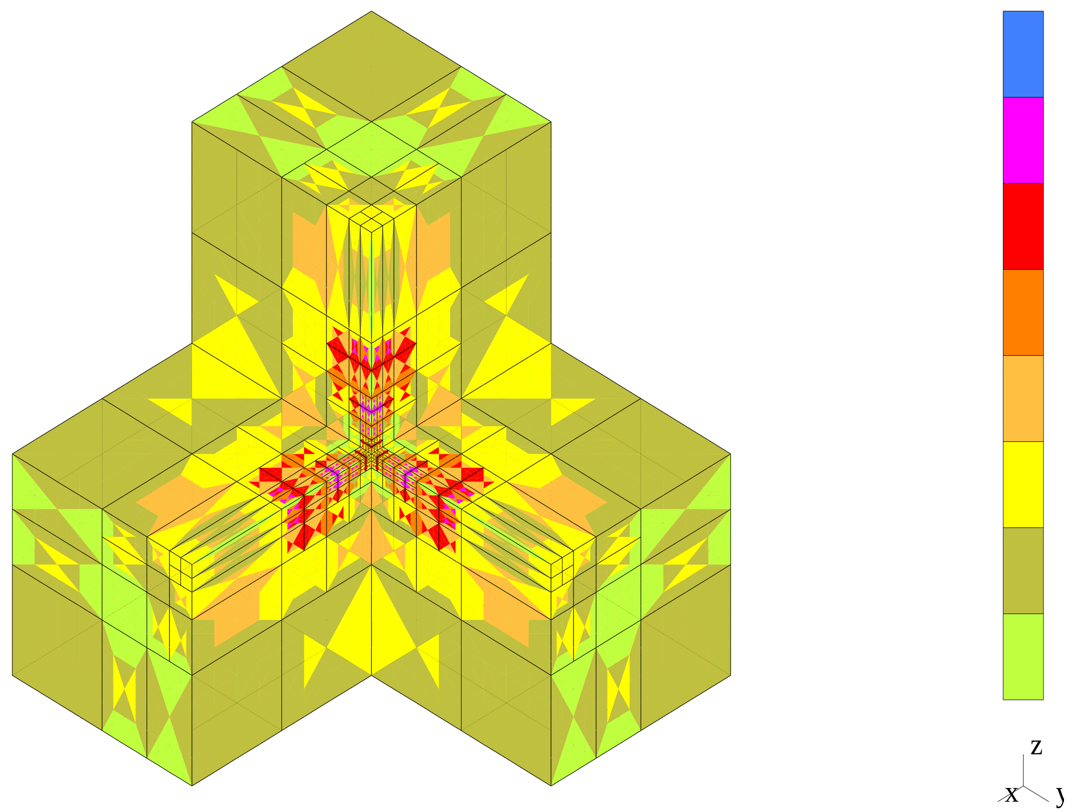

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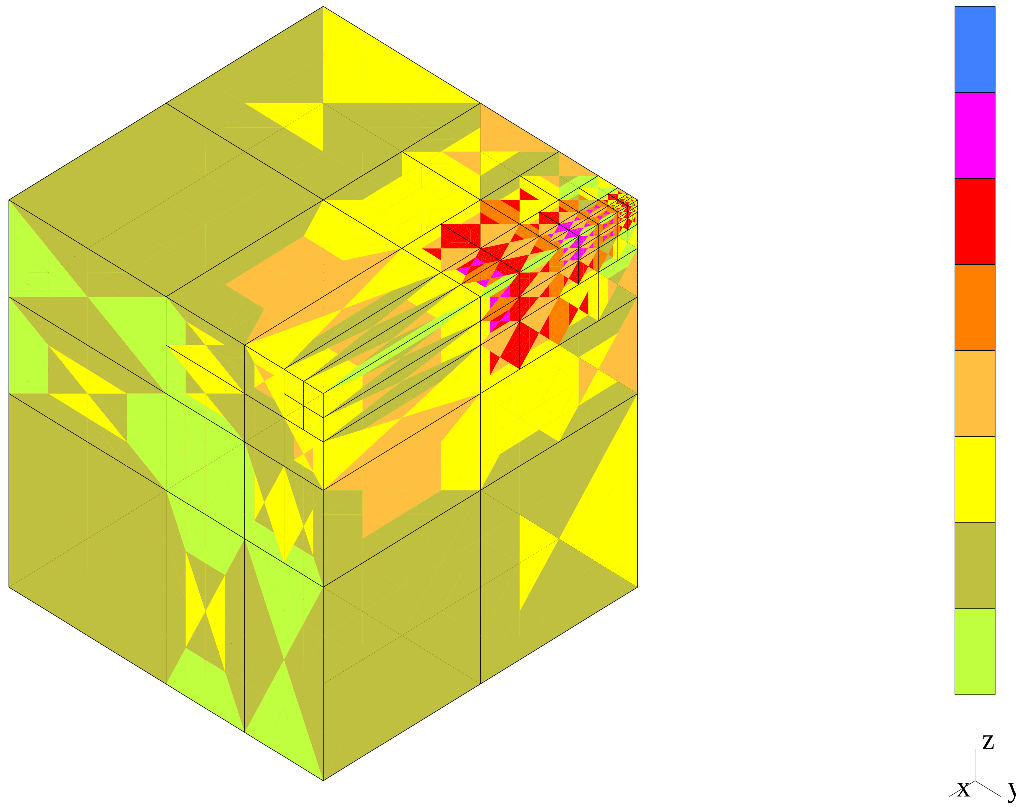

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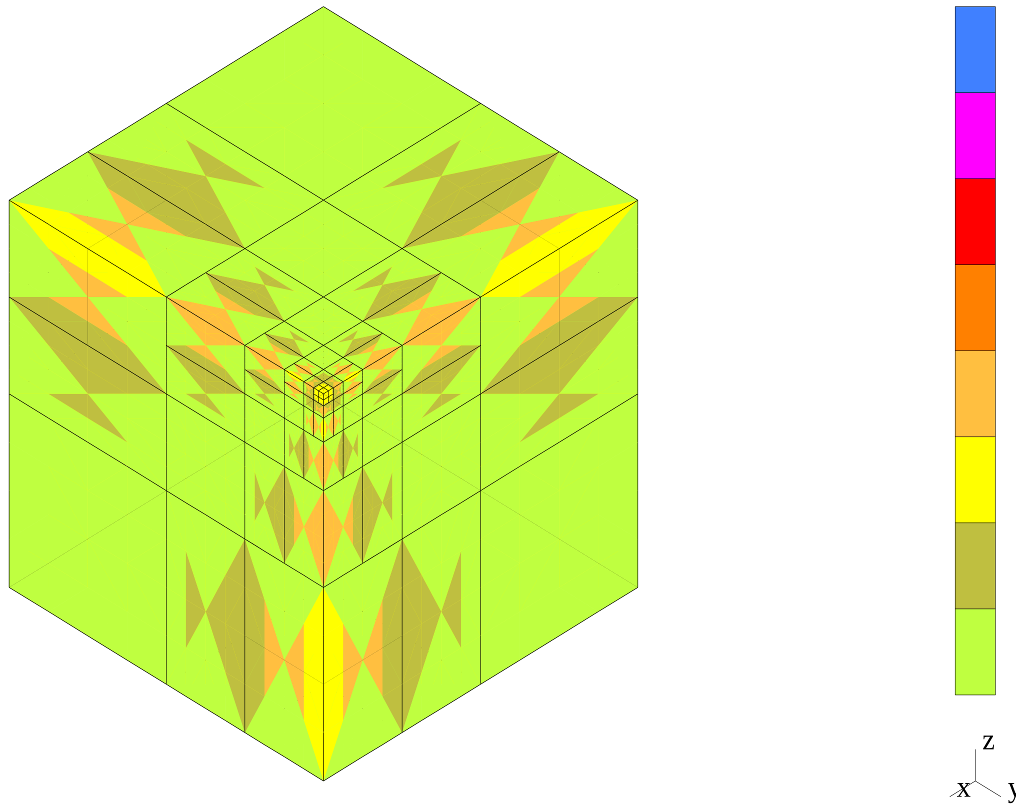

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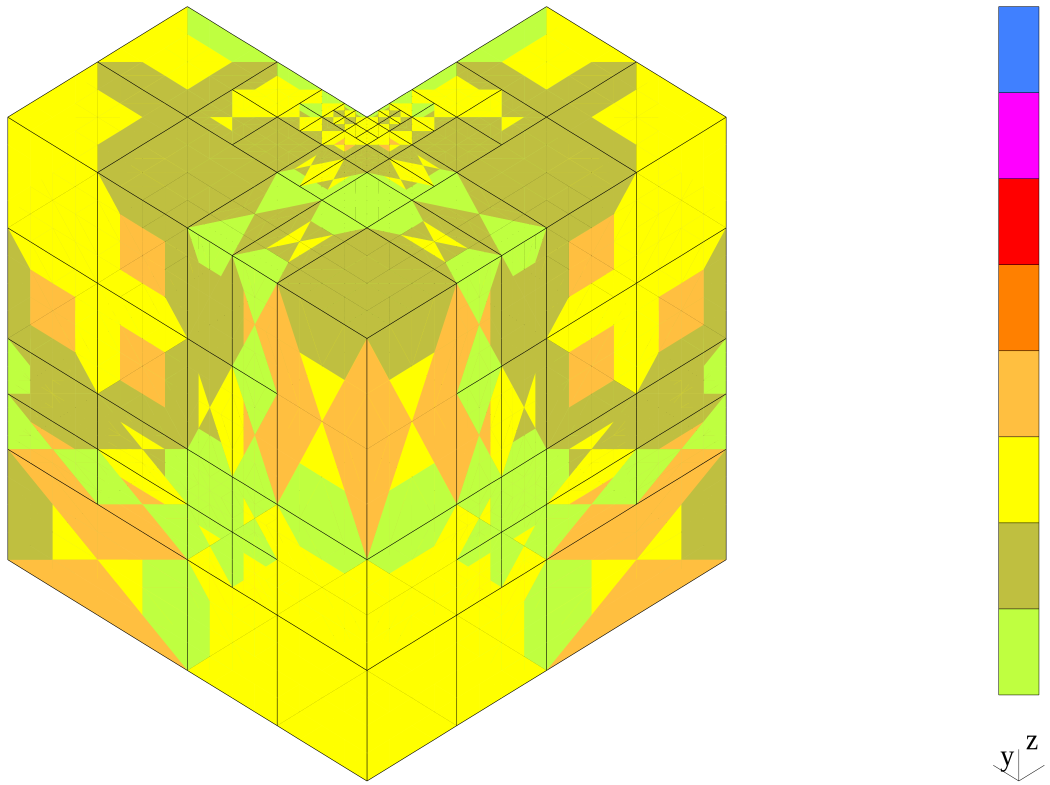
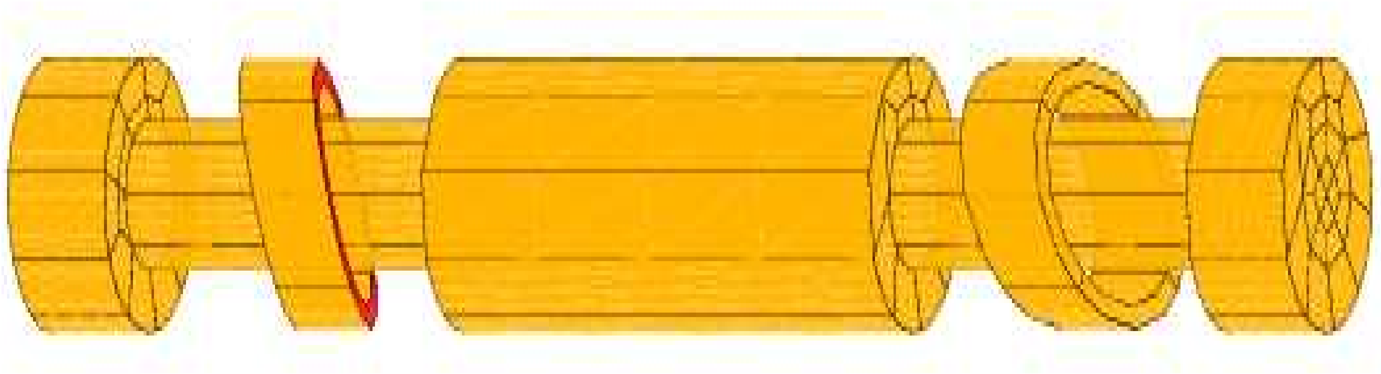


FULLY AUTOMATIC *HP*-ADAPTIVE STRATEGY

Petroleum Engineering Applications



Results are not good. Why?

We are not interested in the energy norm error, but in the solution (or second difference of potential, etc.) at the receiving electrodes.

AUTOMATIC GOAL-ORIENTED *HP*-ADAPTIVITY

What does it mean *Goal-Oriented* Adaptivity?

We consider the following problem (in variational form):

$$\begin{cases} \textbf{Find } \Psi \in V \textbf{ such that:} \\ b(\Psi, \xi) = f(\xi) \quad \forall \xi \in V \ . \end{cases}$$

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$MISLEADING$!!!!!

The problem we *really* want to solve is:

$$\begin{cases} \text{Find } L(\Psi), \text{ where } \Psi \in V \text{ such that:} \\ b(\Psi, \xi) = f(\xi) \quad \forall \xi \in V \ , \end{cases}$$

where $L(\Psi)$ is our goal (for example, $L(\Psi) = \Psi(b) - \Psi(a)$).

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HP goal-oriented adaptivity consists of constructing an optimal grid:

$$\arg \min_{hp:|L(e_{hp})| \leq TOL} N_{hp}$$

AUTOMATIC GOAL-ORIENTED *HP*-ADAPTIVITY

Mathematical Formulation (Goal-Oriented Adaptivity)

We consider the following problem (in variational form):

$$\begin{cases} \text{Find } L(\Psi), \text{ where } \Psi \in V \text{ such that:} \\ b(\Psi, \xi) = f(\xi) \quad \forall \xi \in V \ . \end{cases}$$

We define residual $r_{hp}(\xi) = b(e_{hp}, \xi)$. We seek for solution G of:

$$\begin{cases} \text{Find } G \in V \text{ such that:} \\ r(G) = L(e_{hp}) \ . \end{cases}$$

This is necessarily solved if we find the solution of the *dual* problem:

$$\begin{cases} \text{Find } G \in V \text{ such that:} \\ b(\Psi, G) = L(\Psi) \quad \forall \Psi \in V \ . \end{cases}$$

Notice that $L(e) = b(e, G)$.

AUTOMATIC GOAL-ORIENTED *HP*-ADAPTIVITY

Solution of Dual Problem

Dual problem:

$$\begin{cases} \text{Find } G \in V \text{ such that:} \\ b(\Psi, G) = L(\Psi) \quad \forall \Psi \in V \ . \end{cases}$$

where $L(\Psi) = \sigma(\Psi(b) - \Psi(a))$. But L is NOT a continuous functional !!!!.

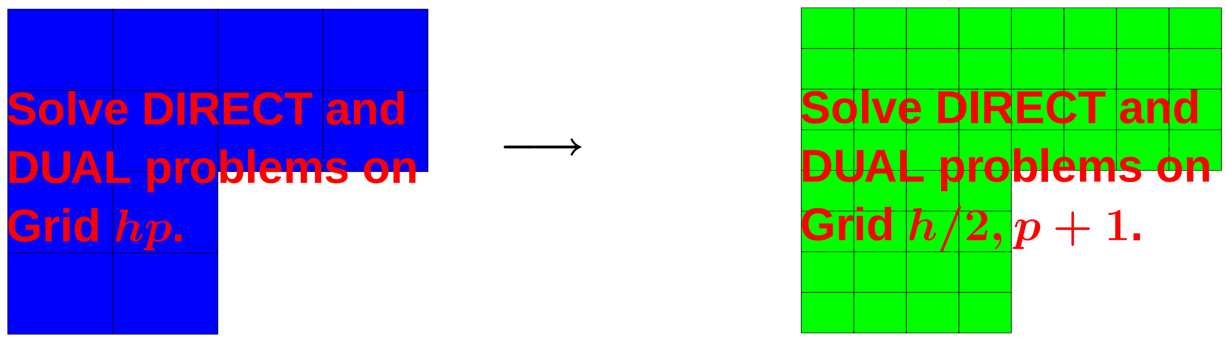
Thus, G CANNOT be computed by solving this dual problem using FEM.

We need a postprocessing formula to obtain a functional $\tilde{L}$ asymptotically equivalent to L.

I. Babuska, A. Miller, *The Post-Processing Approach in the FEM*, 1984.

AUTOMATIC GOAL-ORIENTED *HP*-ADAPTIVITY

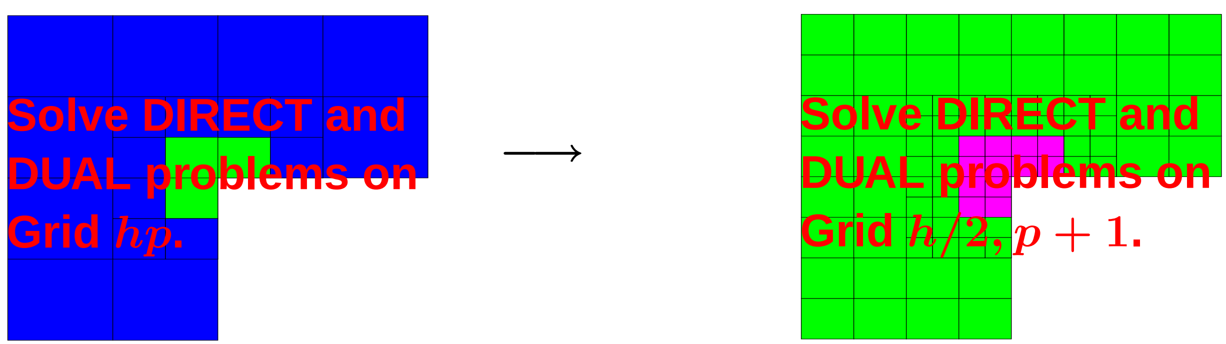
Algorithm for Goal-Oriented Adaptivity



Compute $e = e_{h/2,p+1} - e_{hp}$, and $\epsilon = G_{h/2,p+1} - G_{hp}$.

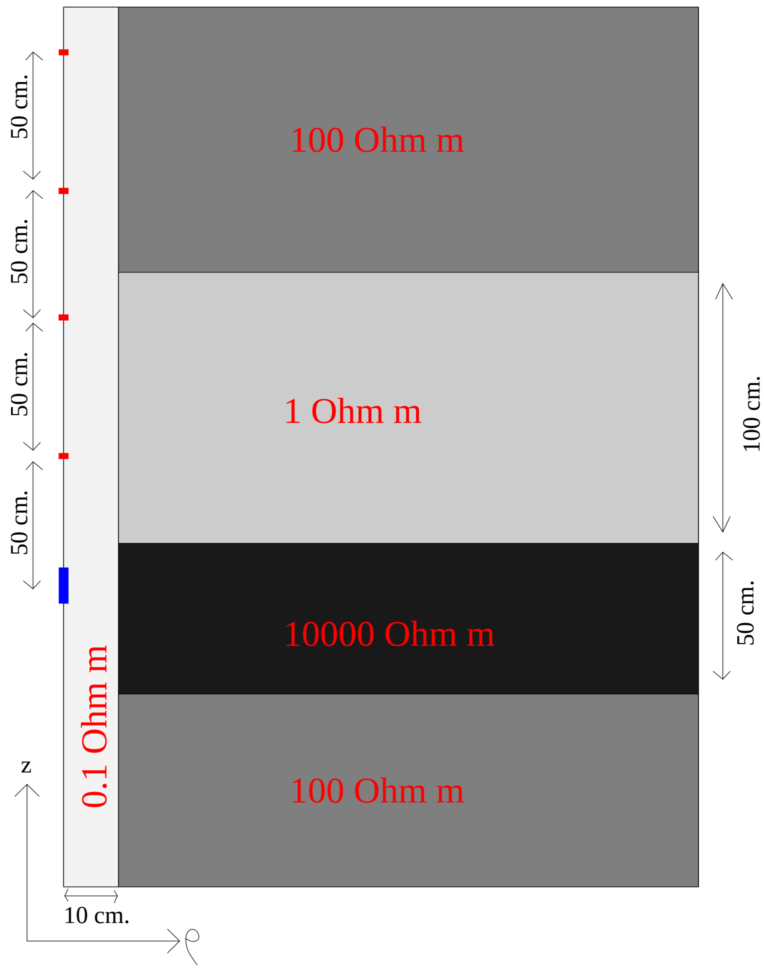
Use estimate $|L(e)| = |b(e, \epsilon)| \leq \sum_K |b_K(e, \epsilon)|$.

Apply the fully automatic hp-adaptive algorithm.



NUMERICAL RESULTS

A Direct Current (DC) Resistivity Logging Problem (Baker-Atlas)



Axisymmetric 3D problem.

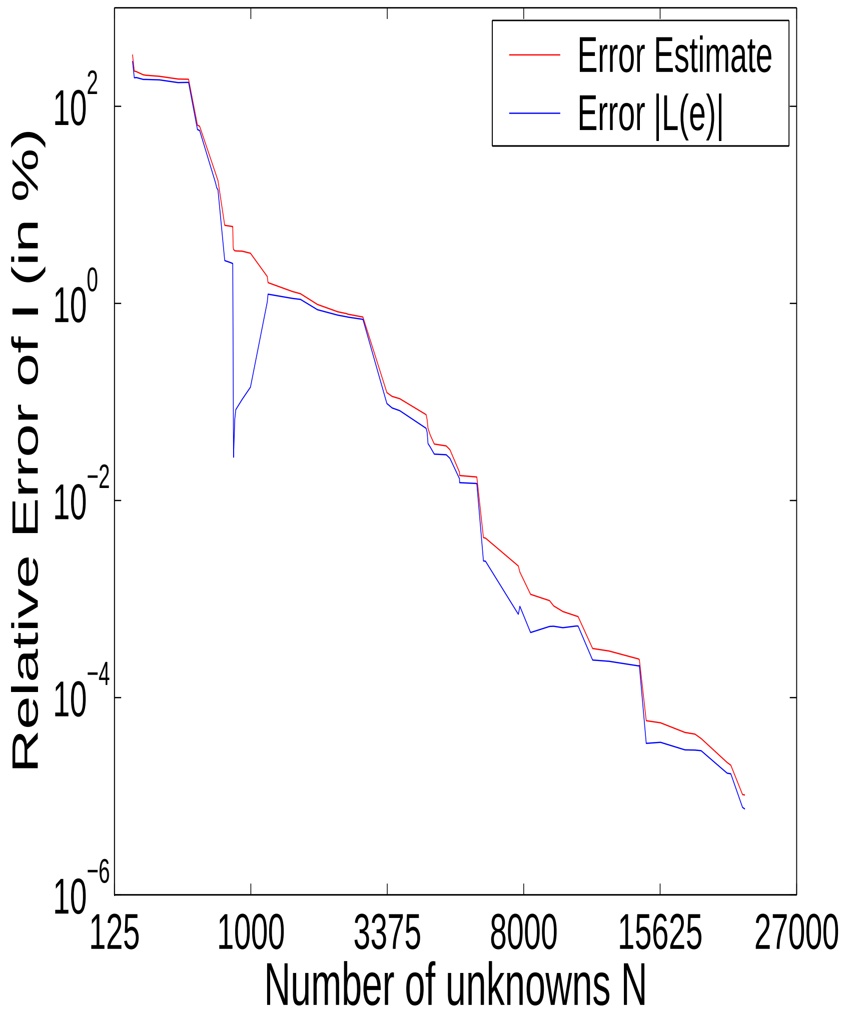
Four different materials.

Material properties varying by up to FIVE orders of magnitude.

Objective:
Determine Electric Current on Receiving Electrodes.

NUMERICAL RESULTS

Convergence History



DC Resistivity Logging Problem with Different Materials.

Distance Between Source and Receiving Electrode: $150cm$.

$|L(e)| \leq \sum_K |b(e,\epsilon)| =$ **Error Estimate.**

Relative Error (in $\%$) vs dB

10^{-6} **% =** 10^{-7} **dB**

10^{-4} **% =** 10^{-5} **dB**

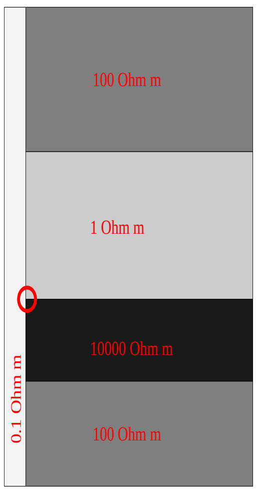
10^{-2} **% =** 10^{-3} **dB**

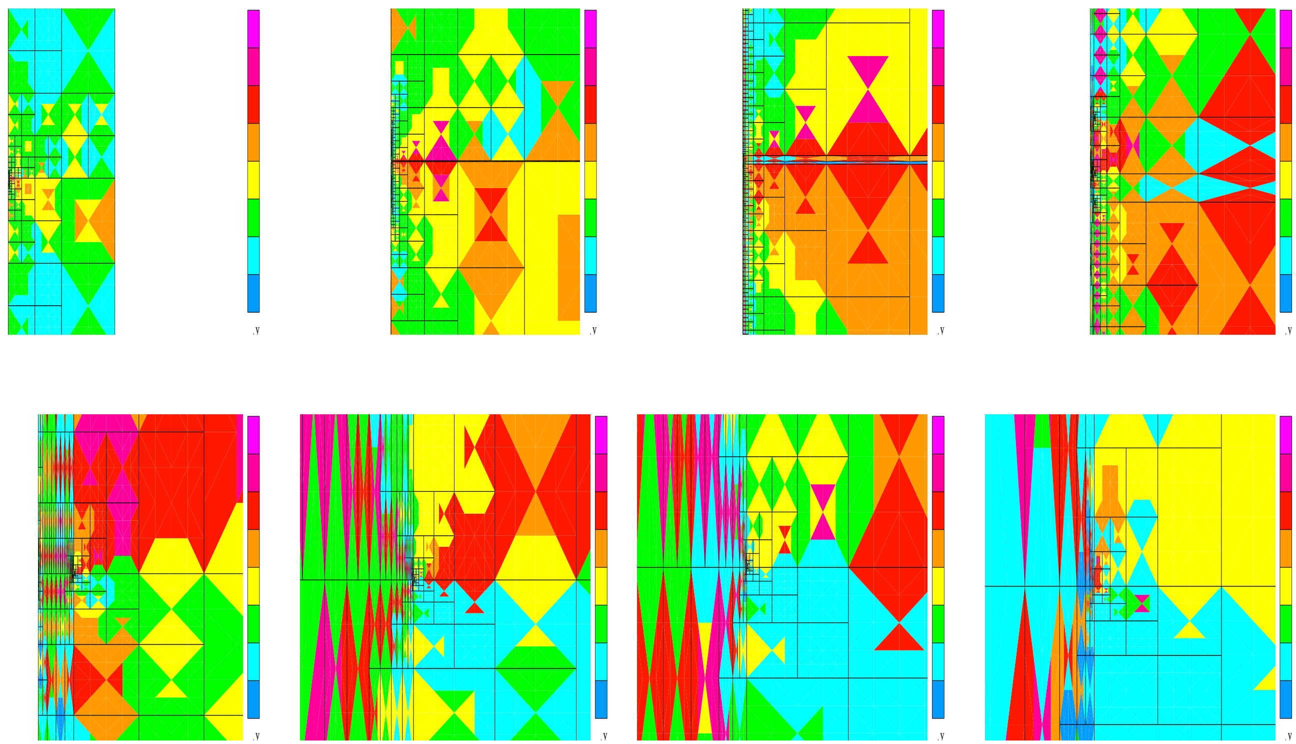
10^{0} **% =** 10^{-1} **dB**

10^{2} **% =** 10^{-1} **dB**

NUMERICAL RESULTS

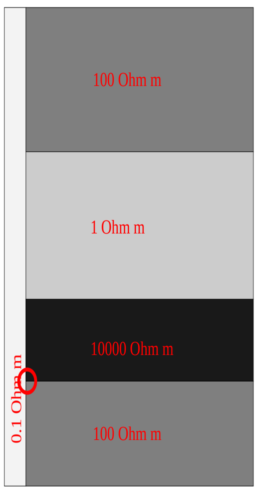
Final hp-grid (Zooms by factor of 10)

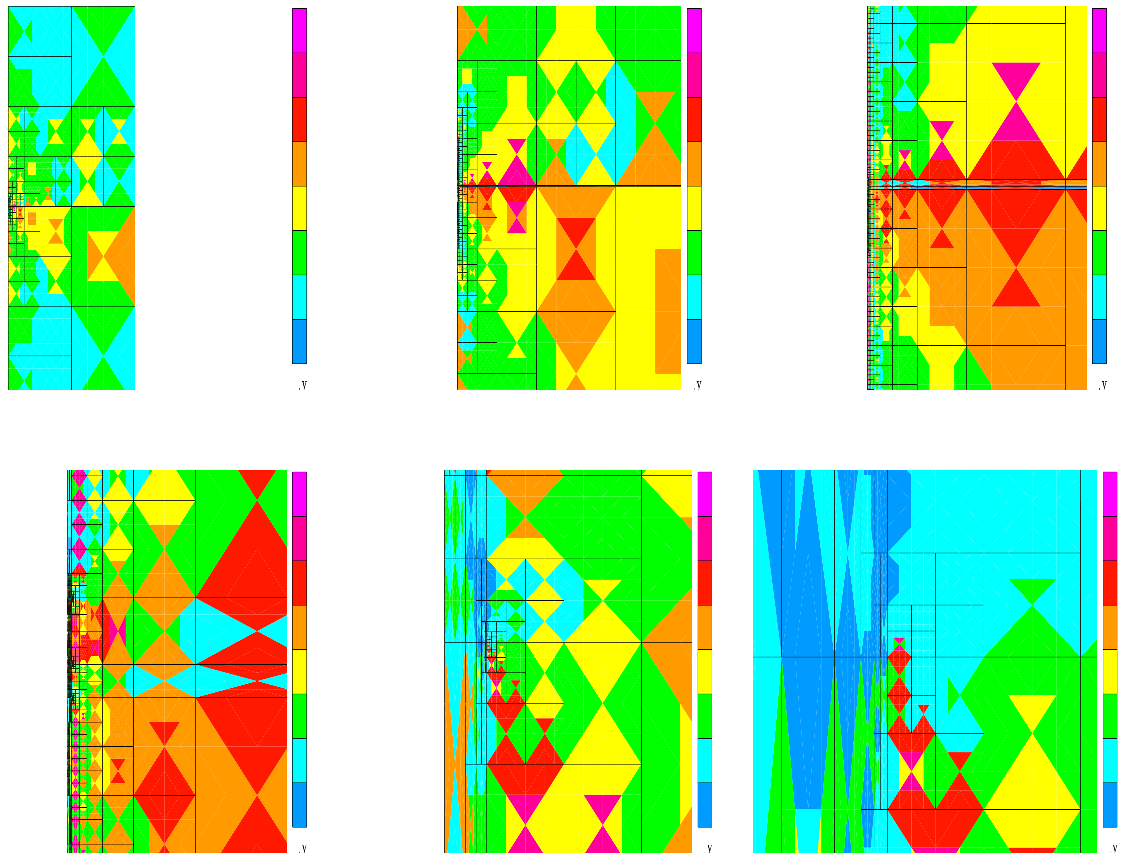




NUMERICAL RESULTS

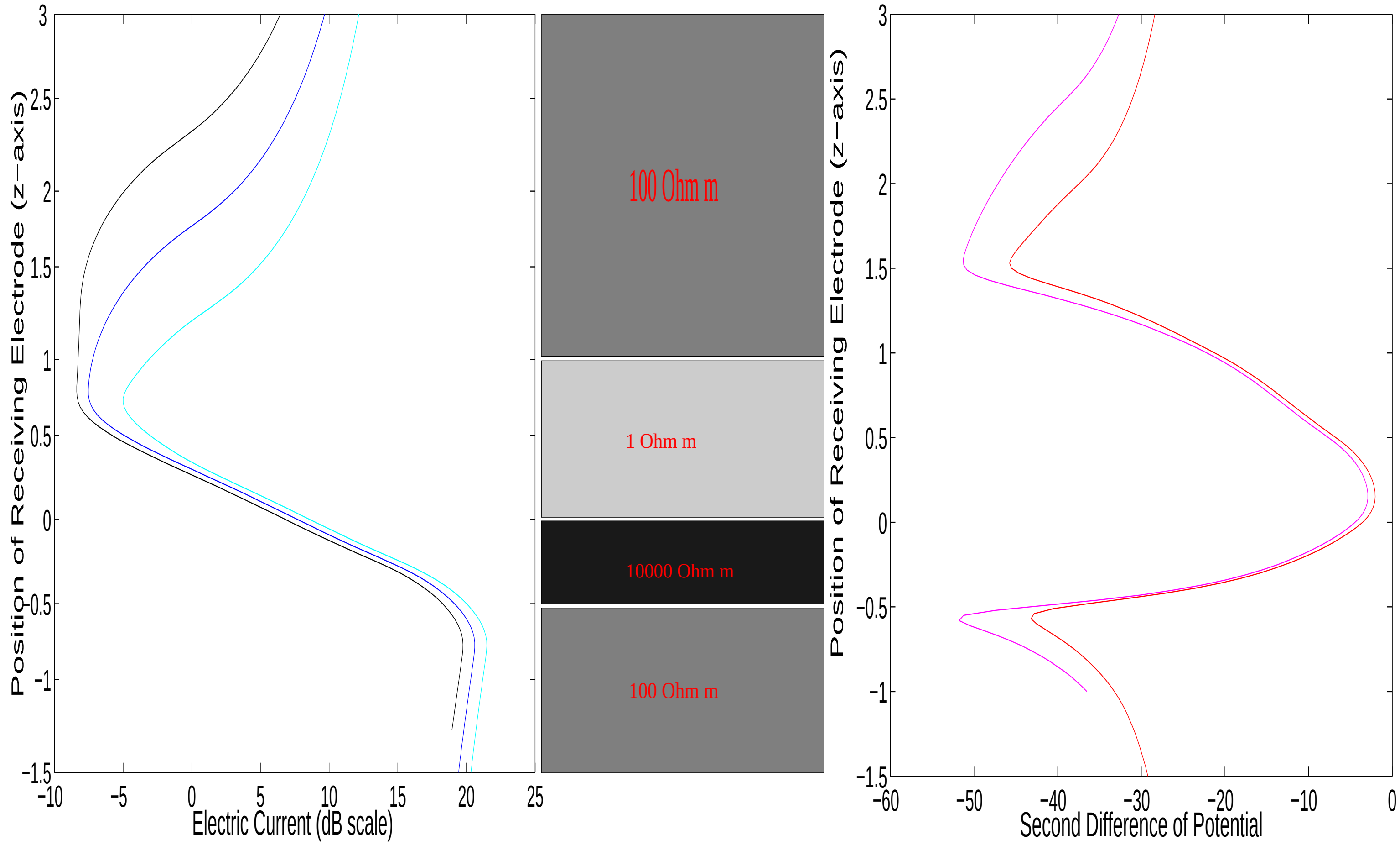
Final hp-grid (Zooms by factor of 10)





NUMERICAL RESULTS

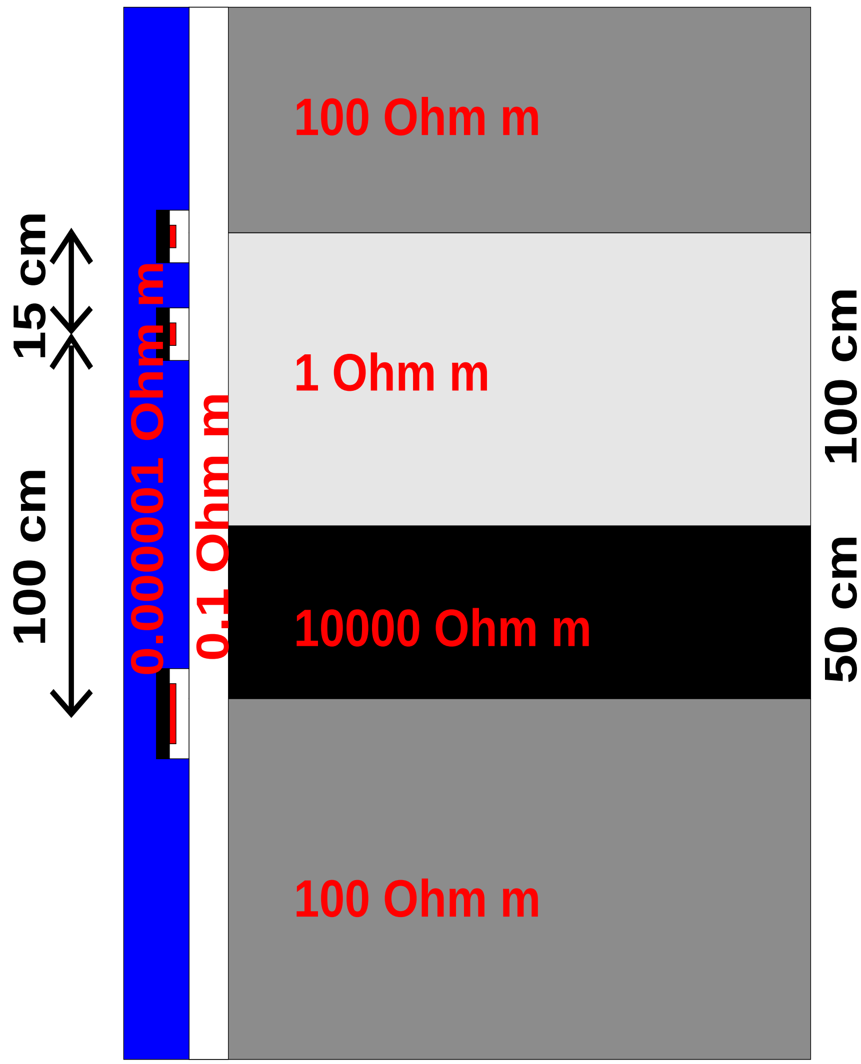
Final Log Obtained by Our Finite Element Software



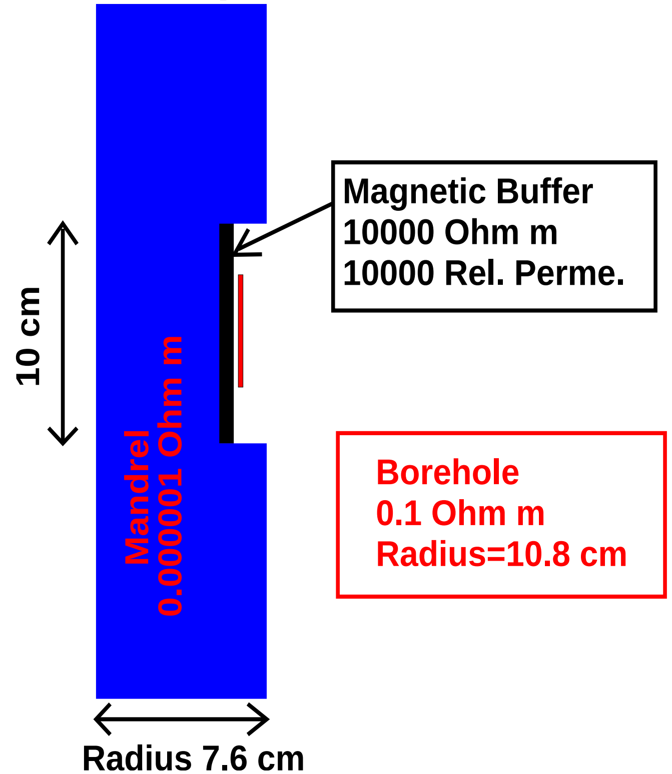
Distance between source and first receiving electrode:

0.5 m -light blue- ; 1.0 m -dark blue- ; 1.5 m -black- **0.5 m -red- ; 1.0 m -magenta-**

NUMERICAL RESULTS



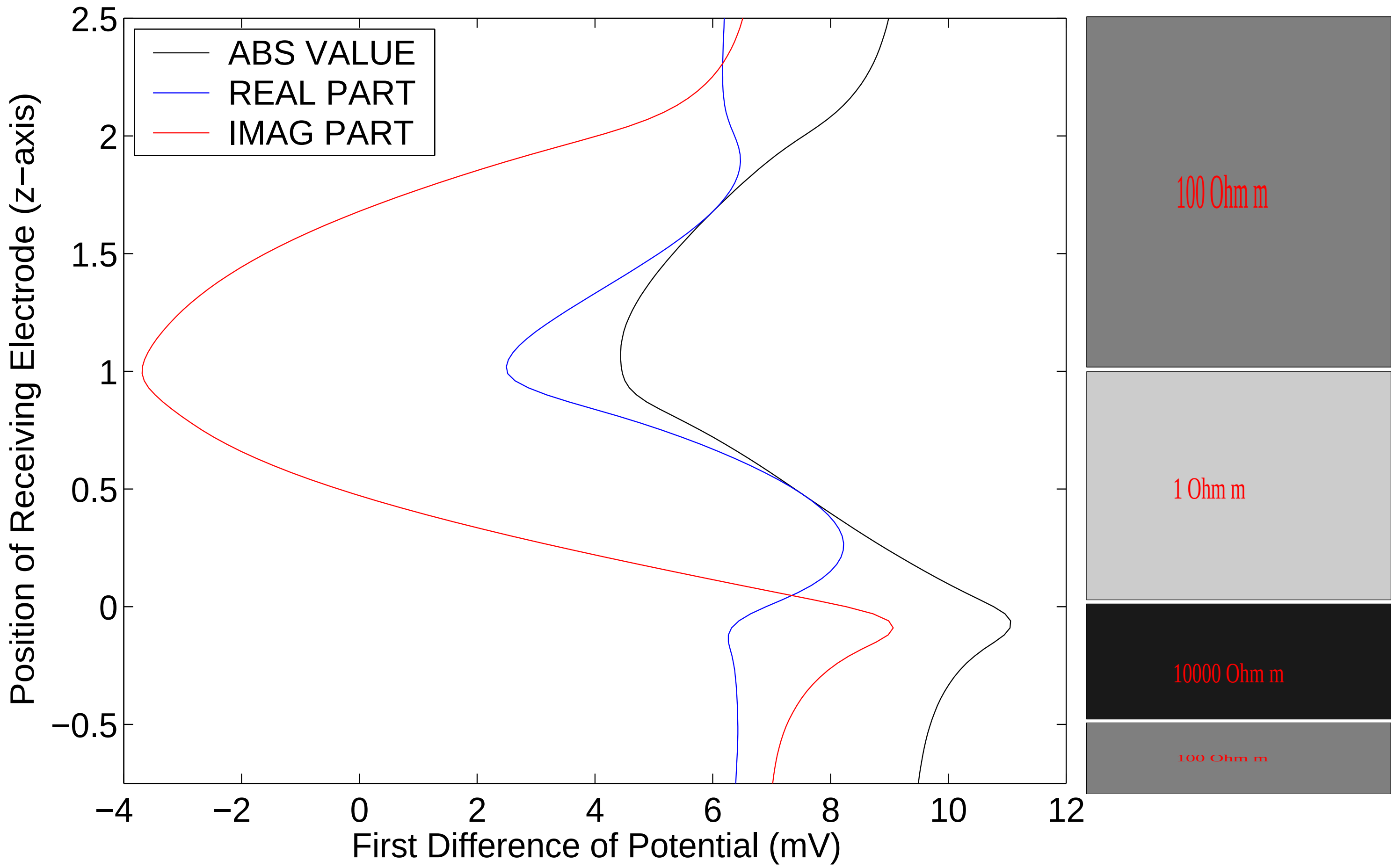
Description of Antennas



Goal: To Compute First Difference of Potential on Receiving Electrodes

NUMERICAL RESULTS

Final Log Obtained by Our Finite Element Software



Frequency: 2 Mhz

CONCLUSIONS AND FUTURE WORK

Conclusions

- **The Fully Automatic Goal-Oriented hp-Adaptive Algorithm converges exponentially in terms of the quantity of interest vs the CPU time.**

- **We accurately simulated challenging Resistivity Logging Problems.**

Future Work

- **To improve performance of the self-adaptive goal-oriented algorithm.**

- **To extend the self-adaptive goal-oriented algorithm to simulate challenging 3D and inverse petroleum engineering problems.**