ADMOS 2005: International Conference on Adaptive Modeling and Simulation

High Accuracy Simulations of Resistivity Logging Instruments Using a Self-Adaptive Goal-Oriented hp Finite Element Method

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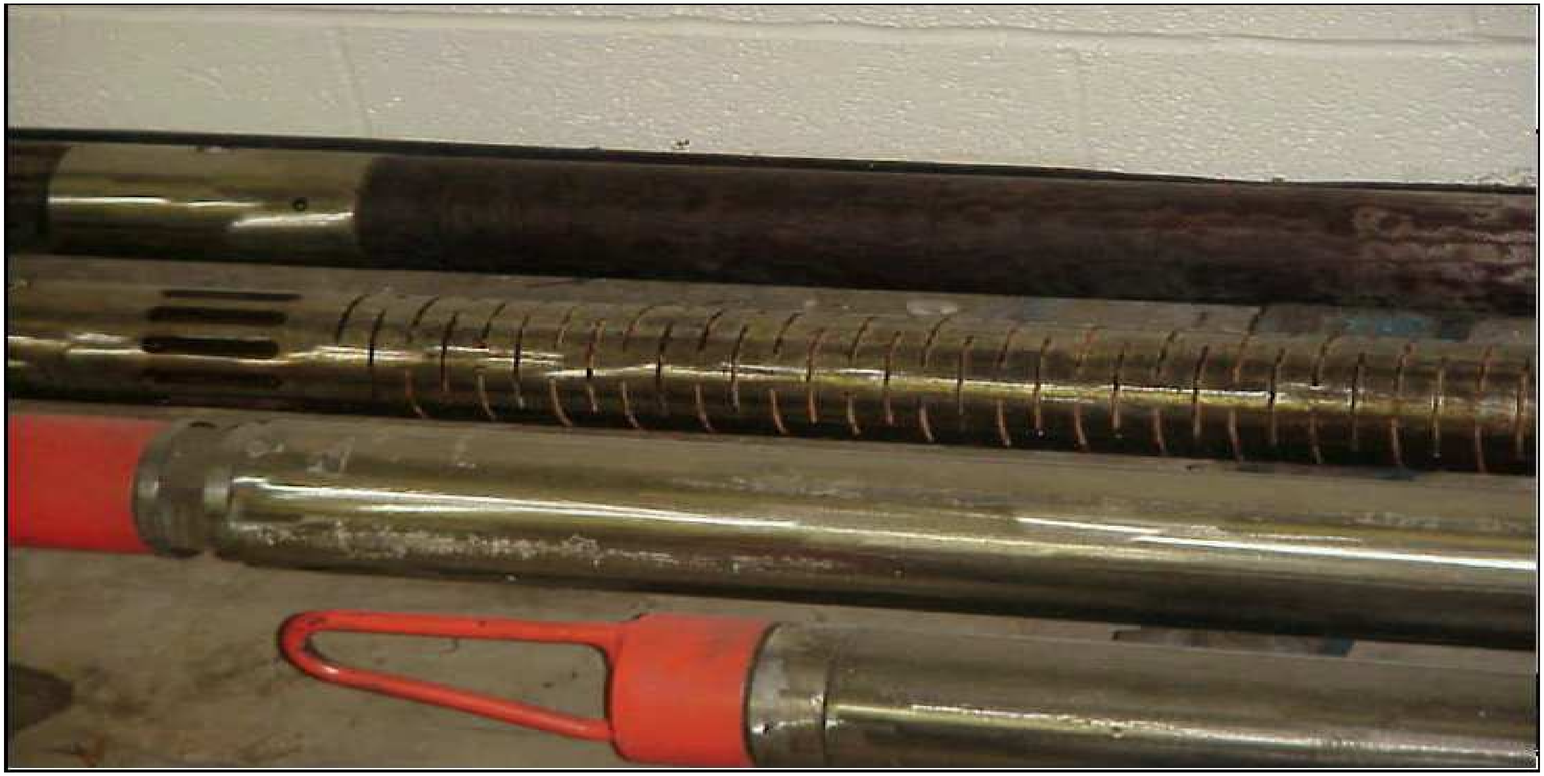
The University of Texas at Austin

OVERVIEW

1. **Motivation: Simulation of Resistivity Logging Instruments.**
2. **Methodology:**
   - **The hp-Finite Element Method (FEM) - Exponential Convergence - .**
   - **Automatic Goal-Oriented Refinements - in the Quantity of Interest -.**
   - **Flexibility, Reliability, High Accuracy and High Performance.**
3. **Numerical Results:**
   - **Simulation of Resistivity Logging Instruments with Casing.**
   - **Simulation of Resistivity Logging Instruments with Mandrel.**
4. **Conclusions and Future Work (3D Problems, Multi-physics).**

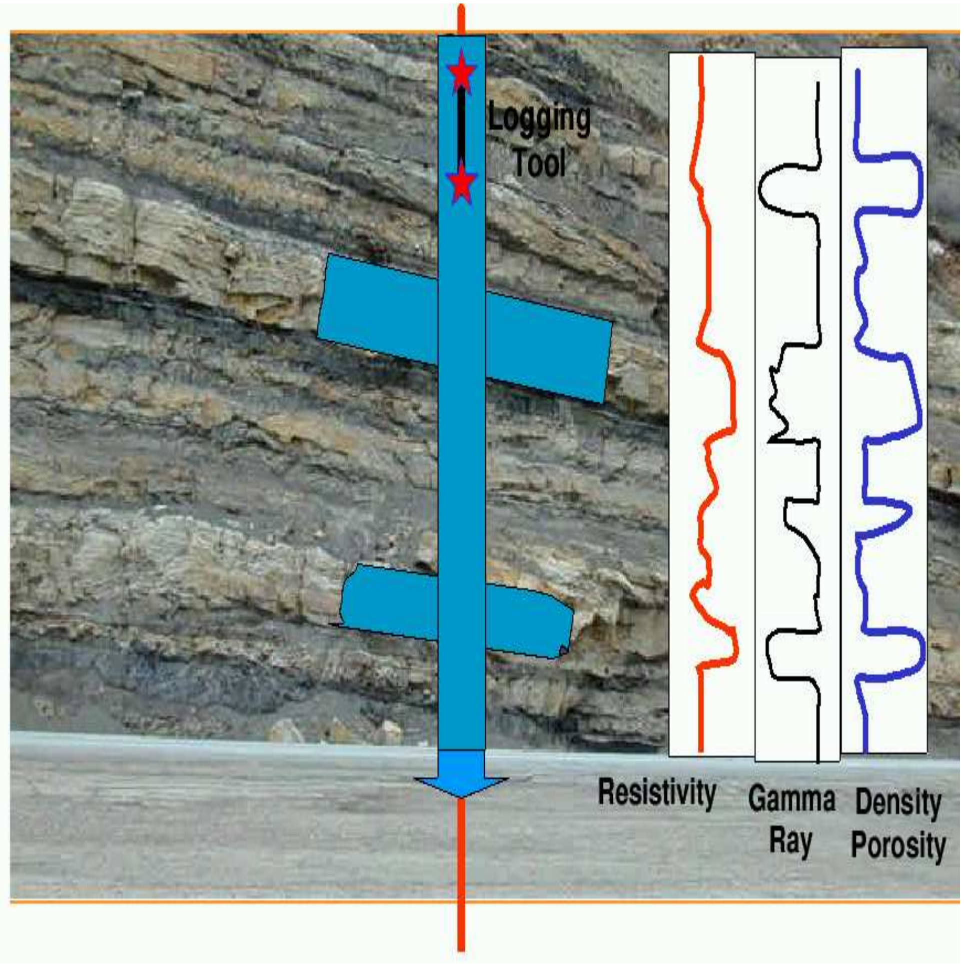
RESISTIVITY LOGGING INSTRUMENTS

Logging Instruments: Definition



RESISTIVITY LOGGING INSTRUMENTS

Utility of Logging Instruments



OBJECTIVES: To determine

- **Payzones (oil and gas).**
- **Amount of oil/gas.**
- **Ability to extract oil/gas.**

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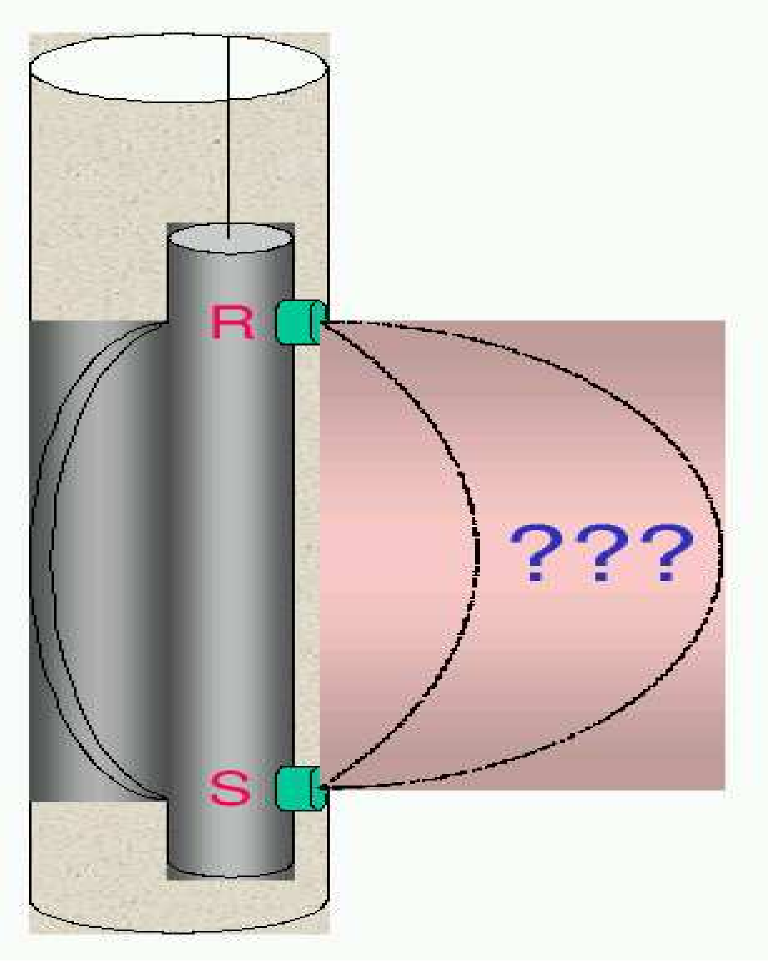
RESISTIVITY LOGGING INSTRUMENTS

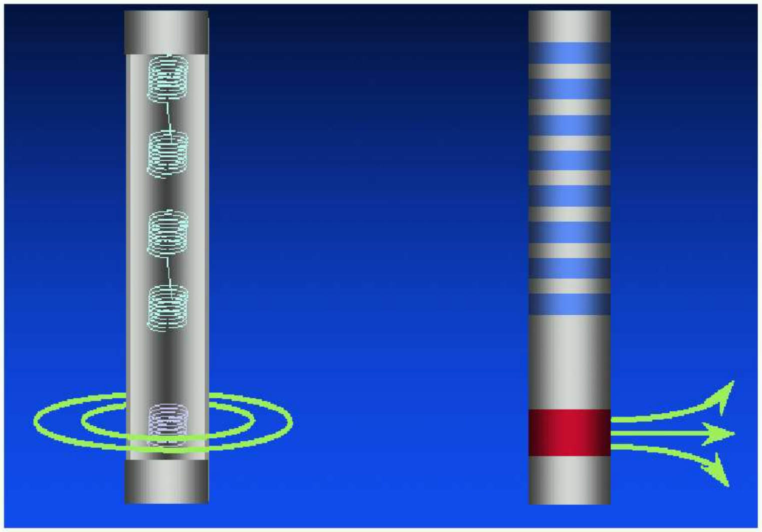
Main Objective: To Solve an Inverse Problem



A software for solving the DIRECT problem is essential in order to solve the INVERSE problem

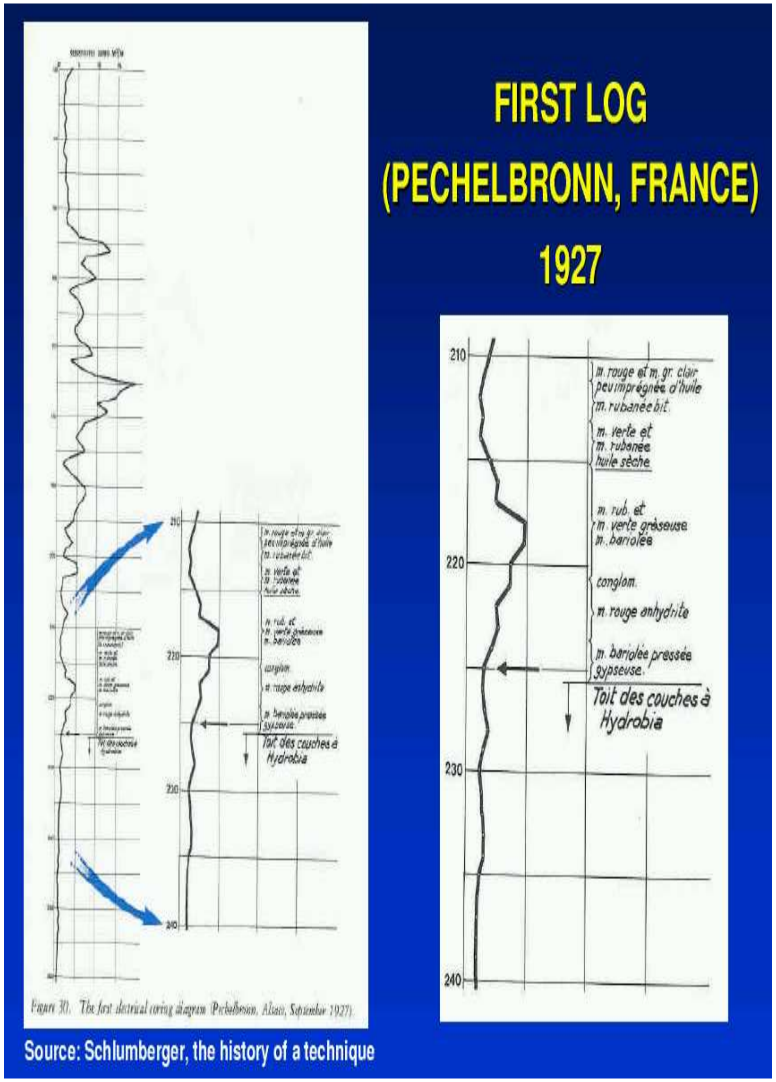
RESISTIVITY LOGGING INSTRUMENTS

Resistivity Logging Instruments



RESISTIVITY LOGGING INSTRUMENTS

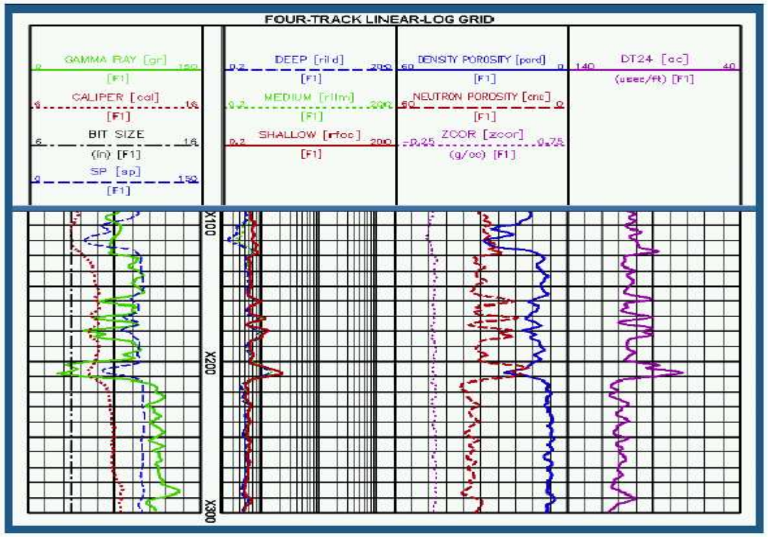
Final Result Obtained from the Logging Instruments

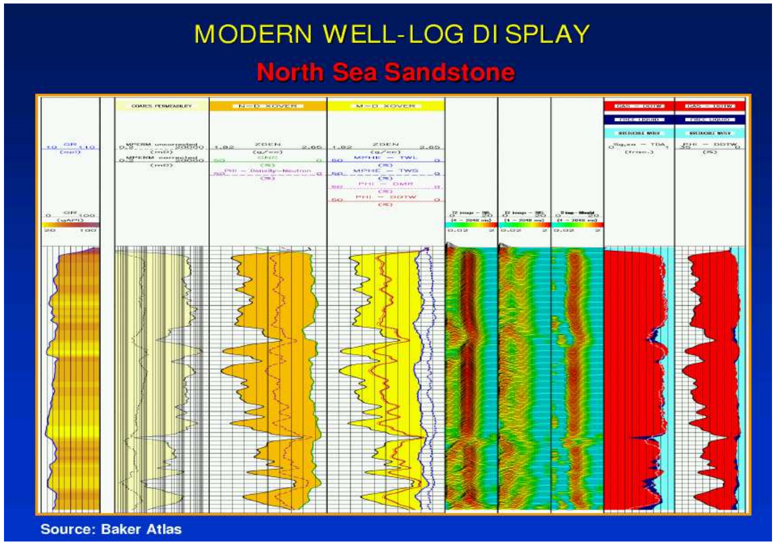


FIRST LOG (PECHELBRONN, FRANCE) 1927

Figure 30. The first electrical coring diagram (Pechelbronn, Alsace, September 1927).

Source: Schlumberger, the history of a technique



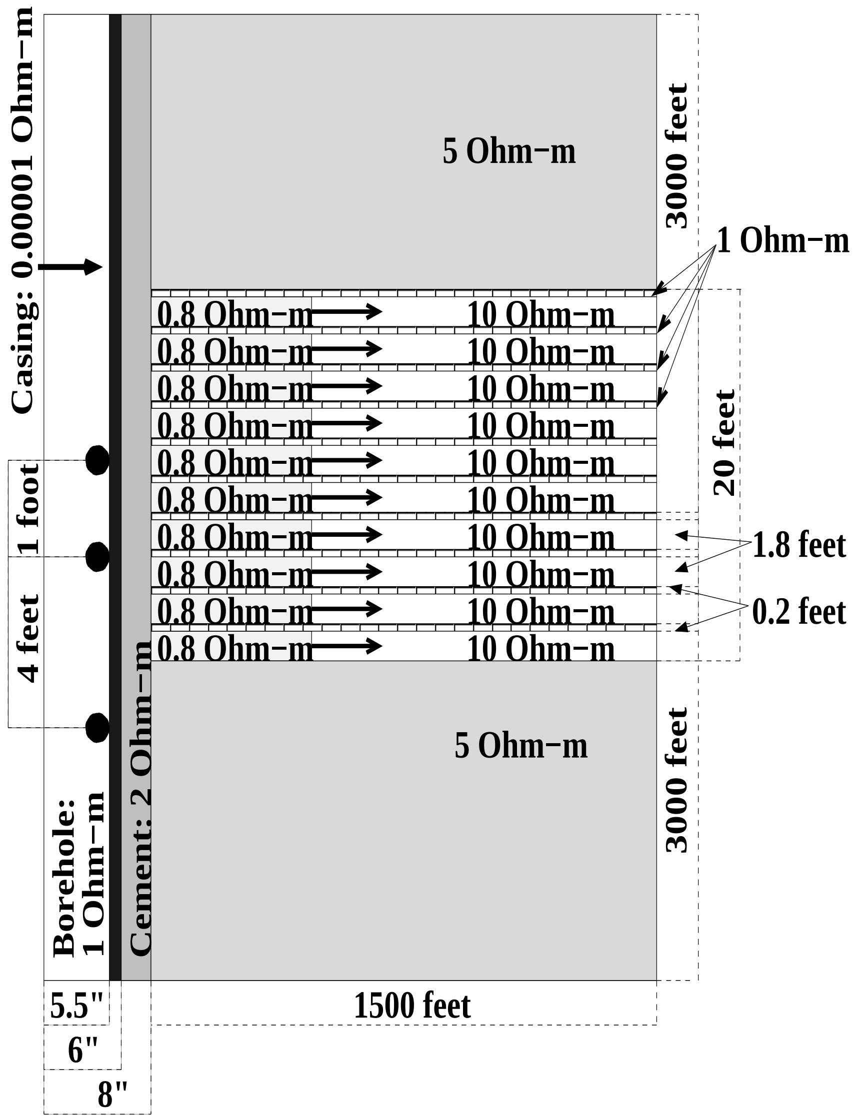


MODERN WELL-LOG DISPLAY

North Sea Sandstone

Source: Baker Atlas

RESISTIVITY LOGGING INSTRUMENTS



Axisymmetric 3D problem.

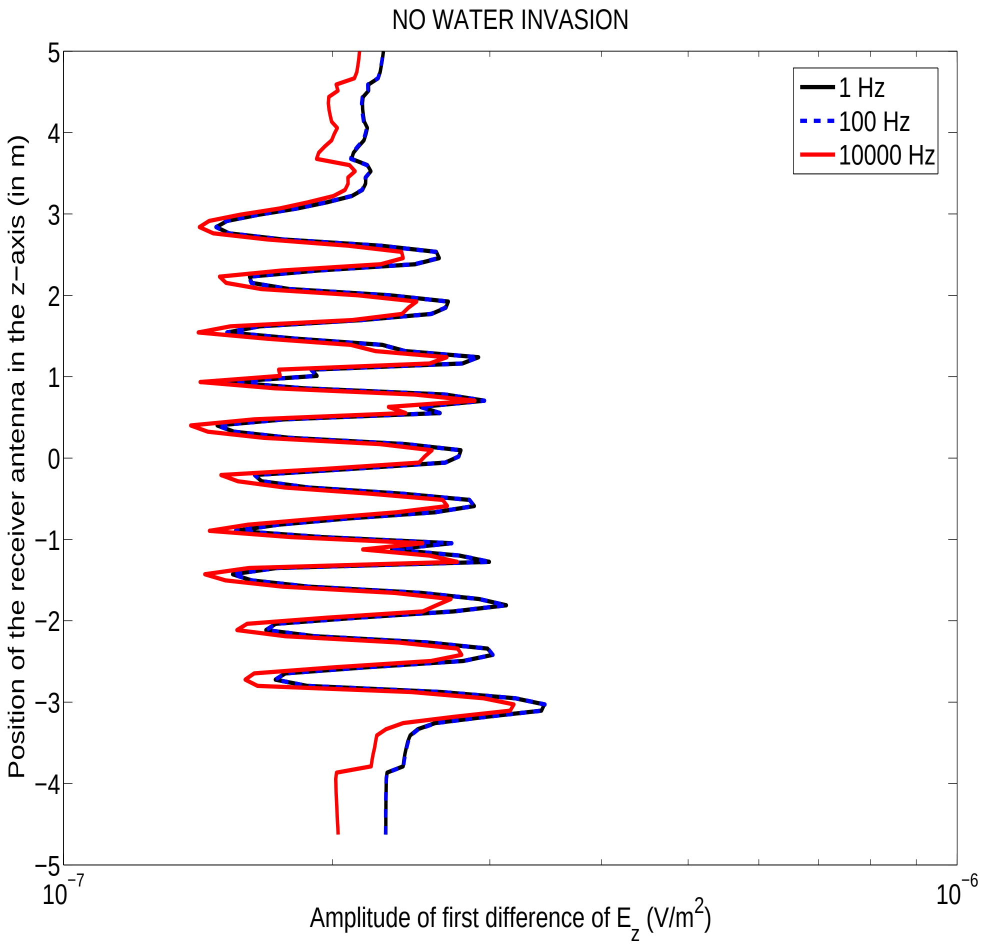
Seven different materials.

Through casing resistivity instrument.

Large variations on resistivity.

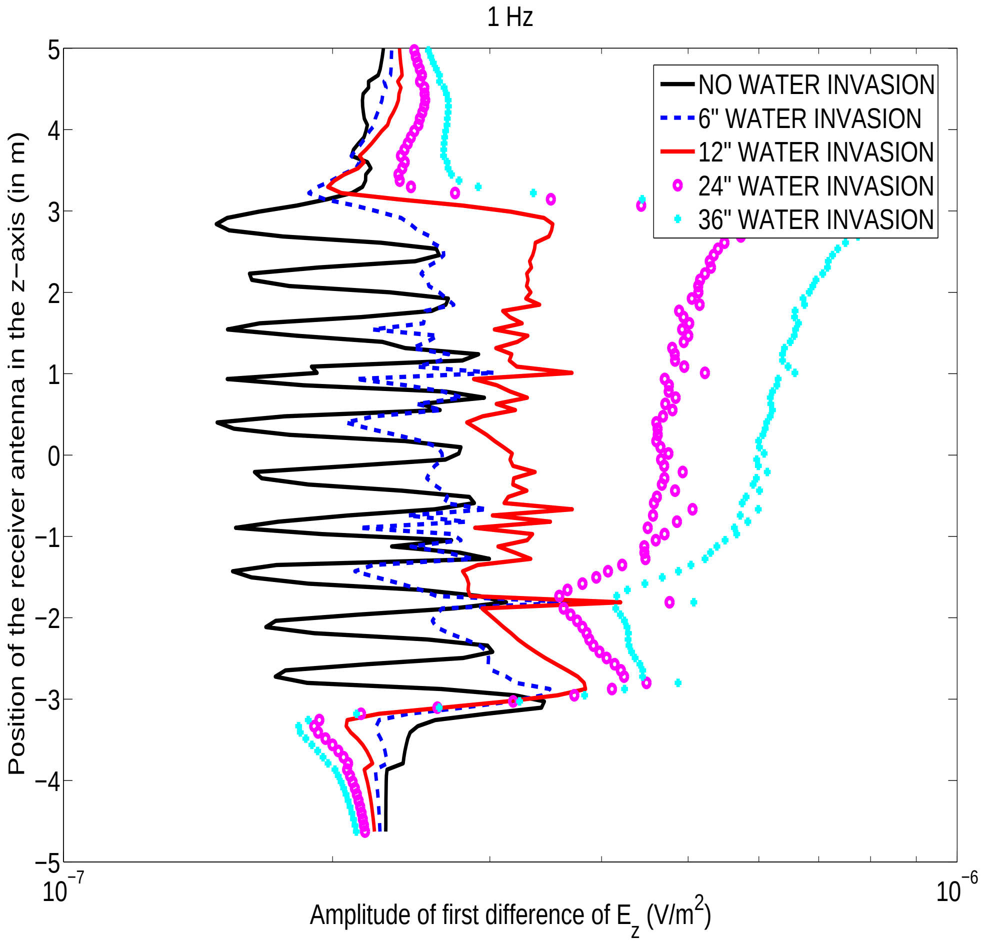
Objective: Study the effect of invasion THROUGH CASING on laminated sands.

RESISTIVITY LOGGING INSTRUMENTS



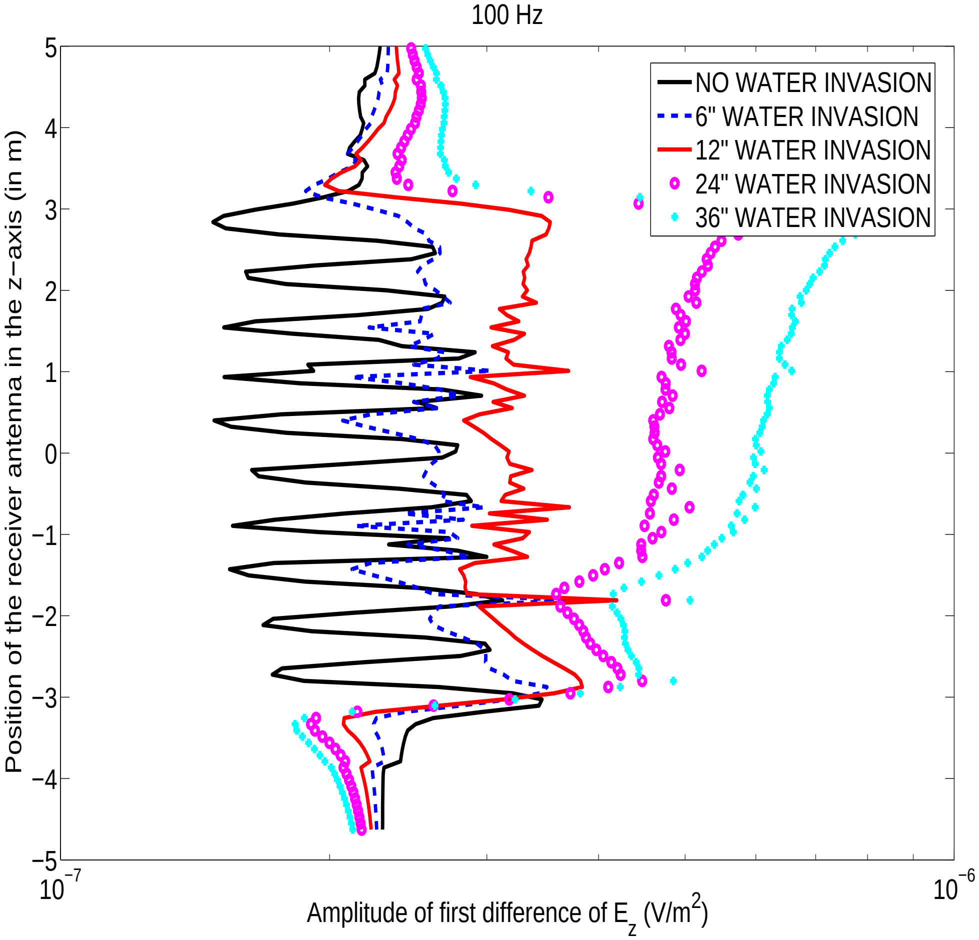
Variations due to frequency are small (below 5%)

RESISTIVITY LOGGING INSTRUMENTS



Variations due to water ivasion are large

RESISTIVITY LOGGING INSTRUMENTS



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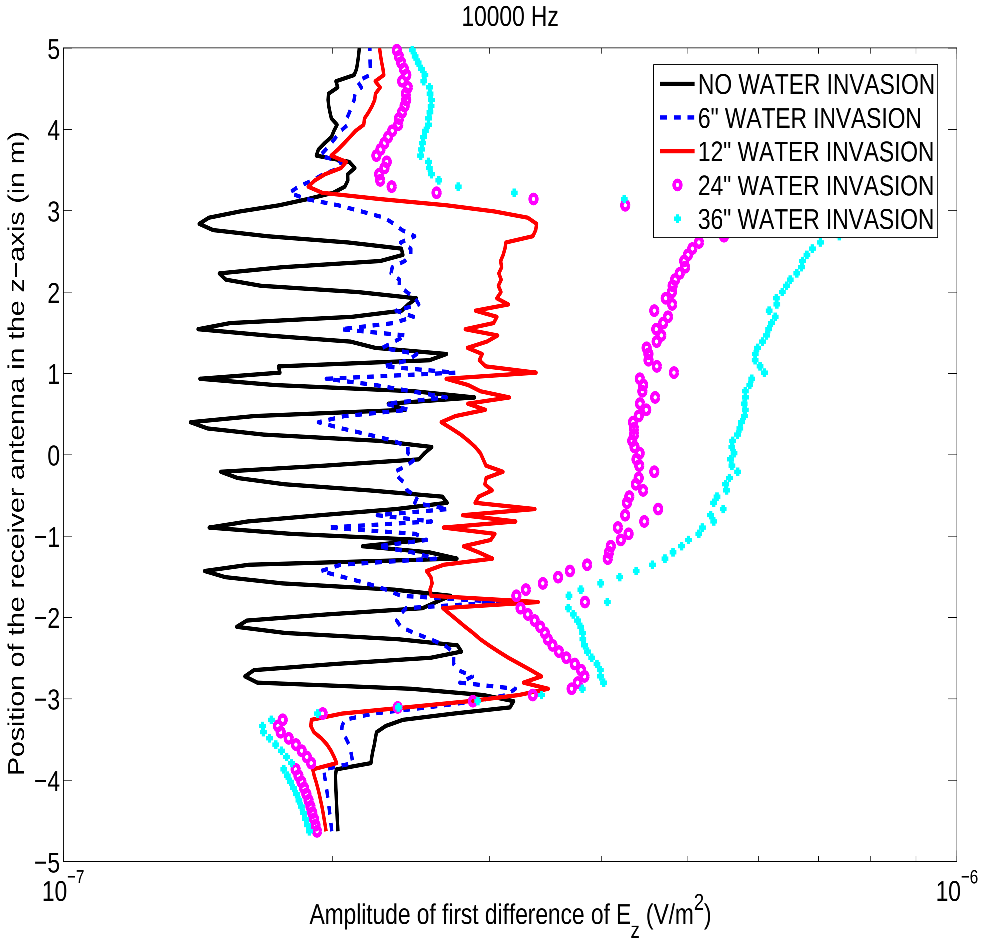

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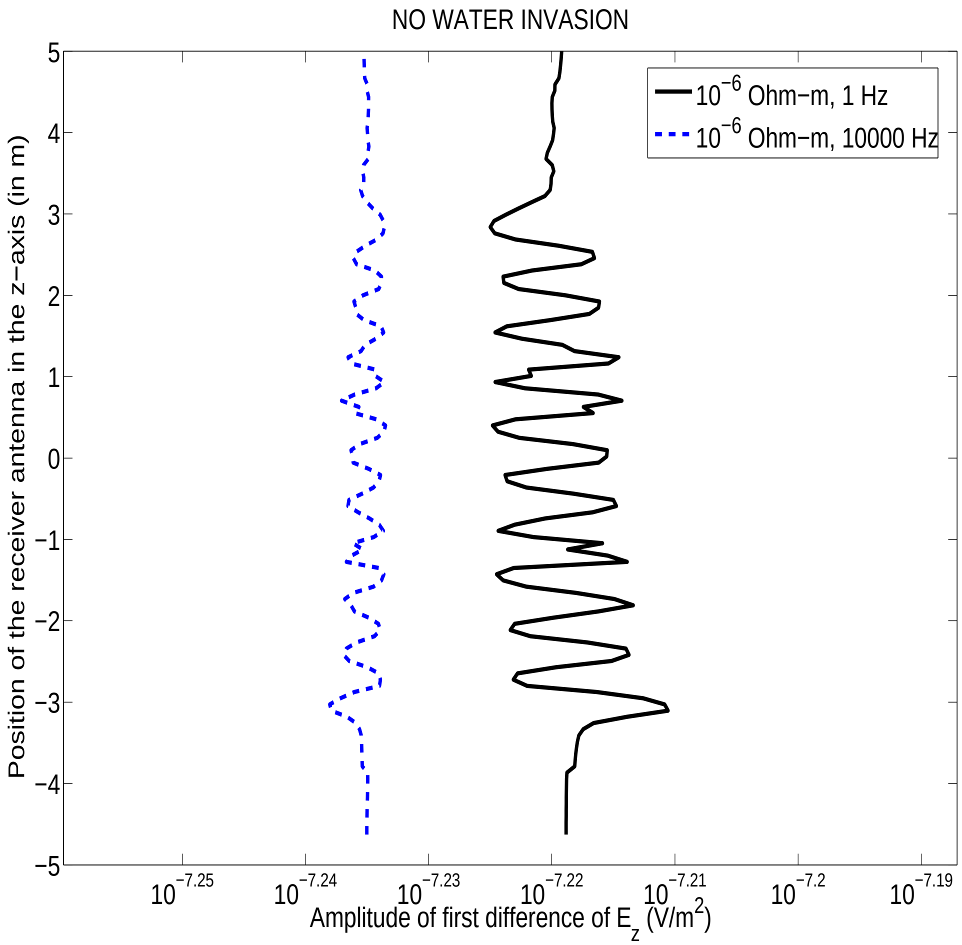
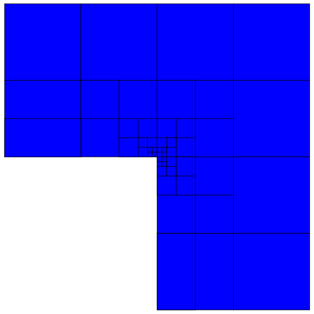


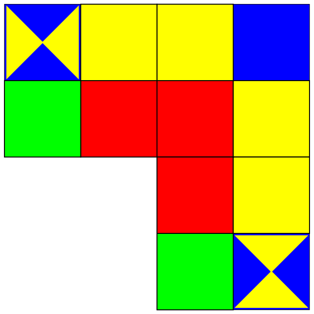
Casing resistivity can be analyzed from different frequency measurements

THE hp-FINITE ELEMENT METHOD (FEM)



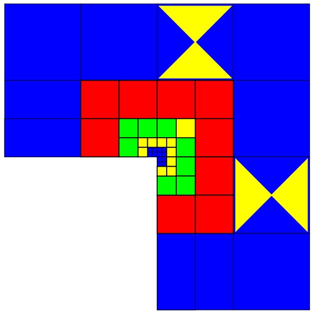
The h-Finite Element Method

1. Convergence limited by the polynomial degree, and large material contrasts.
2. Optimal *h*-grids do NOT converge exponentially in real applications.
3. They may "lock" (100% error).



The p-Finite Element Method

1. Exponential convergence feasible for analytical ("nice") solutions.
2. Optimal *p*-grids do NOT converge exponentially in real applications.
3. If initial *h*-grid is not adequate, the *p*-method will fail miserably.



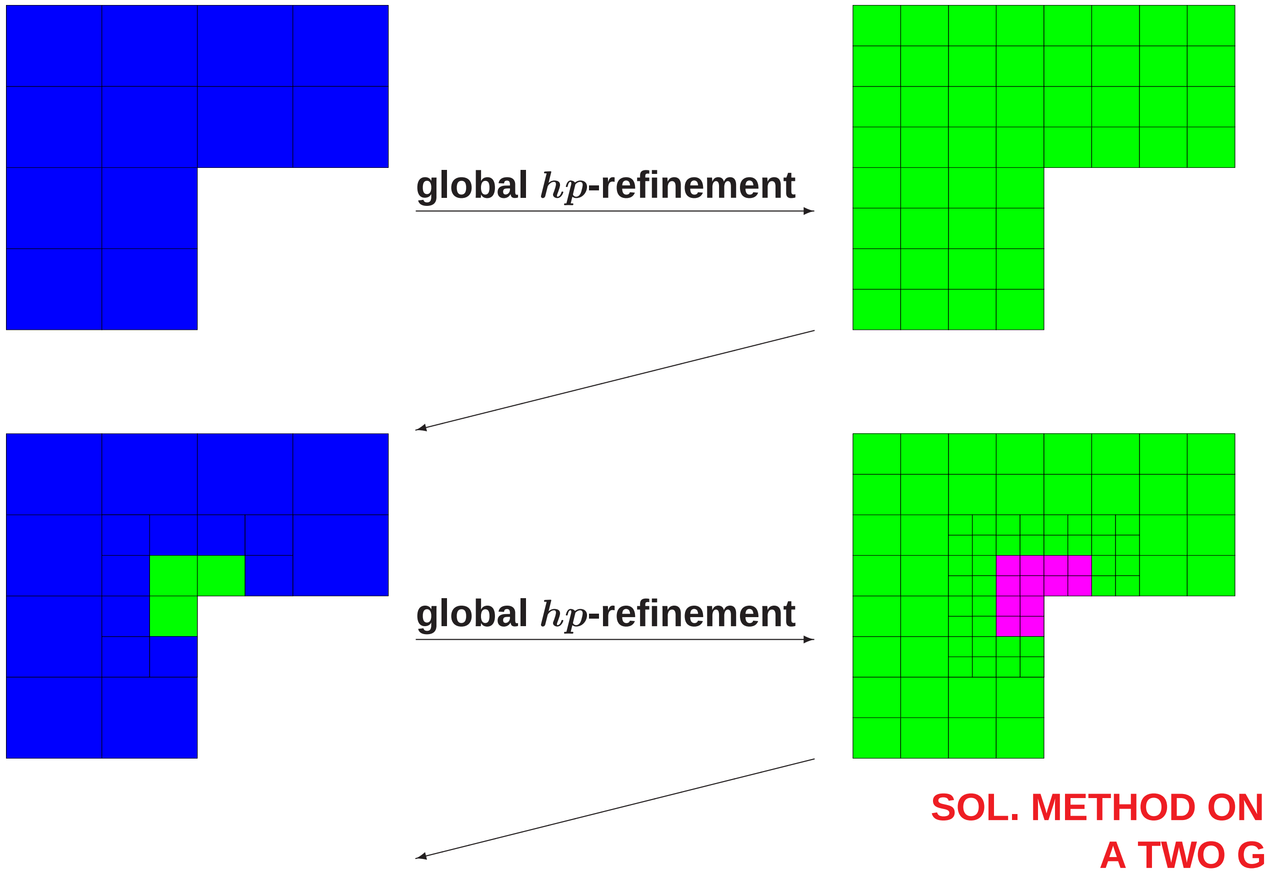
The hp-Finite Element Method

1. Exponential convergence feasible for ALL solutions.
2. Optimal *hp*-grids DO converge exponentially in real applications.
3. If initial *hp*-grid is not adequate, results will still be great.

SELF-ADAPTIVE hp-FEM

Energy norm based fully automatic *hp*-adaptive strategy

Coarse grids (hp) **Fine grids** ($h/2, p+1$)



global hp-refinement

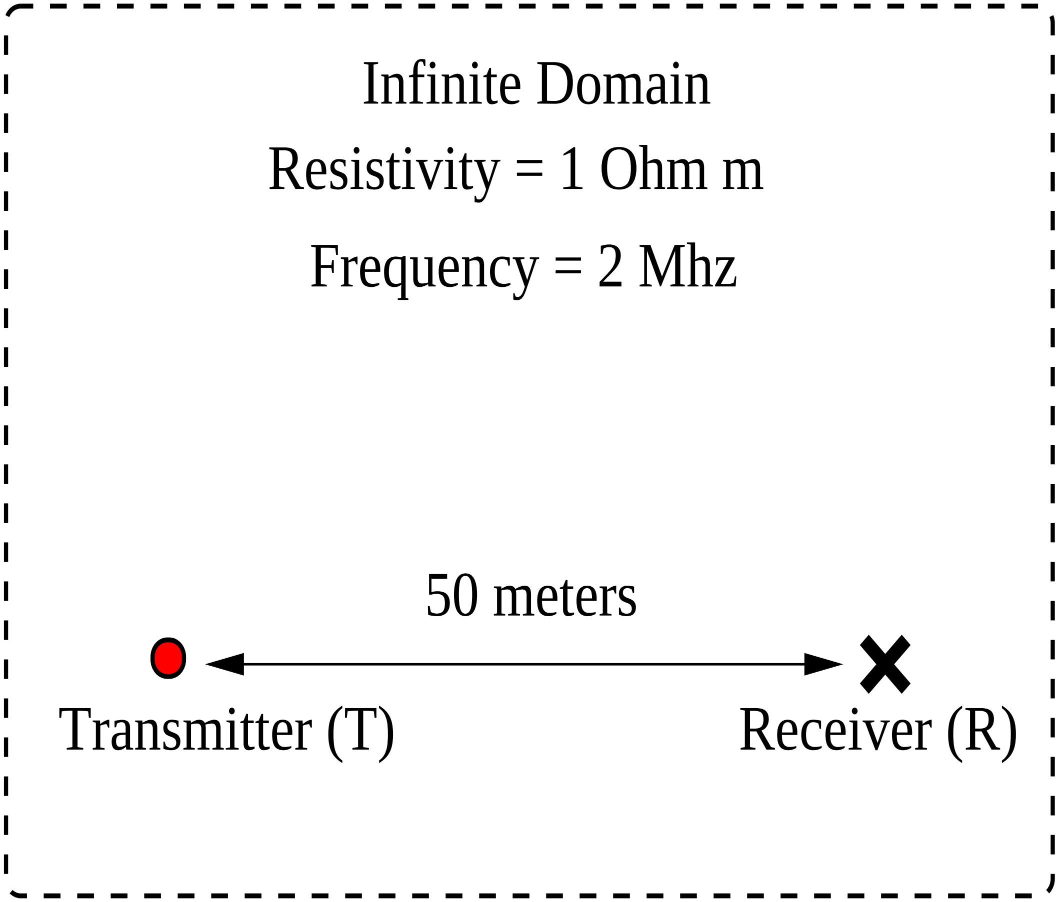
global hp-refinement

SOL. METHOD ON FINE GRIDS:
A TWO GRID SOLVER

SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Motivation (Goal-Oriented Adaptivity)

Test Problem



Infinite Domain

Resistivity = 1 Ohm m

Frequency = 2 Mhz

50 meters

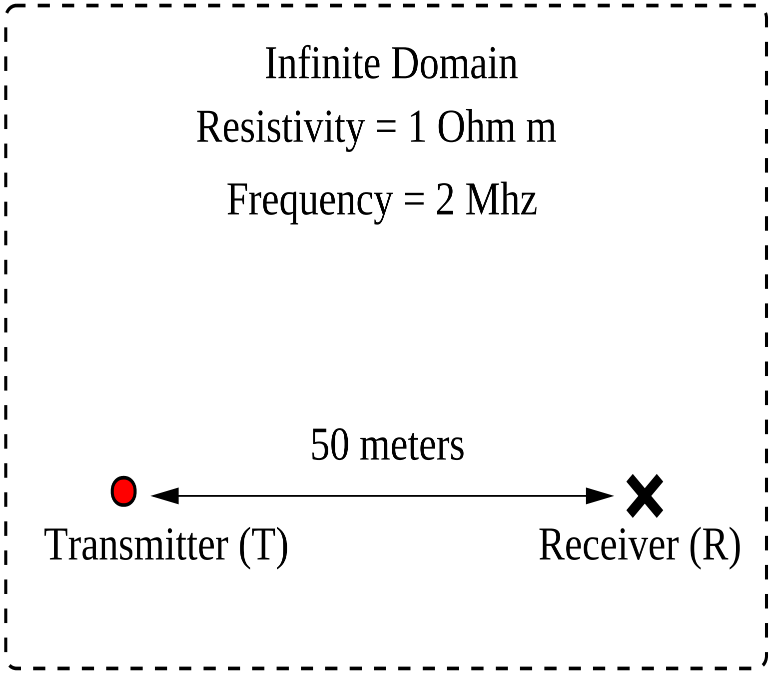
Transmitter (T)

Receiver (R)

SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Motivation (Goal-Oriented Adaptivity)

Test Problem



Infinite Domain

Resistivity = 1 Ohm m

Frequency = 2 Mhz

50 meters

Transmitter (T)

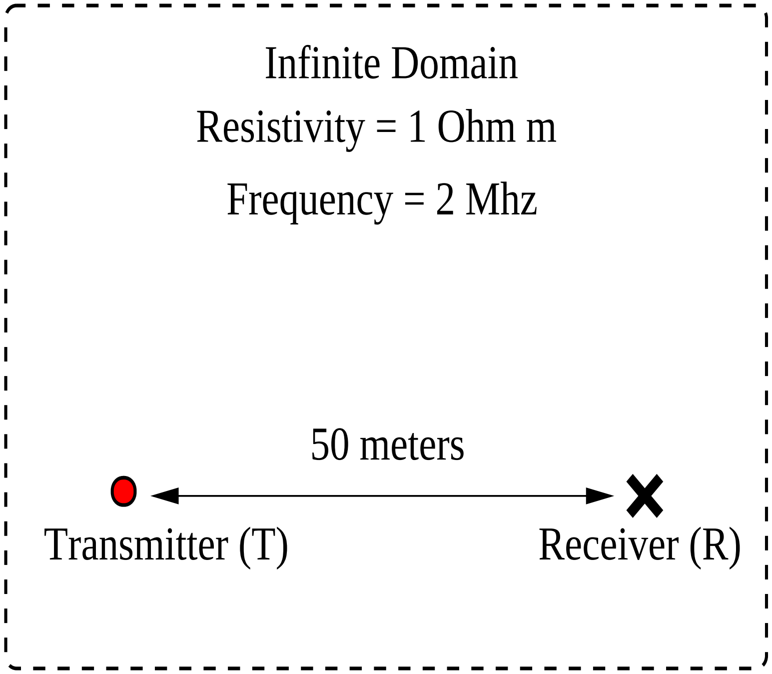
Receiver (R)

- **Solution decays exponentially.**
- $\dfrac{|\boldsymbol{E(T)}|}{|\boldsymbol{E(R)}|} \approx 10^{60}$
- **Results using energy-norm adaptivity:**
  - **Energy-norm error:** 0.001%
  - **Relative error in the quantity of interest** $> 10^{30}$ $\%$**.**

SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Motivation (Goal-Oriented Adaptivity)

Test Problem



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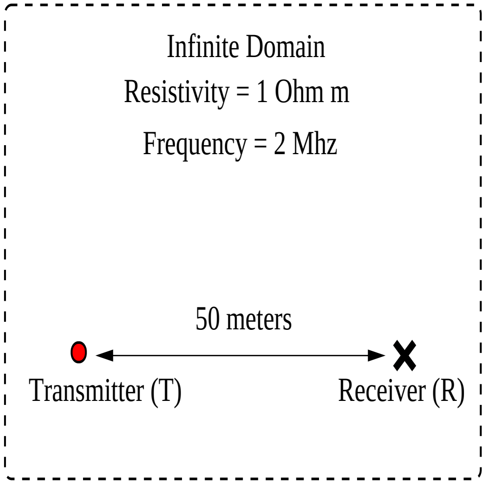
Goal-oriented adaptivity is needed

Becker-Rannacher (1995,1996), Rannacher-Stuttmeier (1997), Cirak-Ramm (1998), Paraschivoiu-Patera (1998), Peraire-Patera (1998), Prudhomme-Oden (1999, 2001), Heuveline-Rannacher (2003), Solin-Demkowicz (2004).

SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Motivation (Goal-Oriented Adaptivity)

Test Problem



Infinite Domain

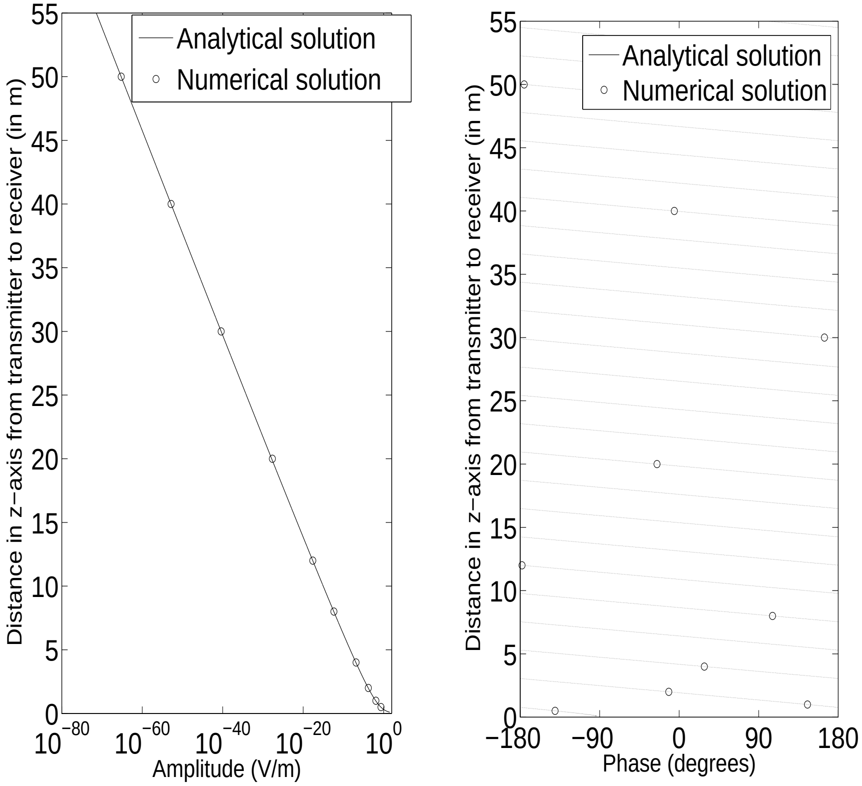
Resistivity = 1 Ohm m

Frequency = 2 Mhz

50 meters

Transmitter (T)

Receiver (R)



Goal-oriented adaptivity is needed

SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Mathematical Formulation (Goal-Oriented Adaptivity)

We consider the following problem (in variational form):

$$\left\{\begin{array}{l}\textbf{Find } L(\Psi)\textbf{, where } \Psi \in V \textbf{ such that :}\\ b(\Psi,\xi) = f(\xi) \quad \forall \xi \in V \ .\end{array}\right.$$

We define residual $r_e(\xi) = b(e,\xi)$. We seek for solution G of:

$$\left\{\begin{array}{l}\textbf{Find } G \in V'' \sim V \textbf{ such that :}\\ G(r_e) = L(e) \ .\end{array}\right.$$

This is necessarily solved if we find the solution of the *dual* problem:

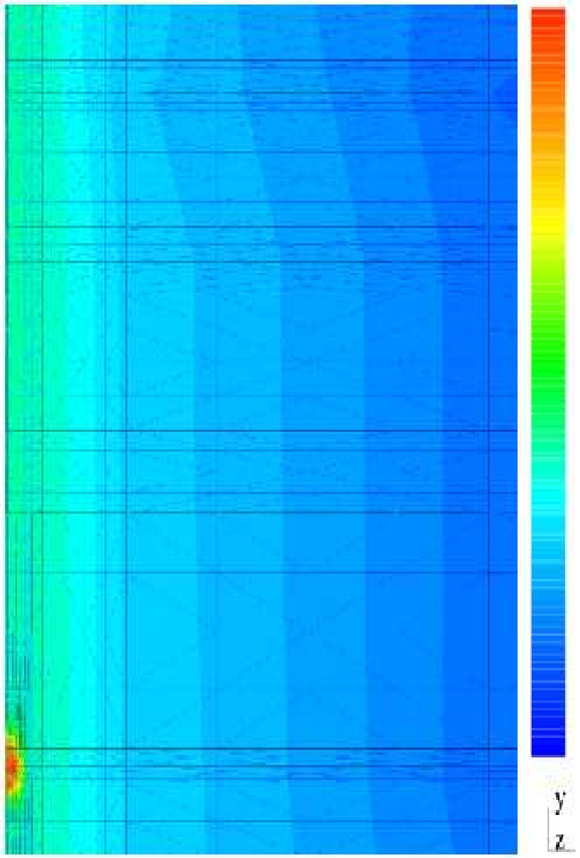
$$\left\{\begin{array}{l}\textbf{Find } G \in V \textbf{ such that :}\\ b(\Psi,G) = L(\Psi) \quad \forall \Psi \in V \ .\end{array}\right.$$

Notice that $L(e) = b(e,G)$.

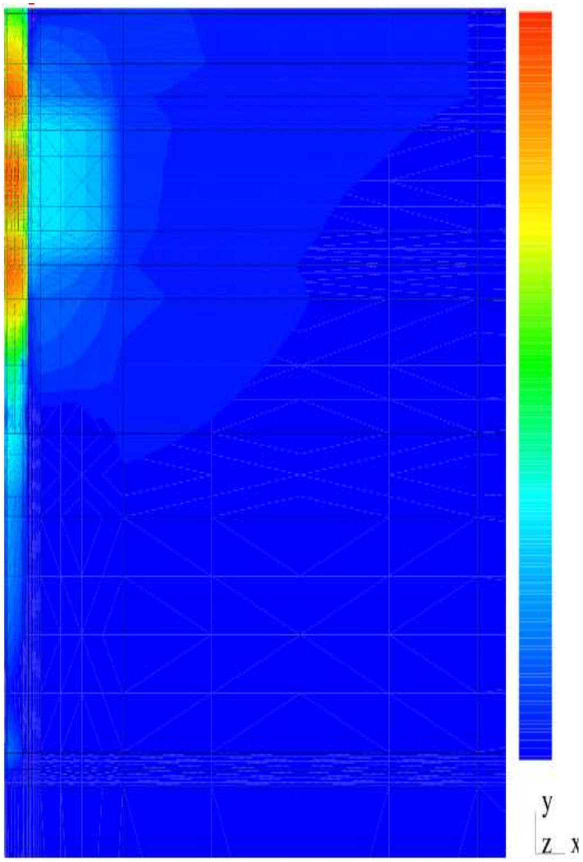
SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Mathematical Formulation (Goal-Oriented Adaptivity)

DIRECT PROBLEM



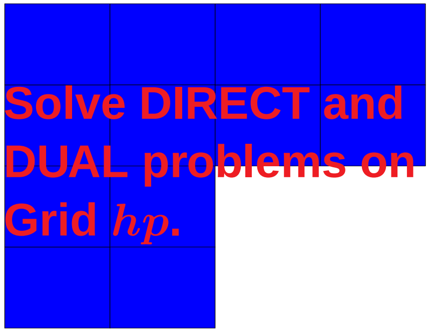
DUAL PROBLEM



$$L(\Psi)=b(\Psi,G)$$

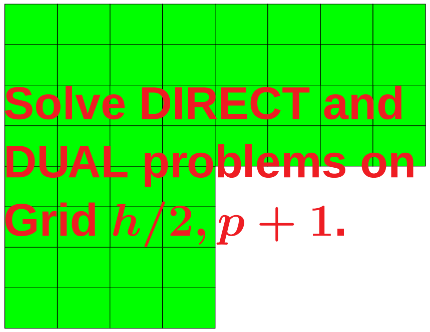
SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Algorithm for Goal-Oriented Adaptivity



Solve DIRECT and DUAL problems on Grid hp.

$\longrightarrow$

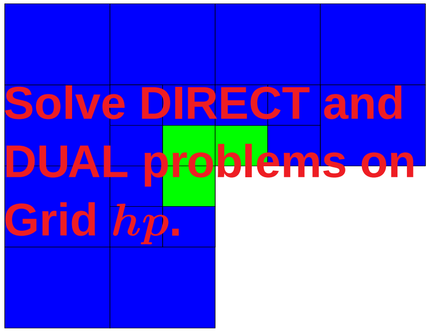


Solve DIRECT and DUAL problems on Grid $h/2, p+1$.

Compute $e = \Psi_{h/2,p+1} - \Psi_{hp}$, and $\tilde{e} = \Psi_{h/2,p+1} - \Pi_{hp}\Psi_{h/2,p+1}$.
Compute $\epsilon = G_{h/2,p+1} - G_{hp}$, and $\tilde{\epsilon} = G_{h/2,p+1} - \Pi_{hp}G_{h/2,p+1}$.

$$|L(e)| = |b(e,\epsilon)| \sim |b(\tilde{e},\tilde{\epsilon})| \leq \sum_K |b_K(\tilde{e},\tilde{\epsilon})| \leq \sum_K \| \tilde{e} \|_{E,K} \| \tilde{\epsilon} \|_{E,K}.$$

Apply the fully automatic hp-adaptive algorithm.



Solve DIRECT and DUAL problems on Grid hp.

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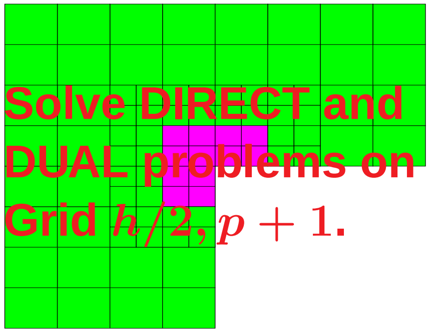
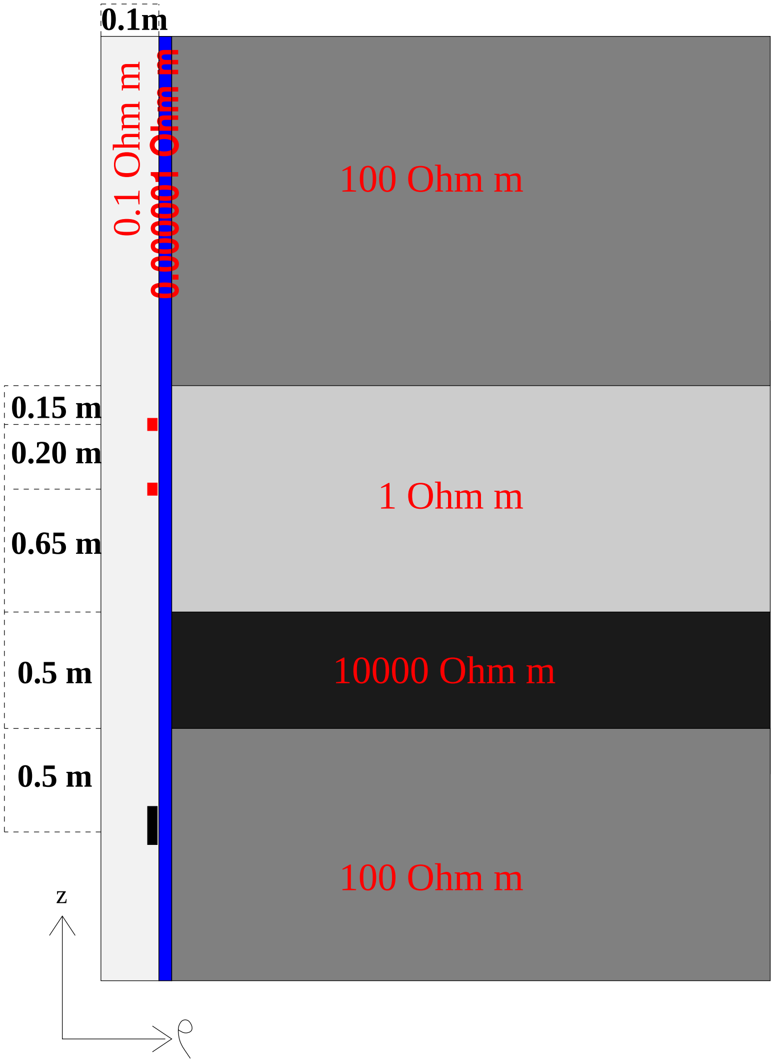


Solve DIRECT and DUAL problems on Grid $h/2, p+1$.

SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Model Problem with Steel Casing



Frequency: 10 Hz - 10 kHz.

Casing resistivity: 10^{-6} Ohm · m.

Casing width: 0.01127 m

Discretization error $<$ 0.1 $\%$

Toroidal antennas

Size (domain): 500m x 4000m

MAIN CHARACTERISTICS OF THE 2D hp-FEM

Flexibility (What Problems Can We Solve?)

Time-Harmonic Maxwell's Equations

$\nabla \times \mathrm{H} = (\bar{\bar{\sigma}} + j\omega\bar{\bar{\epsilon}})\mathrm{E} + \mathrm{J}^{imp}$	**Ampere's law**
$\nabla \times \mathrm{E} = -j\omega\bar{\bar{\mu}}\mathrm{H} - \mathrm{M}^{imp}$	**Faraday's law**
$\nabla \cdot (\bar{\bar{\epsilon}}\mathrm{E}) = \rho$	**Gauss' law of Electricity**
$\nabla \cdot (\bar{\bar{\mu}}\mathrm{H}) = 0$	**Gauss' law of Magnetism**

E-VARIATIONAL FORMULATION:

$$\begin{cases} \textbf{Find } \mathrm{E} \in \mathrm{E}_D + H_D(\mathrm{curl};\Omega) \textbf{ such that:} \\ \displaystyle\int_\Omega (\bar{\bar{\mu}}^{-1}\nabla \times \mathrm{E}) \cdot (\nabla \times \bar{\mathrm{F}})\, dV - \int_\Omega (\bar{\bar{k}}^2\mathrm{E}) \cdot \bar{\mathrm{F}}\, dV = -j\omega \int_\Omega \mathrm{J}^{imp} \cdot \bar{\mathrm{F}}\, dV \\ \displaystyle +j\omega \int_{\Gamma_N} \mathrm{J}^{imp}_{\Gamma_N} \cdot \bar{\mathrm{F}}_t\, dS - \int_\Omega (\bar{\bar{\mu}}^{-1}\mathrm{M}^{imp}) \cdot (\nabla \times \bar{\mathrm{F}})\, dV \quad \forall\, \mathrm{F} \in H_D(\mathrm{curl};\Omega) \end{cases}$$

MAIN CHARACTERISTICS OF THE 2D hp-FEM

Flexibility (What Problems Can We Solve?)

AXISYMMETRIC PROBLEMS

E_ϕ -Variational Formulation (Azimuthal)

$$\left\{\begin{array}{l}
\textbf{Find } \boldsymbol{E_\phi \in E_{\phi,D} + \tilde{H}^1_D(\Omega)} \textbf{ such that:} \\
\int_\Omega (\bar{\bar{\mu}}^{-1}_{\rho,z} \nabla \times \mathrm{E}_\phi) \cdot (\nabla \times \bar{\mathrm{F}}_\phi)\, dV - \int_\Omega (\bar{\bar{k}}^2_\phi \mathrm{E}_\phi) \cdot \bar{\mathrm{F}}_\phi\, dV = -j\omega \int_\Omega J^{imp}_\phi\, \bar{F}_\phi\, dV \\
\qquad +j\omega \int_{\Gamma_N} J^{imp}_{\phi,\Gamma_N}\, \bar{F}_\phi\, dS - \int_\Omega (\bar{\bar{\mu}}^{-1}_{\rho,z} \mathrm{M}^{imp}_{\rho,z}) \cdot \bar{\mathrm{F}}_\phi\, dV \qquad \forall\, F_\phi \in \tilde{H}^1_D(\Omega)
\end{array}\right.$$

$\mathrm{E}_{\rho,z}$ -Variational Formulation (Meridian)

$$\left\{\begin{array}{l}
\textbf{Find } (E_\rho, E_z) \in \mathrm{E}_D + \tilde{H}_D(\mathrm{curl}; \Omega) \textbf{ such that:} \\
\int_\Omega (\bar{\bar{\mu}}^{-1}_{\phi} \nabla \times \mathrm{E}_{\rho,z}) \cdot (\nabla \times \bar{\mathrm{F}}_{\rho,z})\, dV - \int_\Omega (\bar{\bar{k}}^2_{\rho,z} \mathrm{E}_{\rho,z}) \cdot \bar{\mathrm{F}}_{\rho,z}\, dV = \\
\qquad -j\omega \int_\Omega J^{imp}_\rho \bar{F}_\rho + J^{imp}_z \bar{F}_z\, dV + j\omega \int_{\Gamma_N} J^{imp}_{\rho,\Gamma_N} \bar{F}_\rho + J^{imp}_{z,\Gamma_N} \bar{F}_z\, dS \\
\qquad - \int_\Omega (\bar{\bar{\mu}}^{-1}_{\phi} \mathrm{M}^{imp}_{\phi}) \cdot \bar{\mathrm{F}}_{\rho,z}\, dV \qquad \forall\, (F_\rho, F_z) \in \tilde{H}_D(\mathrm{curl}; \Omega)
\end{array}\right.$$

MAIN CHARACTERISTICS OF THE 2D hp-FEM

Flexibility (What Problems Can We Solve?)

- **Physical Devices: Casing, Casing Imperfections, Mandrel, Magnetic Buffers, Insulators, Displacement Currents, Combination of All, etc.**

- **Materials: Isotropic, Anisotropic*.**

- **Sources: Toroidal Antennas, Solenoidal Antennas, Dipoles in Any Direction, Electrodes, Finite Size Antennas, Combination of All, etc.**

- **Logging Instruments: Logging While Drilling (LWD), Laterolog, Normal, Induction, Dielectric Instruments, Cross-well, etc.**

- **Any Frequency (0-10 Ghz).**

ALL AXISYMMETRIC RESISTIVITY LOGGING PROBLEMS

MAIN CHARACTERISTICS OF THE 2D hp-FEM

Reliability (Can We Trust the Solutions?)

- **Comparison Against Analytical Results.**
  1. Exact solution in a homogeneous media.
  2. Exact solution in a homogeneous media with a mandrel.
  3. Exact solution in a homogeneous media with casing.
- **Verification of Physical Properties.**
  1. Reciprocity principle (Gregory Itskovich).
  2. Discrete divergence free approximation for edge elements.
- **Numerical Verifications.**
  1. Different size of domain and antennas.
  2. Comparison against other numerical software (Yang Wei).
  3. Error control provided by the fine grid solution.
  4. Comparison between continuous elements vs. edge elements.

MAIN CHARACTERISTICS OF THE 2D hp-FEM

Reliability (Can We Trust the Solutions?)

Problem with casing at 10 kHz.

Continuous Elements

Quantity of Interest	Real Part	Imag Part
COARSE GRID	0.1516098429E-08	-0.1456374493E-08
FINE GRID	0.1516094029E-08	-0.1456390824E-08

Edge Elements

Quantity of Interest	Real Part	Imag Part
COARSE GRID	0.1516060872E-08	-0.1456337248E-08
FINE GRID	0.1516093804E-08	-0.1456390864E-08

Error control provided by the fine grid solution.

MAIN CHARACTERISTICS OF THE 2D hp-FEM

Reliability (Can We Trust the Solutions?)

Problem with casing at 10 kHz.

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Comparison between continuous elements vs. edge elements.

MAIN CHARACTERISTICS OF THE 2D hp-FEM

Reliability (Can We Trust the Solutions?)

- **Comparison Against Analytical Results.**
  1. Exact solution in a homogeneous media.
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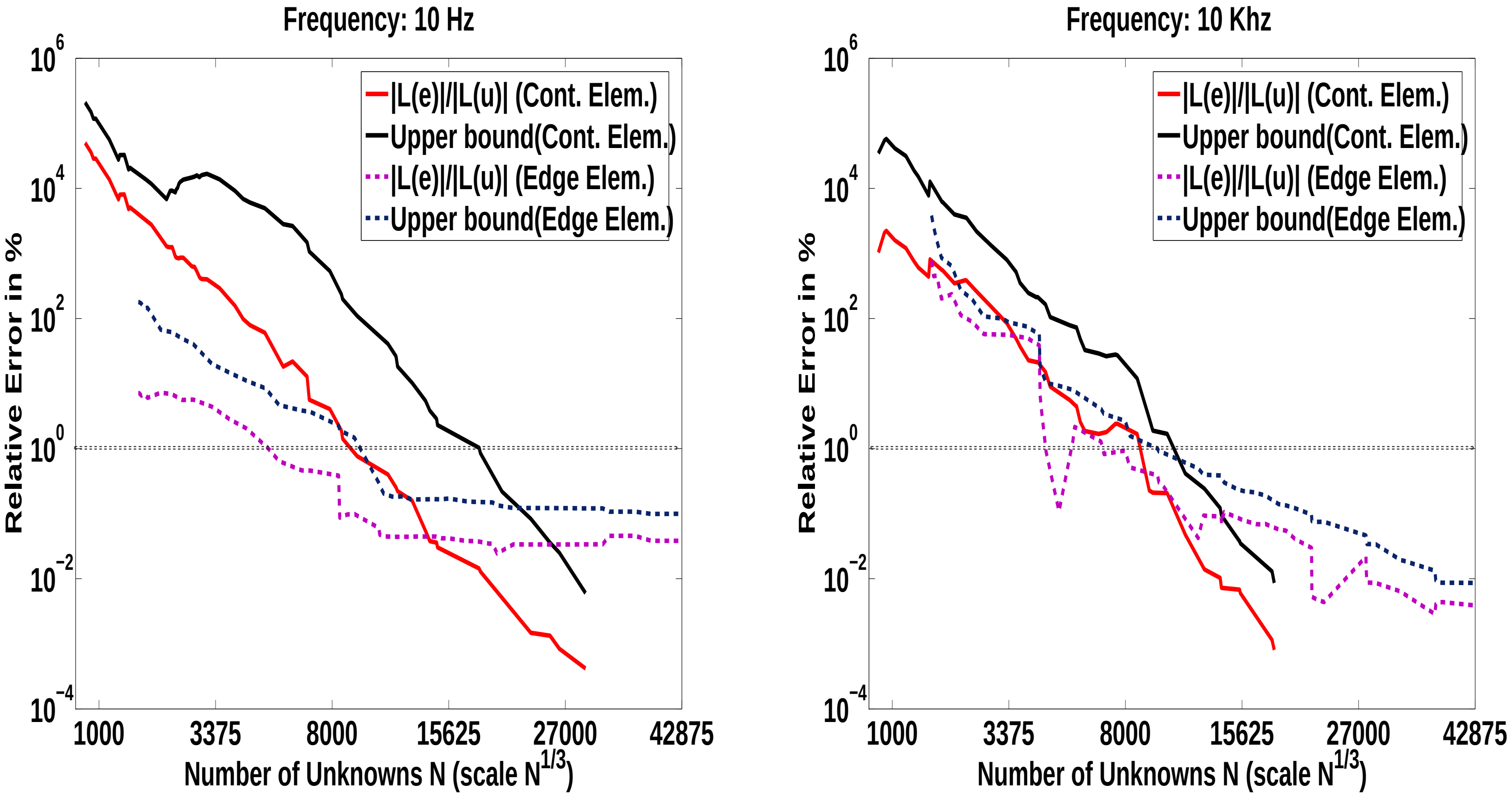
HIGHLY RELIABLE SOFTWARE

MAIN CHARACTERISTICS OF THE 2D hp-FEM

Accuracy (Are the Solutions Accurate?)

Problem with Casing (Convergence Curve)



EXTREMELY ACCURATE SOFTWARE

MAIN CHARACTERISTICS OF THE 2D hp-FEM

Performance (How Fast Can We Solve the Problems?)

80 Vert. Pos.	$10^{-6}\Omega \cdot m$	$10^{-5}\Omega \cdot m$
Toroid (10 Khz)	19’ 46”	16’ 28”
Ring of Vert. Dipoles (10 Khz)	22’ 47”	17’ 02”
Ring of Horiz. Dipoles (10 Khz)	19’ 25”	13’ 25”
Electrodes (0 Hz)	10’ 10”	8’ 35”

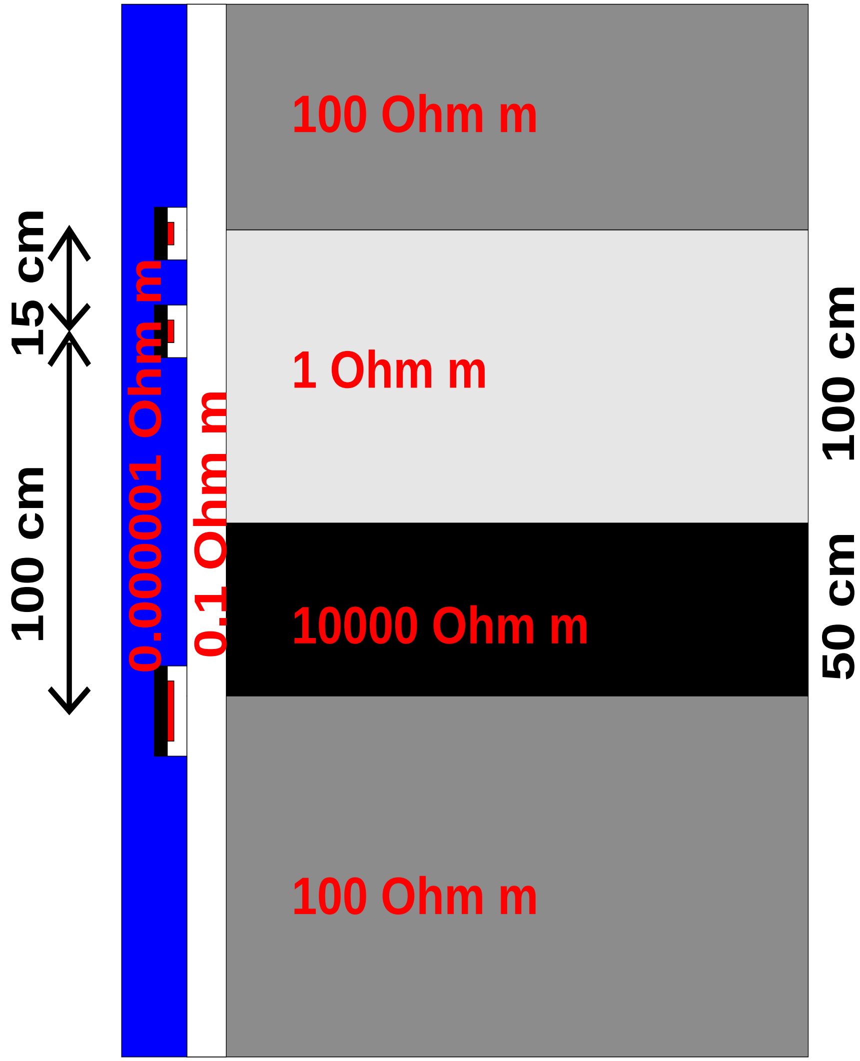
IBM Power 4 compiler 1.3 Ghz (4 years old)

Possible improvements in performance:

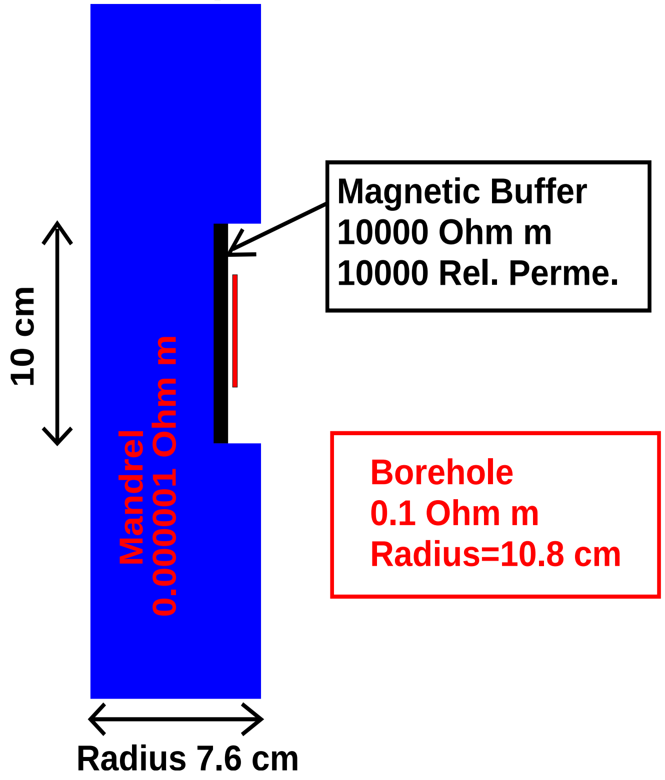
- To use a 3.4 Ghz processor.
- To execute the code in 8 processors (10 positions per processor).
- To improve implementation.

HIGH PERFORMANCE SOFTWARE

SIMULATION OF LOGGING INSTRUMENTS



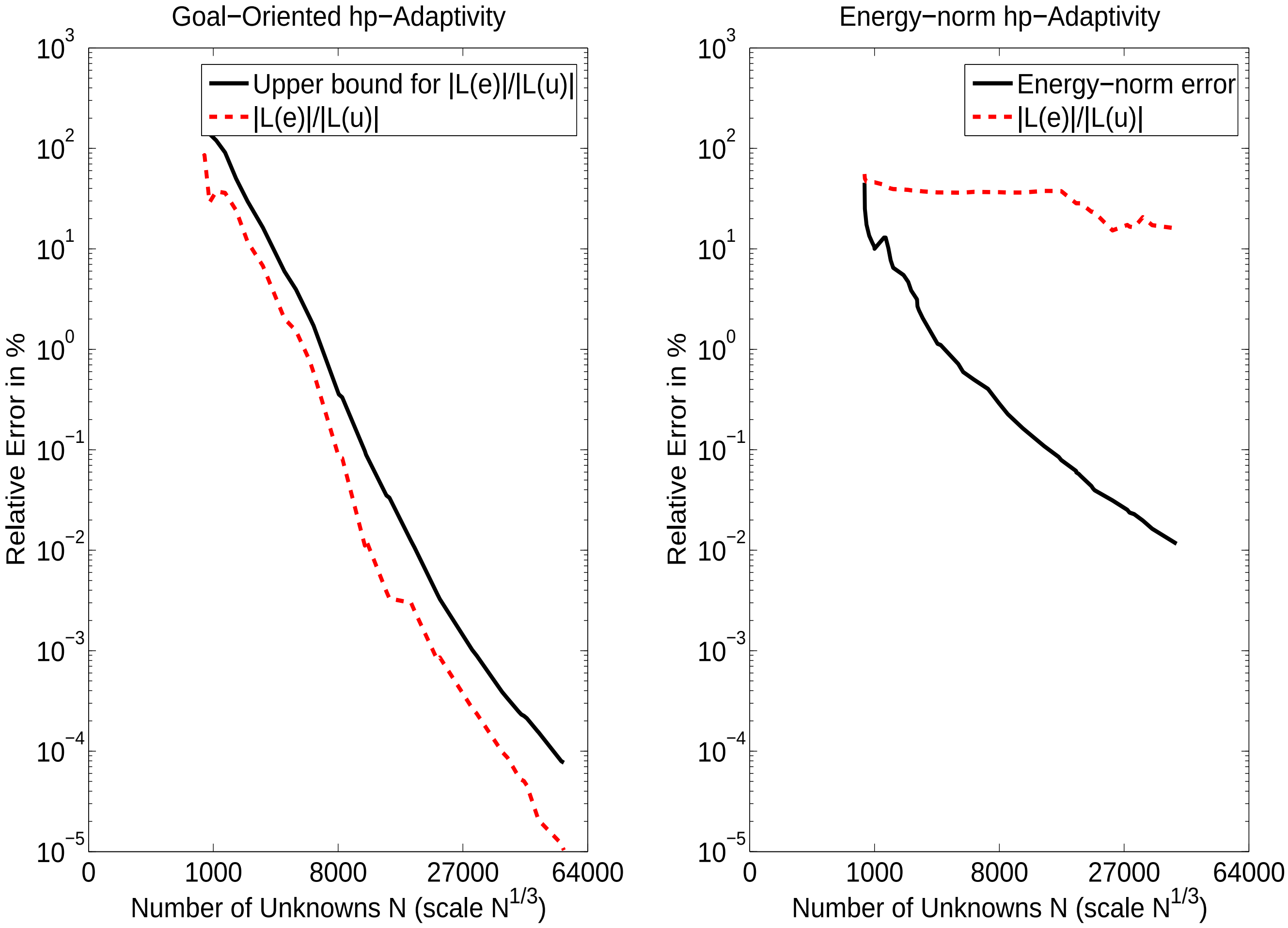
Description of Antennas



Goal: To Study the Effect of Invasion, Anistotropy, and Magnetic Permeability.

SIMULATION OF LOGGING INSTRUMENTS

First. Vert. Diff. E_ϕ (solenoid). Position: 0.475m



SIMULATION OF LOGGING INSTRUMENTS

Goal-Oriented vs. Energy-norm hp-Adaptivity

Problem with Mandrel at 2 Mhz.

Continuous Elements (Goal-Oriented Adaptivity)

Quantity of Interest	Real Part	Imag Part
COARSE GRID	-0.1629862203E-01	-0.4016944732E-02
FINE GRID	-0.1629862347E-01	-0.4016944223E-02

Continuous Elements (Energy-norm Adaptivity)

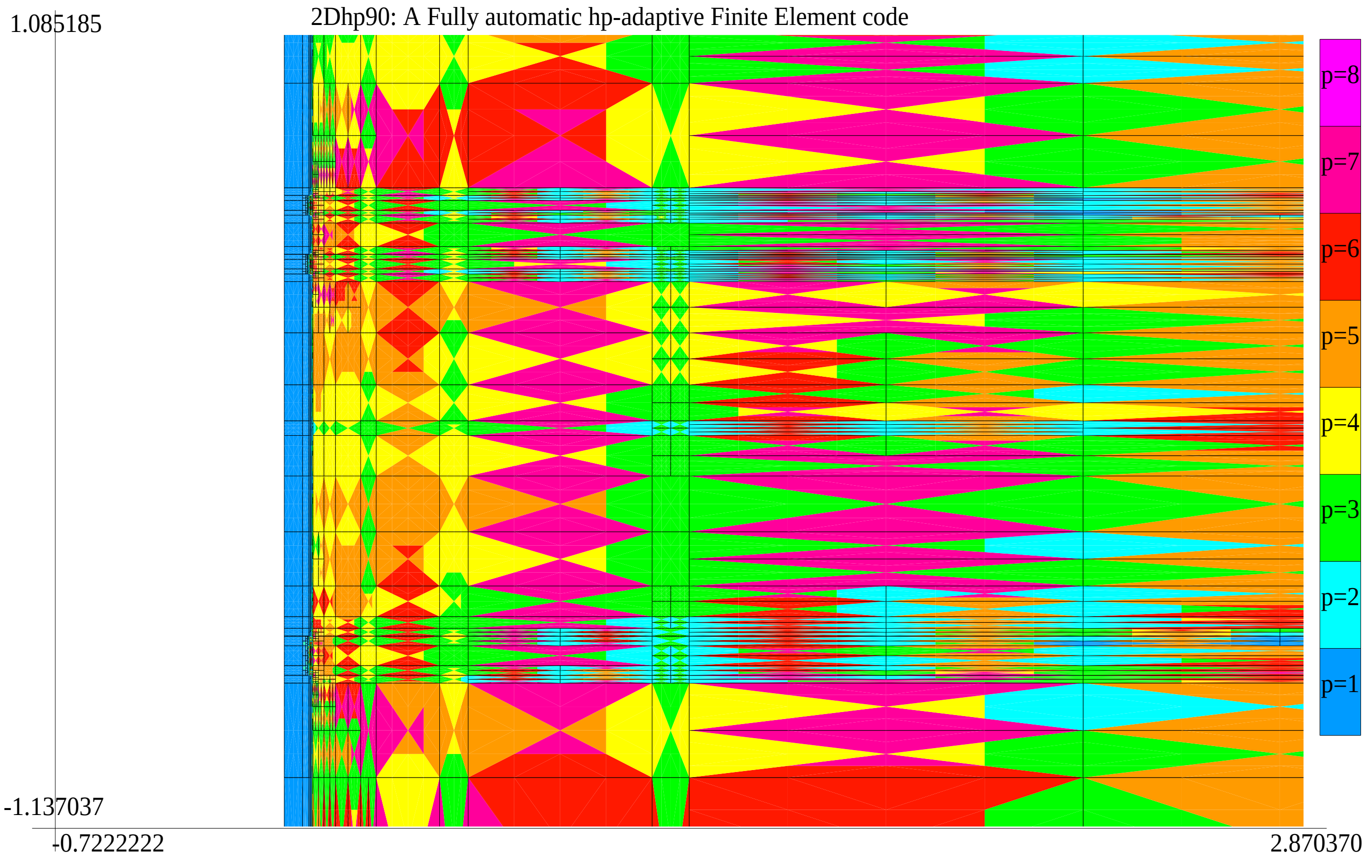
Quantity of Interest	Real Part	Imag Part
0.01% ENERGY ERROR	-0.1382759158E-01	-0.2989492851E-02

It is critical to use GOAL-ORIENTED adaptivity.

SIMULATION OF LOGGING INSTRUMENTS

First. Vert. Diff. E_ϕ (solenoid). Position: 0.475m

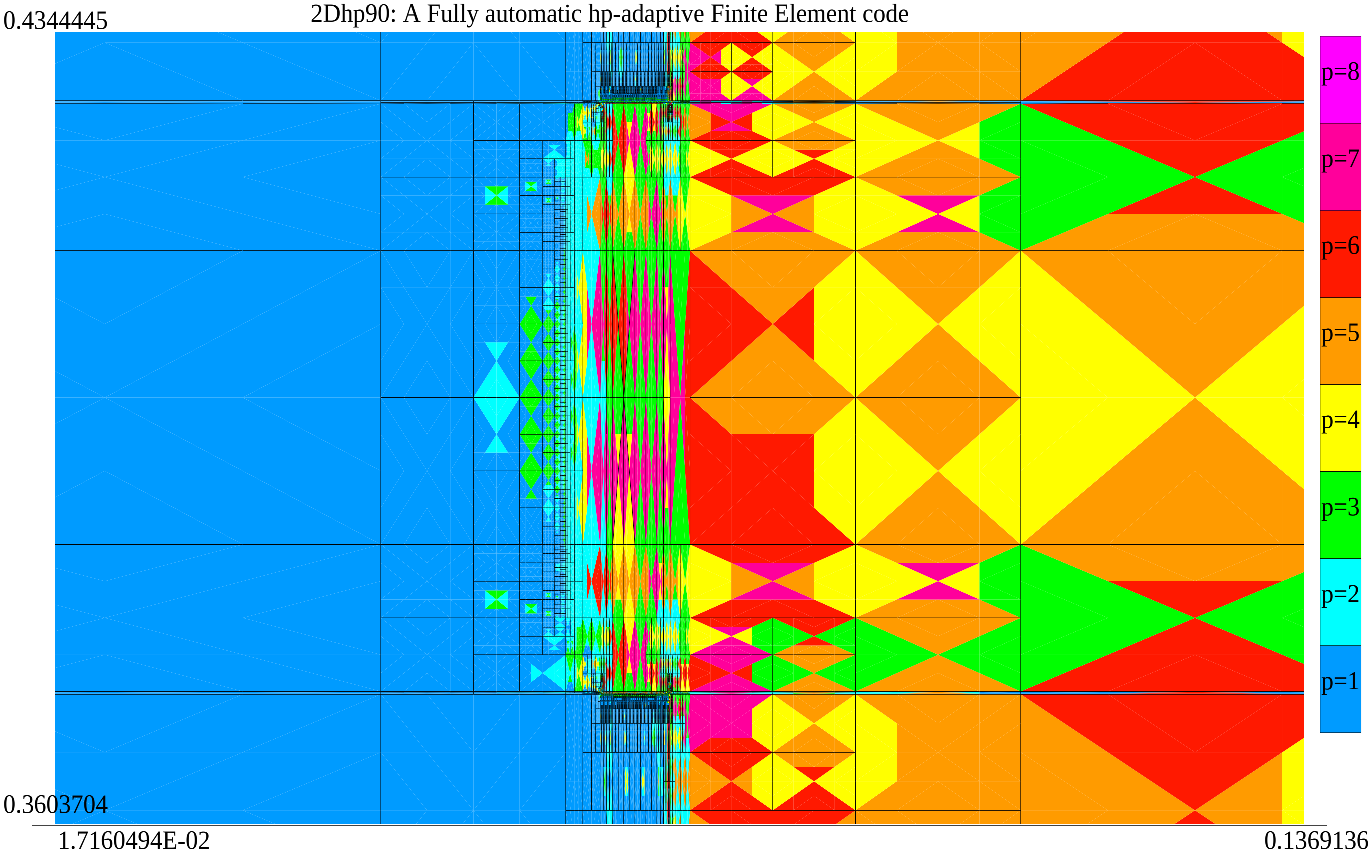
GOAL-ORIENTED HP-ADAPTIVITY



SIMULATION OF LOGGING INSTRUMENTS

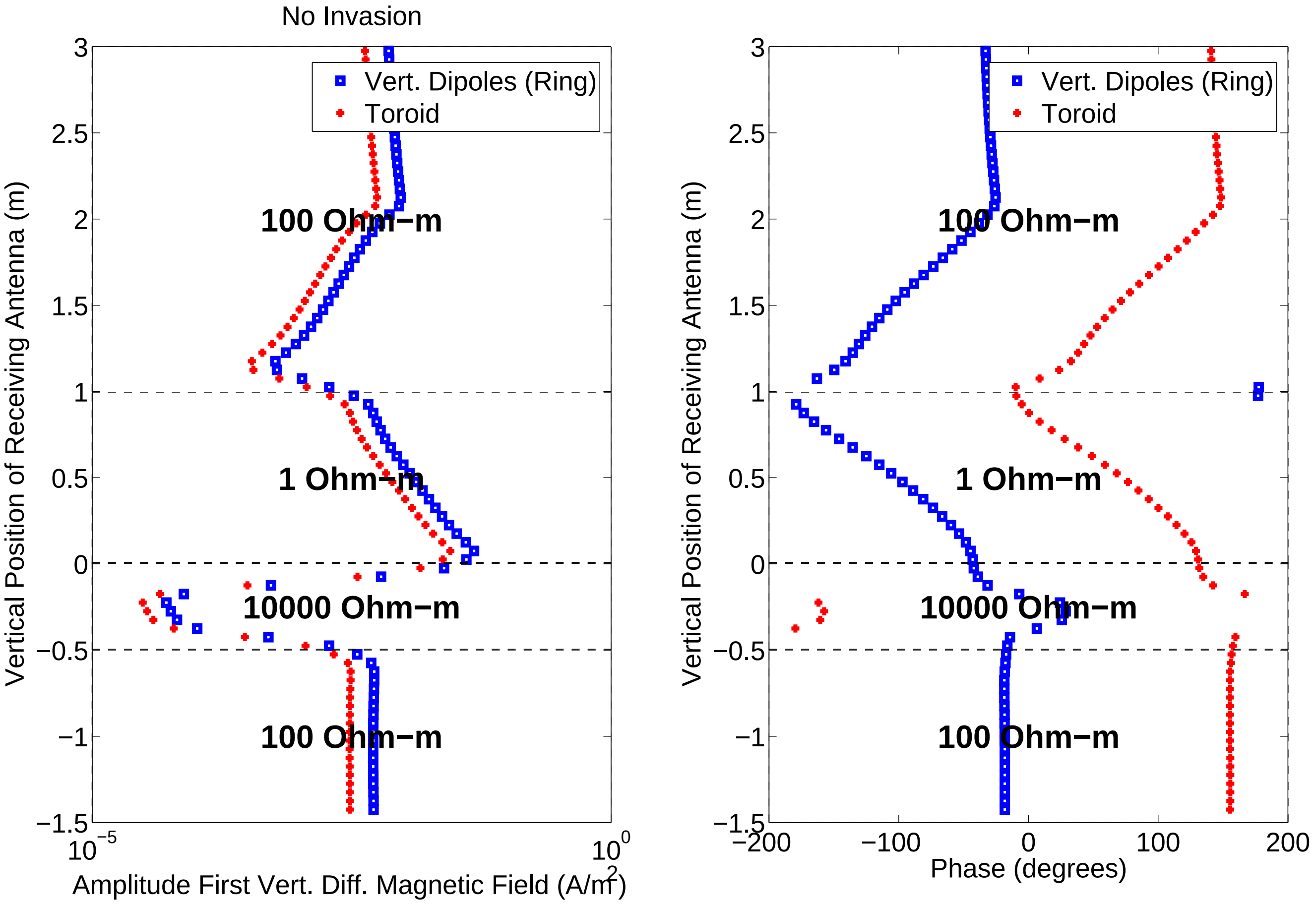
First. Vert. Diff. E_ϕ (solenoid). Position: 0.475m

GOAL-ORIENTED HP-ADAPTIVITY (ZOOM TOWARDS FIRST RECEIVER ANTENNA)



SIMULATION OF LOGGING INSTRUMENTS

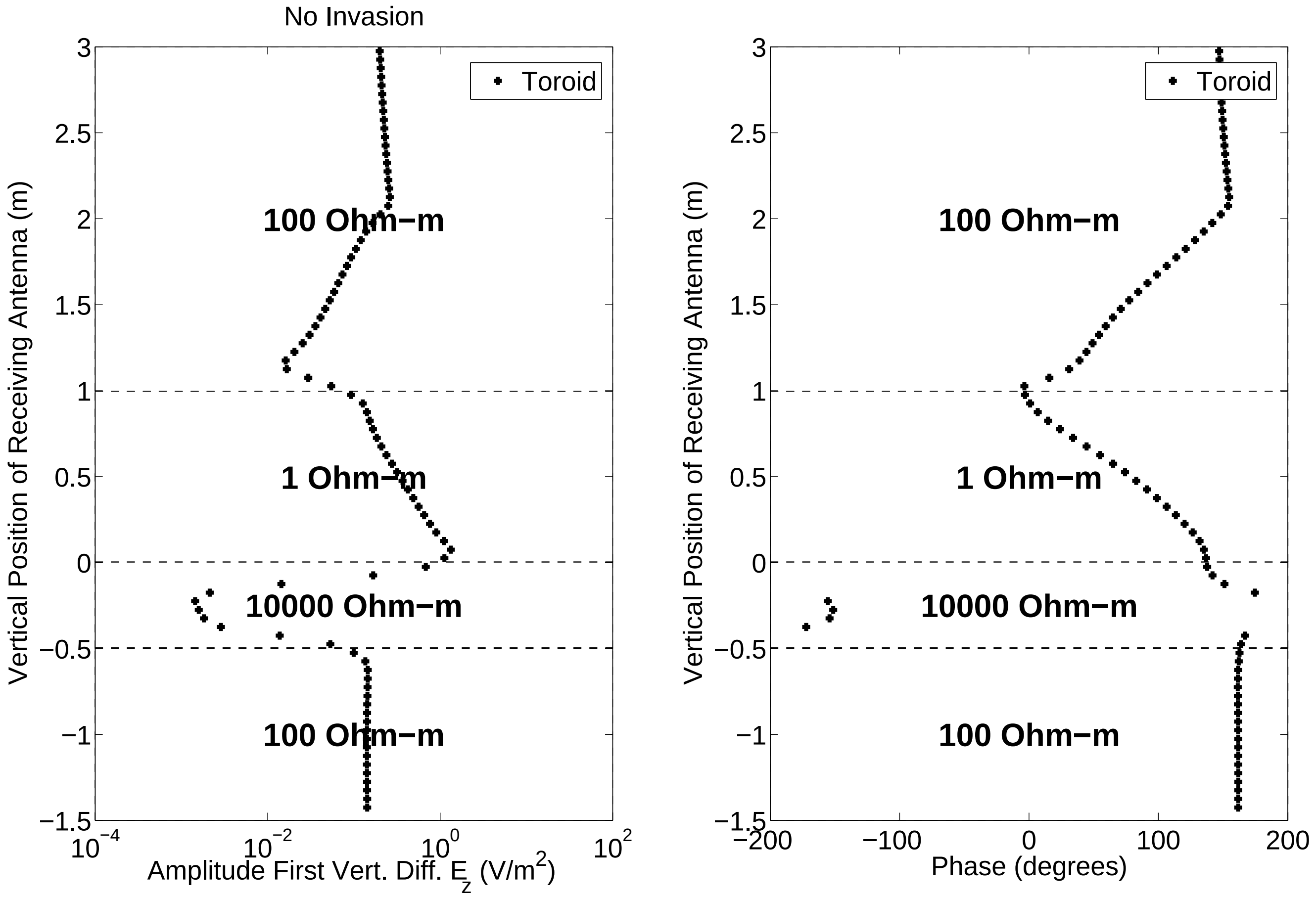
First Vert. Diff. H_ϕ for different antennas



In LWD instruments, we obtain similar results using toroids or a ring of vert. dipoles

SIMULATION OF LOGGING INSTRUMENTS

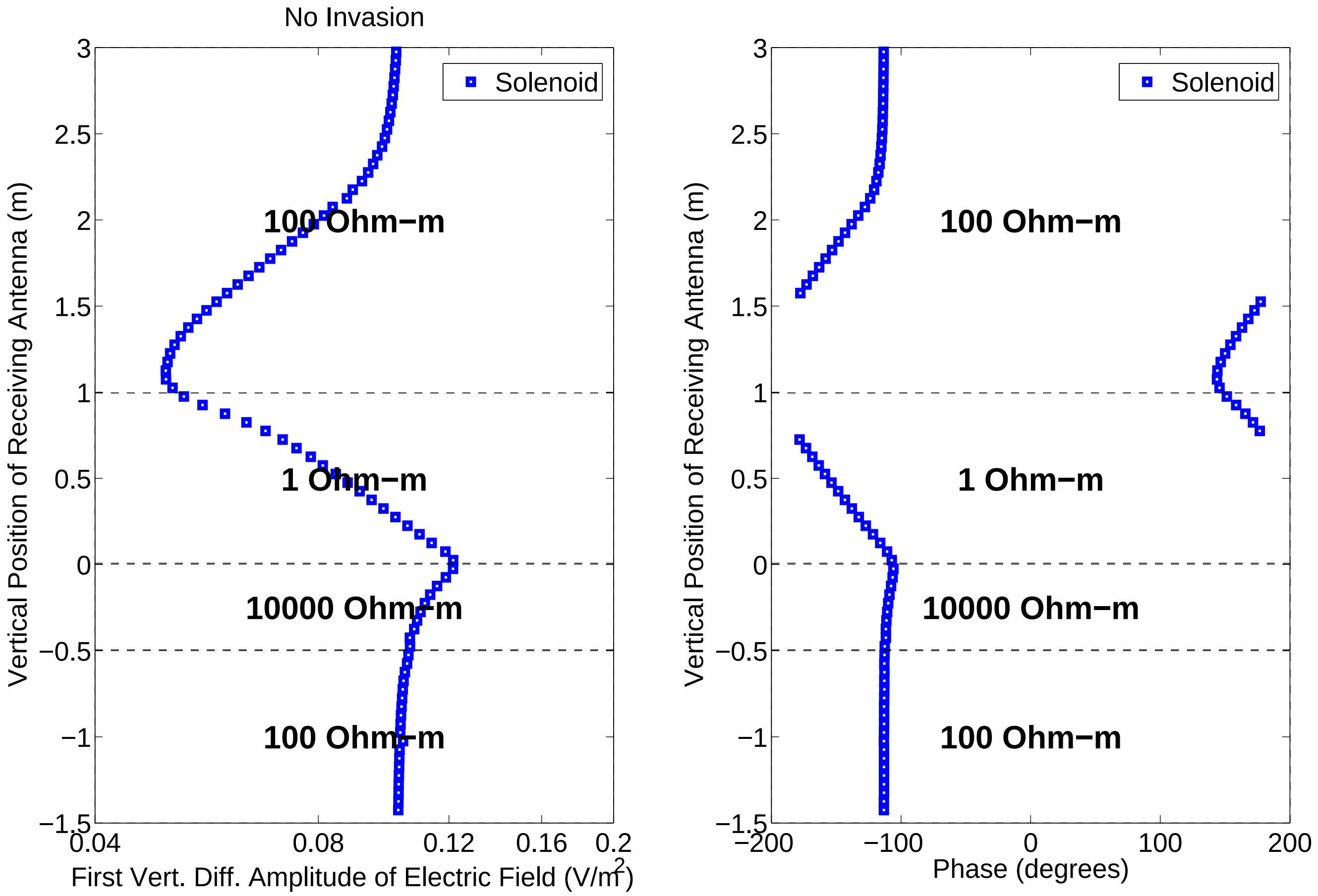
First Vert. Diff. E_z for a toroid antenna



Toroids are adequate for identifying highly resistive layers

SIMULATION OF LOGGING INSTRUMENTS

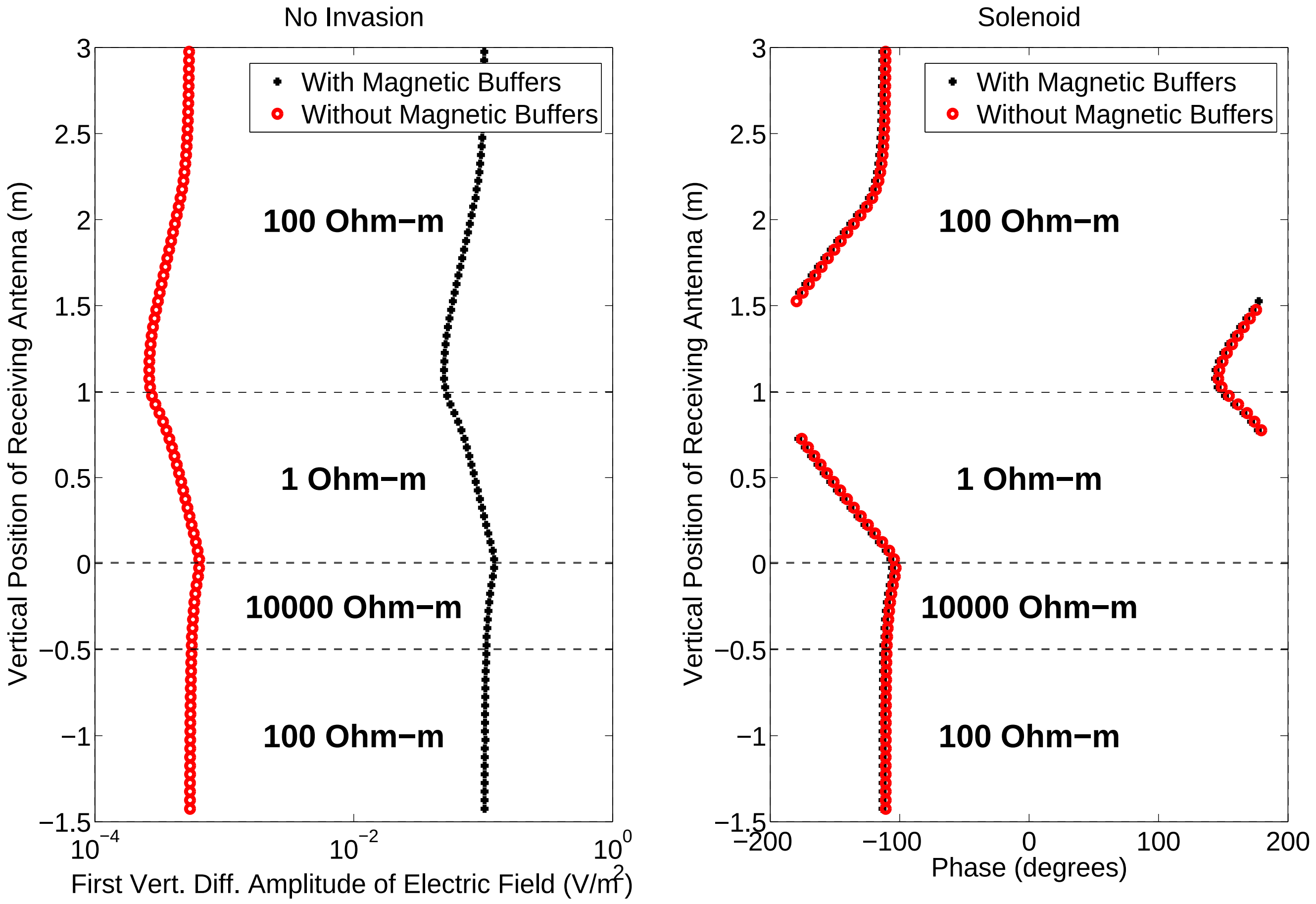
First Vert. Diff. E_ϕ for a solenoid antenna



Solenoids are adequate for identifying low resistive layers

SIMULATION OF LOGGING INSTRUMENTS

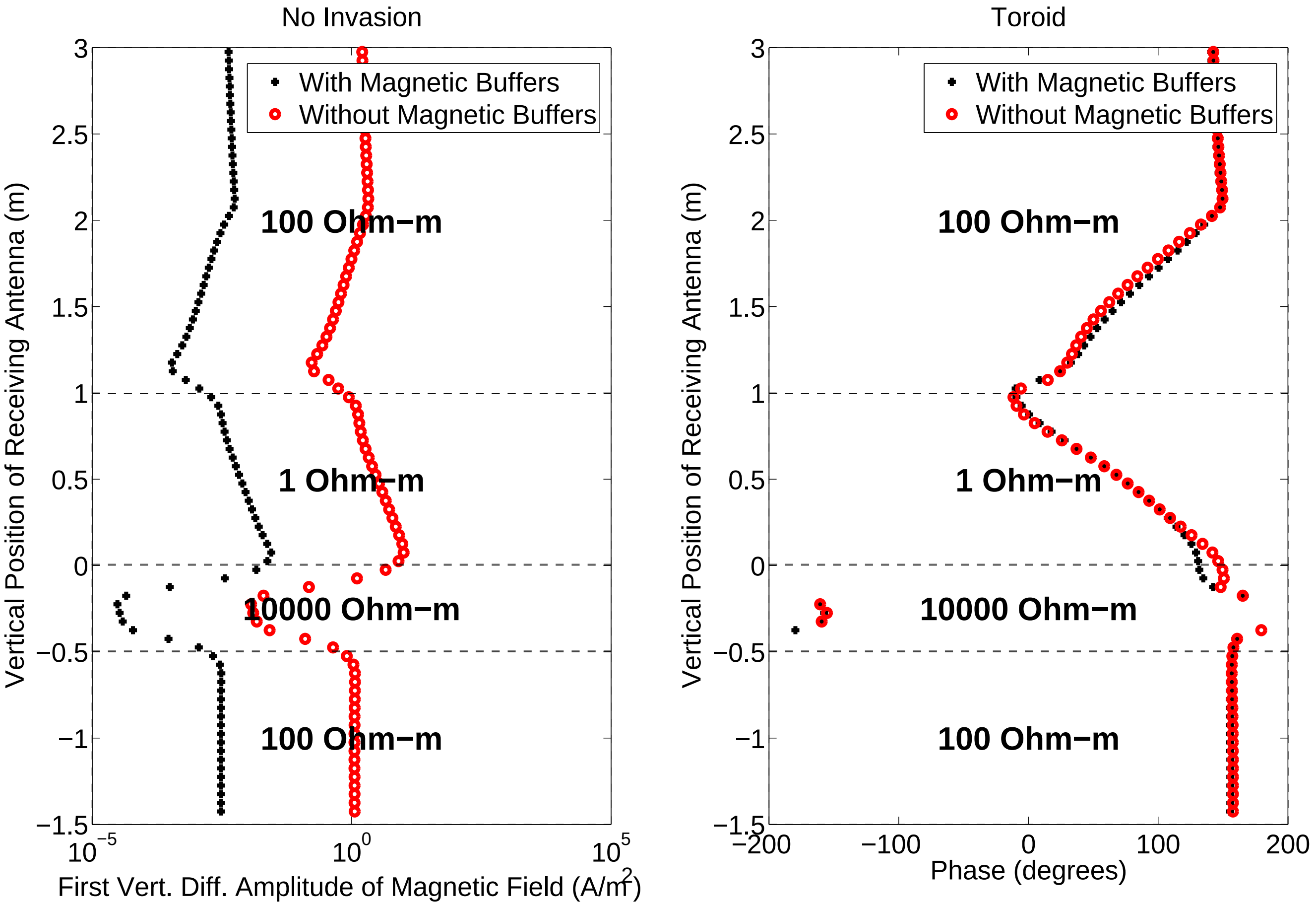
Use of Magnetic Buffers (E_ϕ for a solenoid)



Use of magnetic buffers strengthen the signal in combination with solenoids

SIMULATION OF LOGGING INSTRUMENTS

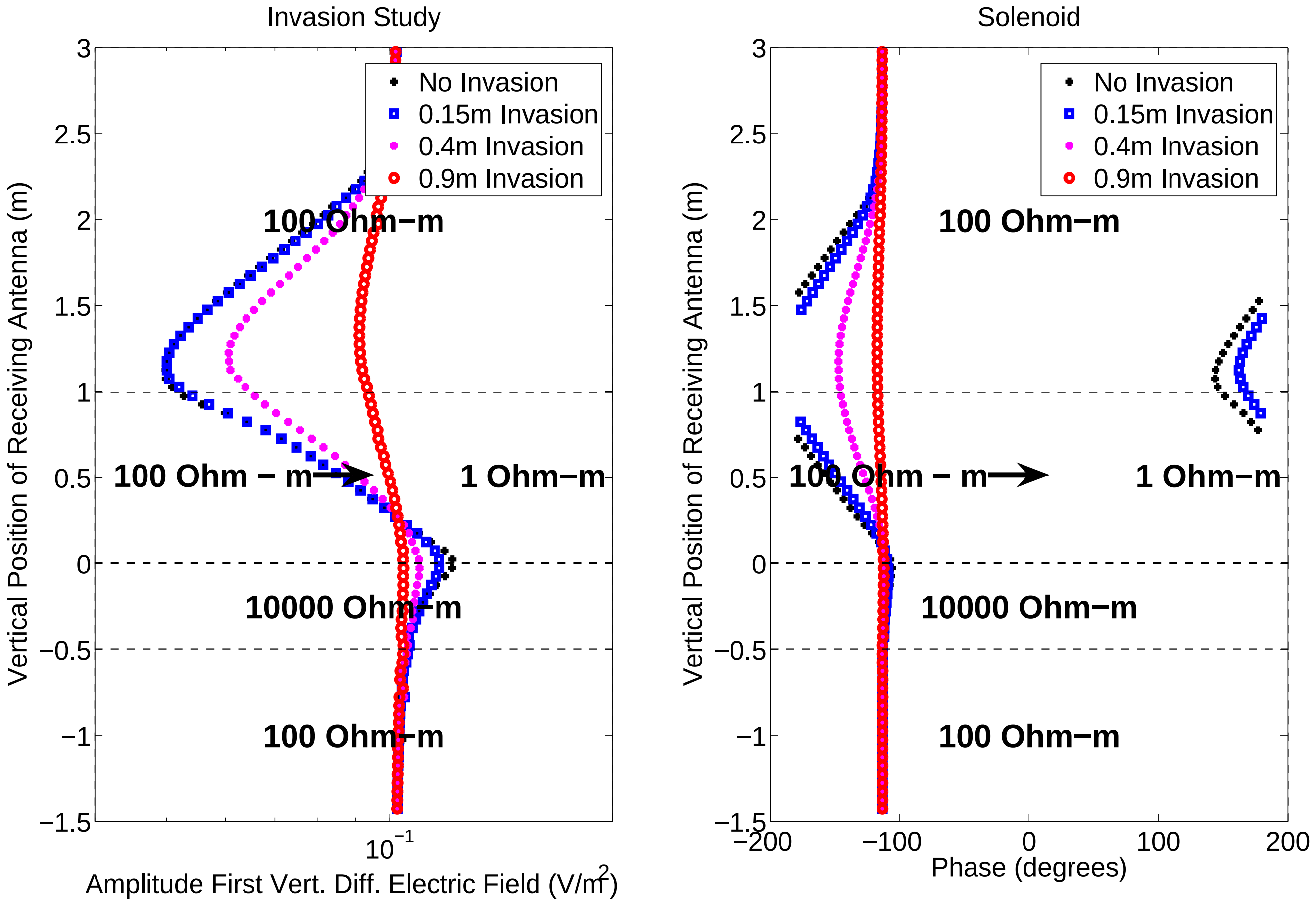
Use of Magnetic Buffers (H_ϕ for a toroid)



However, magnetic buffers weaken the signal in combination with toroids

SIMULATION OF LOGGING INSTRUMENTS

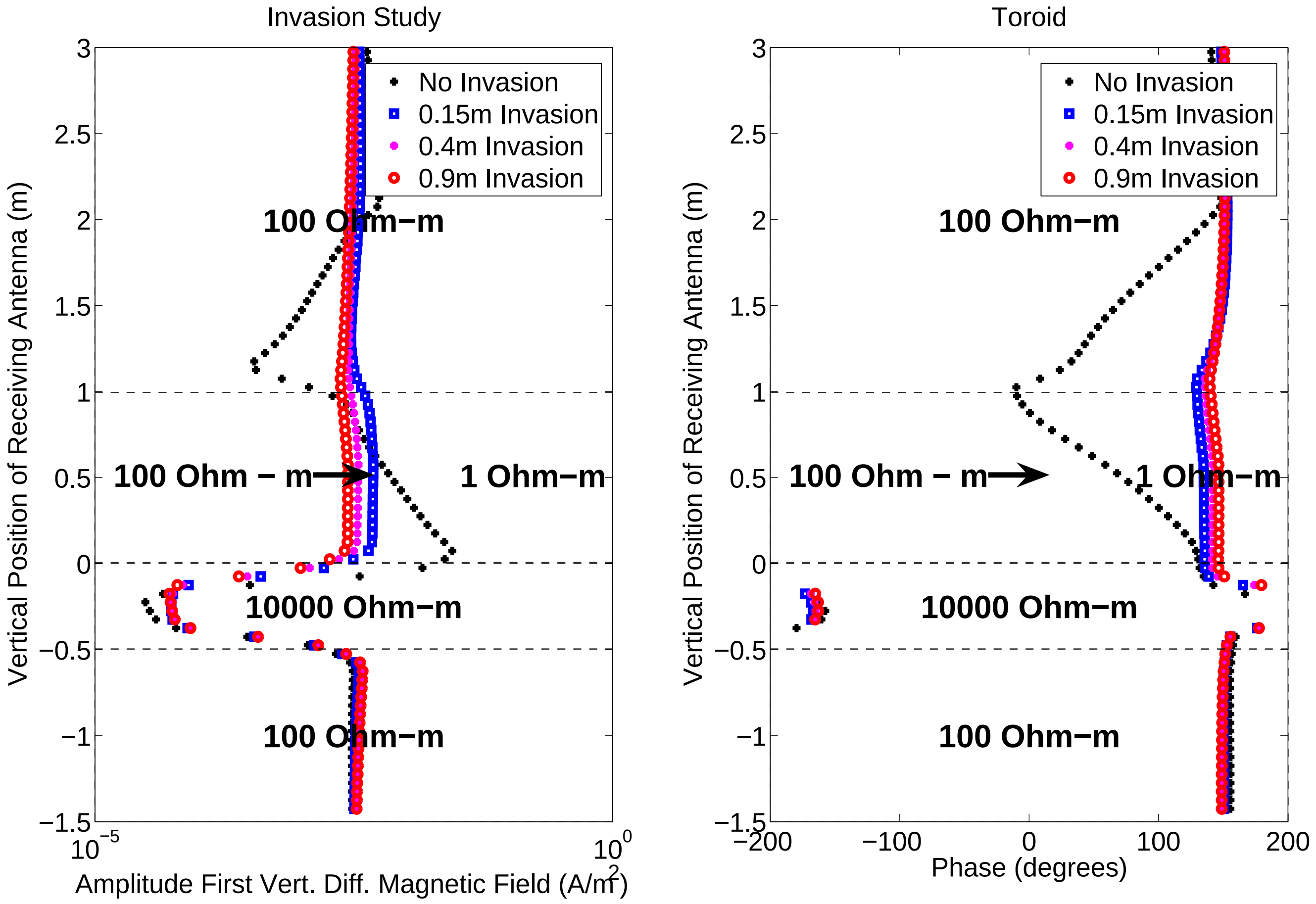
Invasion study (E_ϕ for a solenoid)



Large invasion effects can be sensed using solenoids

SIMULATION OF LOGGING INSTRUMENTS

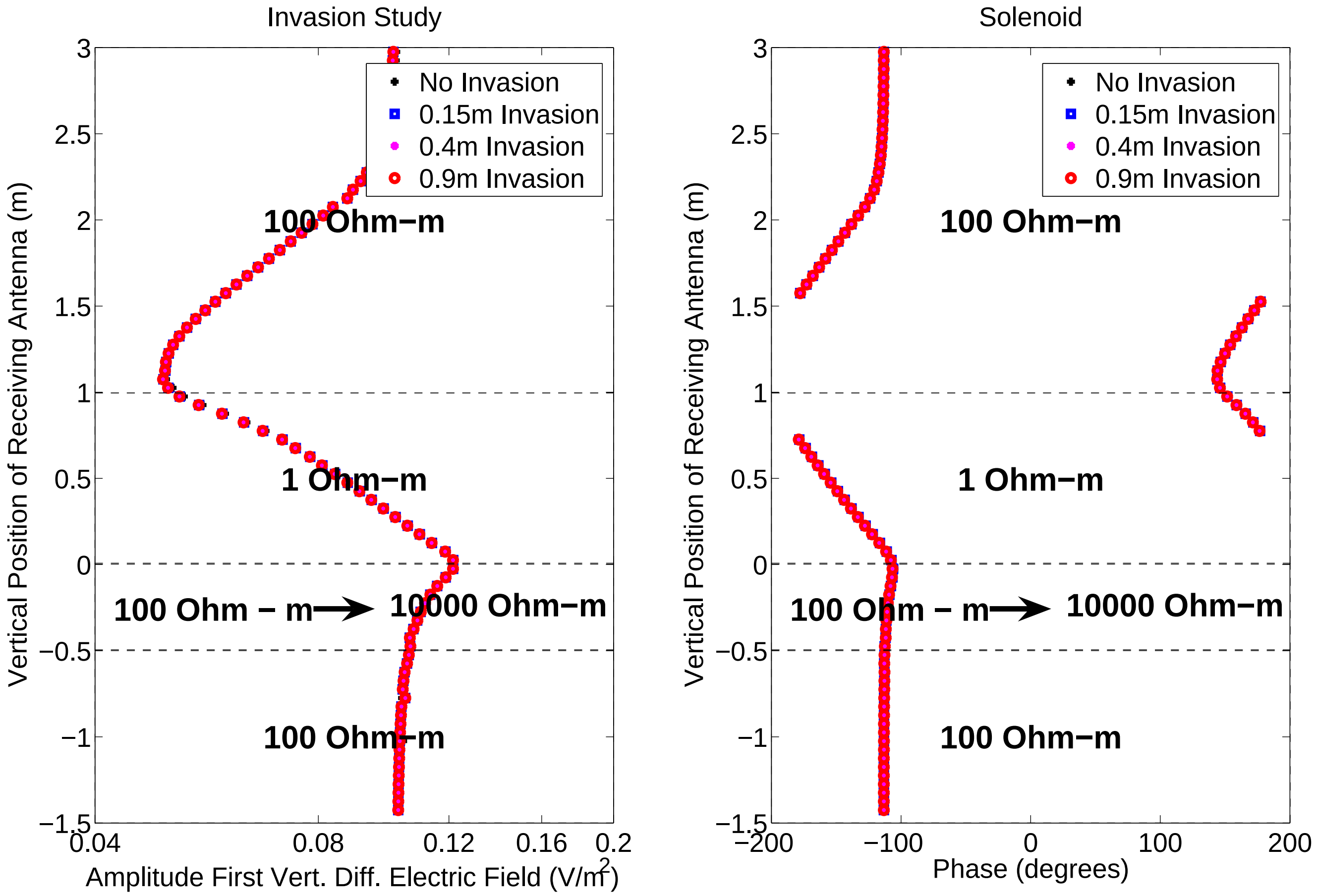
Invasion study (H_ϕ for a toroid)



Small invasion effects can be sensed using toroids

SIMULATION OF LOGGING INSTRUMENTS

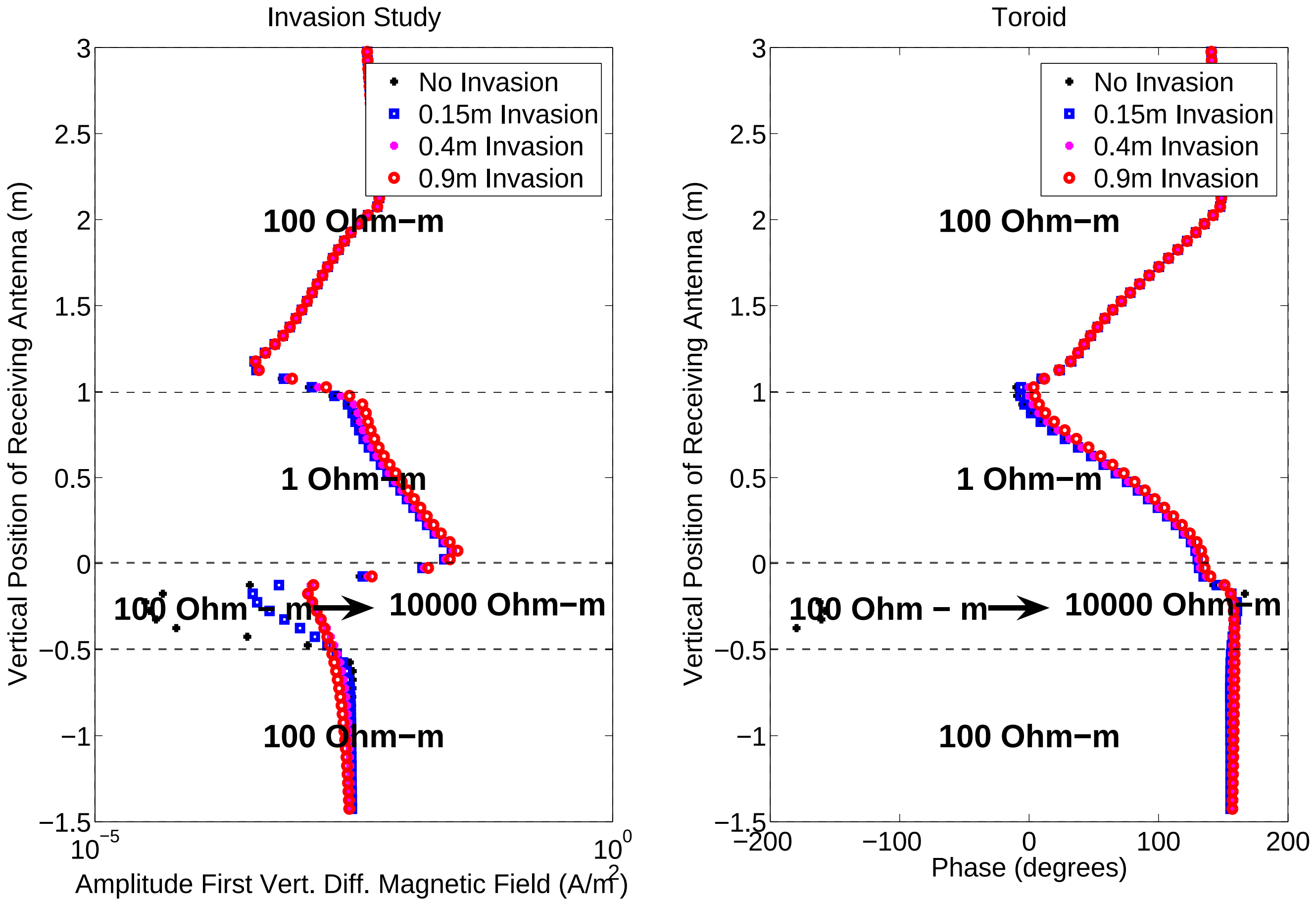
Invasion study (E_ϕ for a solenoid)



Invasion in resistive layers cannot be sensed using solenoids

SIMULATION OF LOGGING INSTRUMENTS

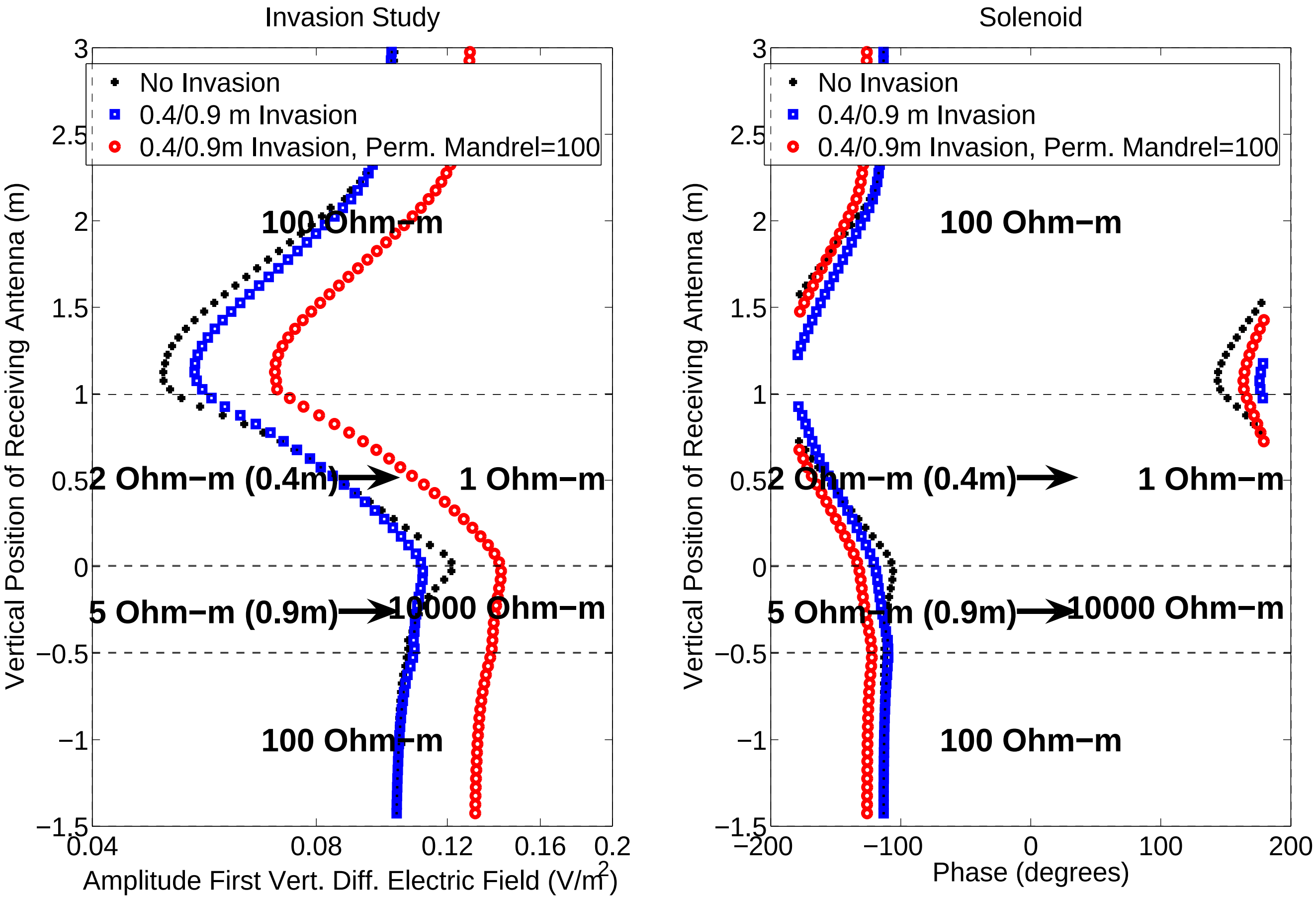
Invasion study (H_ϕ for a toroid)



Invasion in resistive layers should be studied using toroids

SIMULATION OF LOGGING INSTRUMENTS

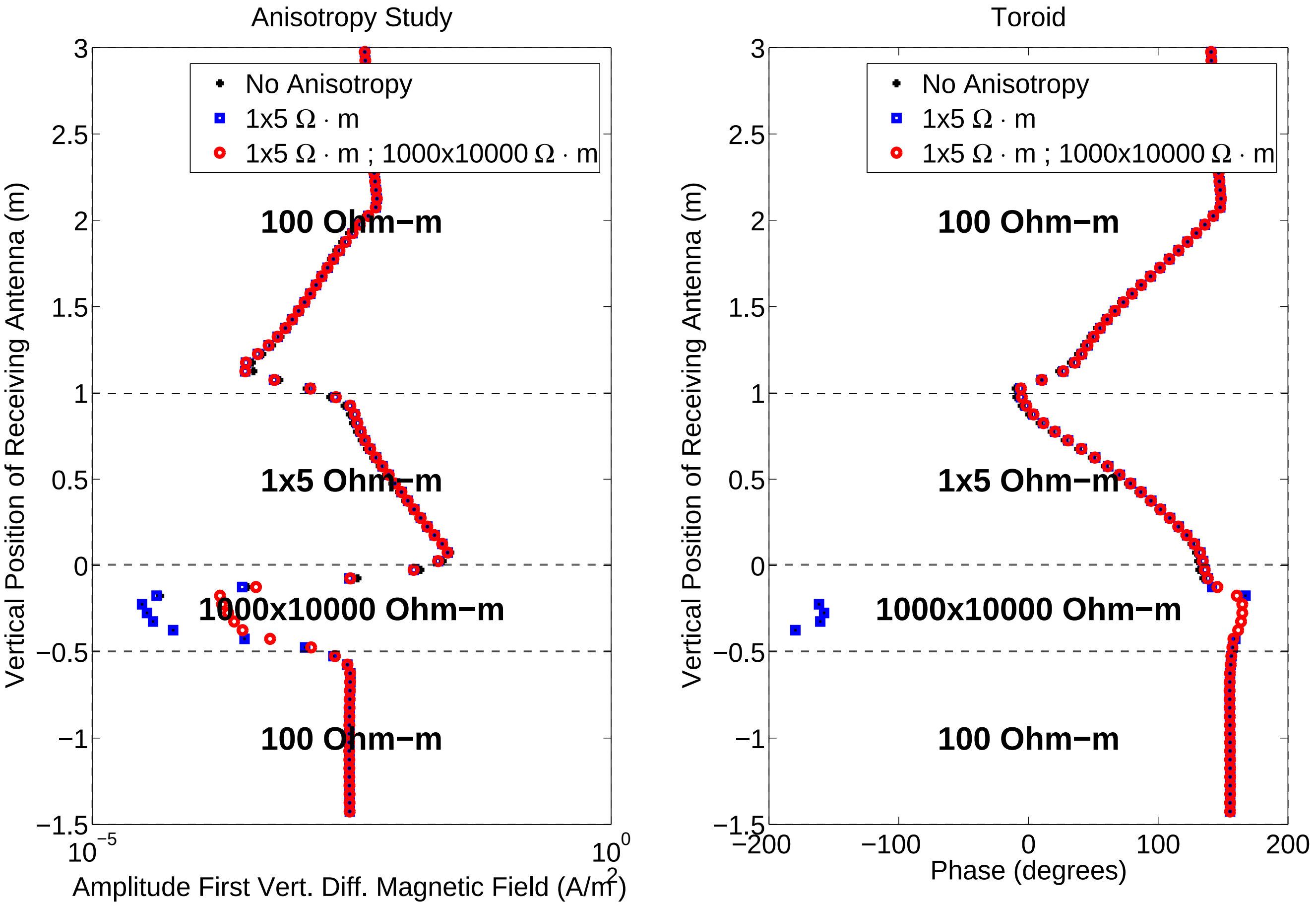
Invasion and mandrel magnetic permeab. (E_ϕ)



The effect of magnetic permeability on the mandrel is similar to the effect of magnetic buffers

SIMULATION OF LOGGING INSTRUMENTS

Anisotropy (H_ϕ)



Anisotropy effects may be important when studying resistive layers

CONCLUSIONS AND FUTURE WORK

- **The self-adaptive goal-oriented hp-adaptive strategy converges exponentially in terms of a user-prescribed quantity of interest vs. the CPU time.**

- **We obtain fast, reliable and accurate solutions for problems with a large dynamic range and high material constrasts.**

Future Work

- **To apply the self-adaptive goal-oriented hp-FEM to 3D problems for simulation of deviated wells.**

- **To apply the self-adaptive goal-oriented hp-FEM for inversion of 2D multi-physic problems.**