Final Ph.D. Progress Report

Integration of hp-adaptivity with a Two Grid Solver: Applications to Electromagnetics.

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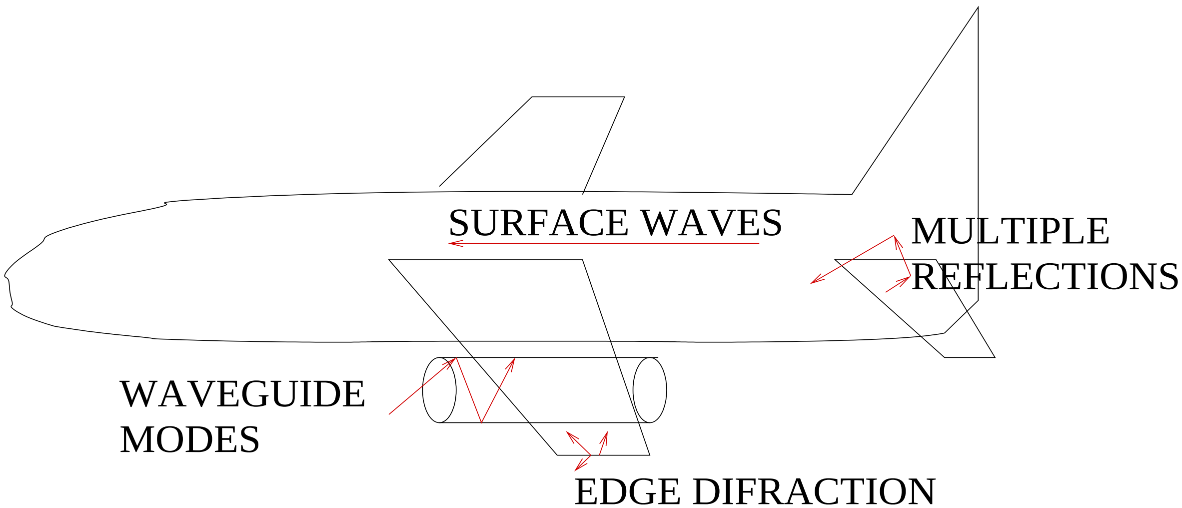
Institute for Computational Engineering and Sciences (ICES)
The University of Texas at Austin

OVERVIEW

2. MOTIVATION

Radar Cross Section (RCS) Analysis

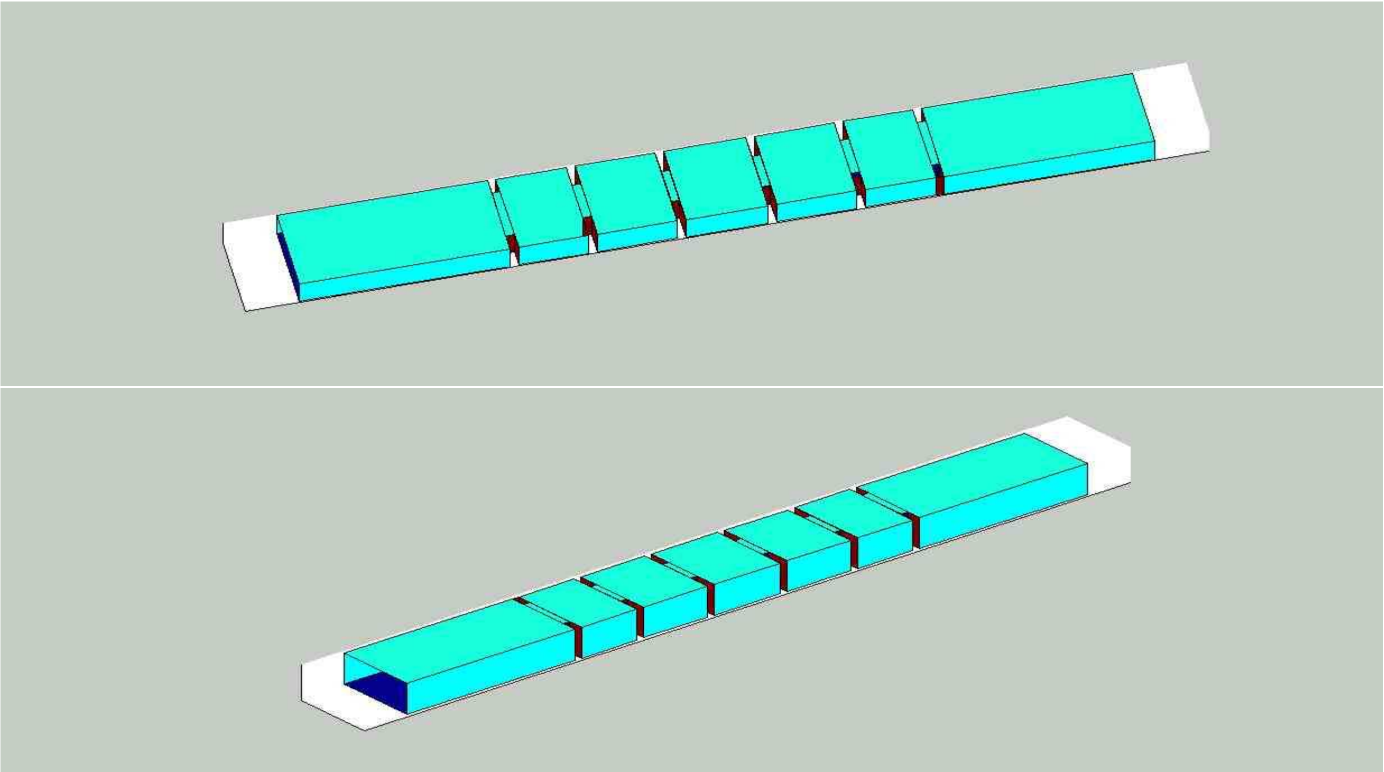


RCS$=4\pi\frac{\text{Power scattered to receiver per unit solid angle}}{\text{Incident power density}} = \lim_{r\to\infty} 4\pi r^2 \frac{|\boldsymbol{E}^s|}{|\boldsymbol{E}^i|}$.

Goal: Determine the RCS of a plane.

2. MOTIVATION

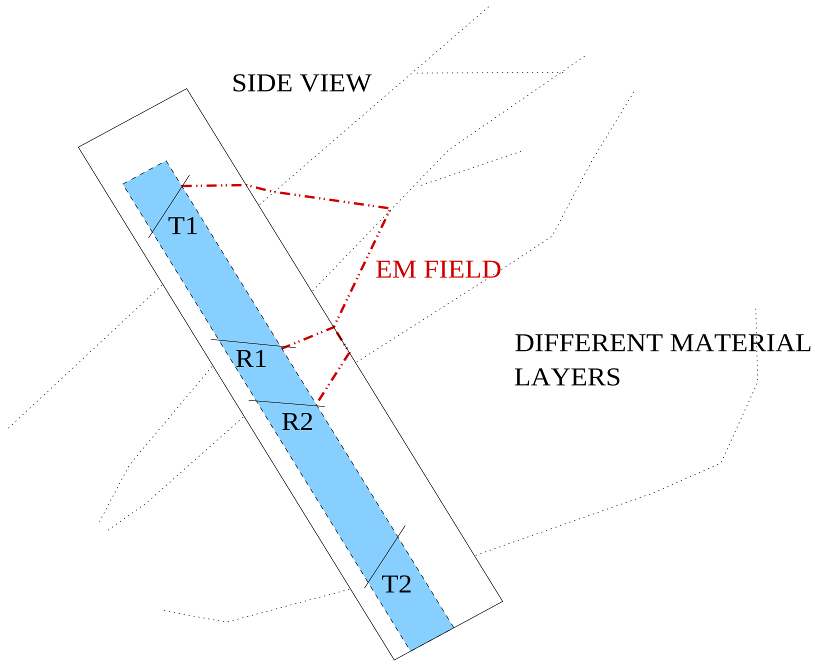
Waveguide Design



Goal: Determine electric field intensity at the ports.

2. MOTIVATION

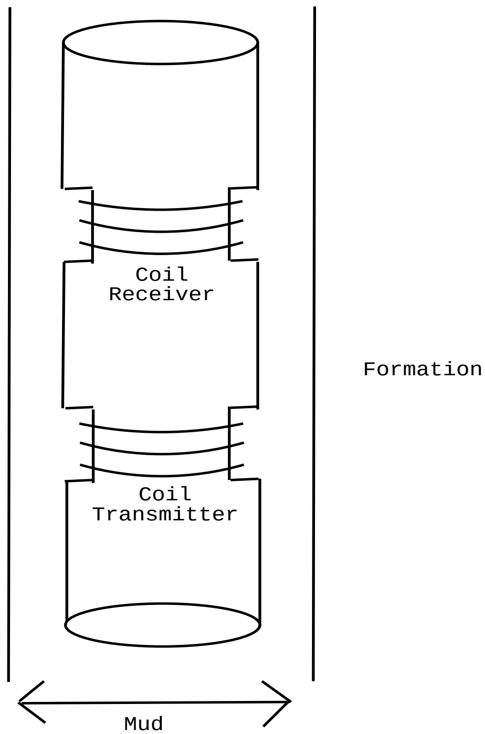
Modeling of a Logging While Drilling (LWD) electromagnetic measuring device



Goal: Determine EM field at the receiver antennas.

2. MOTIVATION

Modeling of a Logging While Drilling (LWD) electromagnetic measuring device



Simplest case:
ONE COIL TRANSMITTER

Goal: Determine EM field at the receiver antennas.

3. MAXWELL'S EQUATIONS

Time Harmonic Maxwell's Equations:

$$\nabla \times \mathbf{E} = -j\mu\omega\mathbf{H}$$
$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E} + \sigma\mathbf{E} + \mathbf{J}^{imp}$$

Reduced Wave Equation:

$$\boldsymbol{\nabla} \times \left(\frac{1}{\mu}\boldsymbol{\nabla} \times \boldsymbol{E}\right) - (\omega^2\epsilon - j\omega\sigma)\boldsymbol{E} = -j\omega\boldsymbol{J}^{imp}\,,$$

Boundary Conditions (BC):

- **Dirichlet BC at a PEC surface:**

$$\mathbf{n} \times \boldsymbol{E}^s = -\mathbf{n} \times \boldsymbol{E}^{inc} \qquad \mathbf{n} \times \boldsymbol{E} = 0$$

- **Neumann continuity BC at a material interface:**

$$\mathbf{n} \times \frac{1}{\mu}\nabla \times \boldsymbol{E}^s = -\mathbf{n} \times \frac{1}{\mu}\nabla \times \boldsymbol{E}^{inc} \qquad \mathbf{n} \times \frac{1}{\mu}\nabla \times \boldsymbol{E} = -j\omega\mathbf{J}_S^{imp}$$

- **Silver Müller radiation condition at ∞:**

$$\mathbf{e}_r \times (\nabla \times \boldsymbol{E}^s) - jk_0 \times \boldsymbol{E}^s = O(r^{-2})$$

3. MAXWELL'S EQUATIONS

Variational formulation

The reduced wave equation in Ω,

$$\nabla \times \left(\frac{1}{\mu} \nabla \times E \right) - (\omega^2 \epsilon - j\omega\sigma) E = -j\omega J^{imp} \, ,$$

A variational formulation

$$\left\{ \begin{array}{l} \text{Find } \mathrm{E} \in H_D(\mathrm{curl};\Omega) \text{ such that} \\ \displaystyle\int_\Omega \frac{1}{\mu} (\nabla \times \mathrm{E}) \cdot (\nabla \times \bar{F}) dx - \int_\Omega (\omega^2 \epsilon - j\omega\sigma) E \cdot \bar{\mathrm{F}} dx = \\ \qquad \displaystyle -j\omega \left\{ \int_\Omega J^{imp} \cdot \bar{F} dx + \int_{\Gamma_2} J_S^{imp} \cdot \bar{F} dS \right\} \quad \text{for all } F \in H_D(\mathrm{curl};\Omega) \, . \end{array} \right.$$

A stabilized variational formulation (using *a Lagrange multiplier*):

$$\left\{ \begin{array}{l} \text{Find } E \in H_D(\mathrm{curl};\Omega), p \in H_D^1(\Omega) \text{ such that} \\ \displaystyle\int_\Omega \frac{1}{\mu} (\nabla \times E)(\nabla \times \bar{F}) dx - \int_\Omega (\omega^2 \epsilon - j\omega\sigma) E \cdot \bar{F} dx - \int_\Omega (\omega^2 \epsilon - j\omega\sigma) \nabla p \cdot \bar{F} dx = \\ \qquad \displaystyle -j\omega \left\{ \int_\Omega J^{imp} \cdot \bar{F} dx + \int_{\Gamma_2} J_S^{imp} \cdot \bar{F} dS \right\} \quad \forall F \in H_D(\mathrm{curl};\Omega) \\ \displaystyle -\int_\Omega (\omega^2 \epsilon - j\omega\sigma) E \cdot \nabla \bar{q} \, dx = -j\omega \left\{ \int_\Omega J^{imp} \cdot \nabla \bar{q} \, dx + \int_{\Gamma_2} J_S^{imp} \cdot \nabla \bar{q} dS \right\} \quad \forall q \in H_D^1(\Omega) \, . \end{array} \right.$$

3. MAXWELL'S EQUATIONS

De Rham diagram

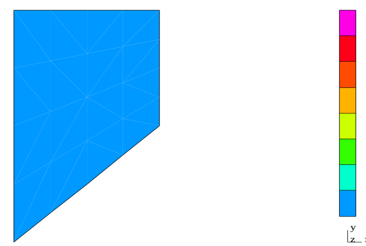
De Rham diagram is critical to the theory of FE discretizations of Maxwell's equations.

$$
\begin{array}{ccccccccccc}
\mathbb{R} & \longrightarrow & W & \xrightarrow{\nabla} & Q & \xrightarrow{\nabla\times} & V & \xrightarrow{\nabla\circ} & L^2 & \longrightarrow & 0 \\
\big\downarrow id & & \big\downarrow \Pi & & \big\downarrow \Pi^{\mathrm{curl}} & & \big\downarrow \Pi^{\mathrm{div}} & & \big\downarrow P & & \\
\mathbb{R} & \longrightarrow & W^{\mathrm{p}} & \xrightarrow{\nabla} & \mathrm{Q}^{\mathrm{p}} & \xrightarrow{\nabla\times} & \mathrm{V}^{\mathrm{p}} & \xrightarrow{\nabla\circ} & W^{\mathrm{p}-1} & \longrightarrow & 0\,.
\end{array}
$$

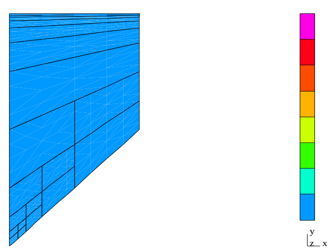
This diagram relates two exact sequences of spaces, on both continuous and discrete levels, and corresponding interpolation operators.

4. *HP*-ADAPTIVITY

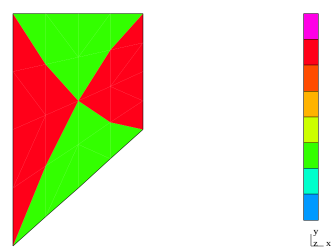
Different refinement strategies for finite elements:



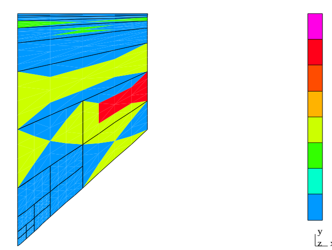
Given initial grid



***h*-refined grid**



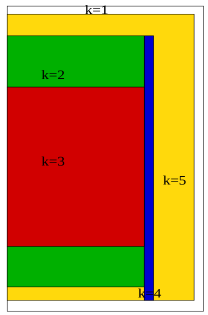
***p*-refined grid**



***hp*-refined grid**

4. *HP*-ADAPTIVITY

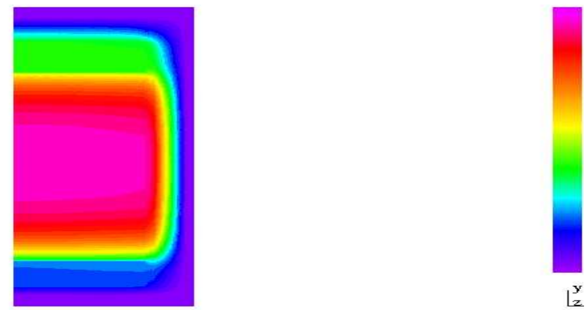
Orthotropic heat conduction example



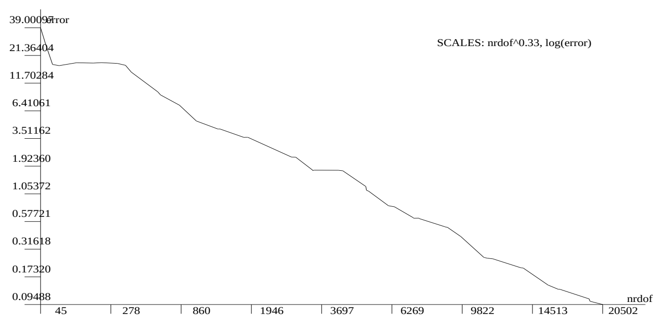
Equation: $\nabla(\mathrm{K}\nabla u) = f^{(k)}$

$$\mathrm{K} = \mathrm{K}^{(k)} = \begin{bmatrix} K_x^{(k)} & 0 \\ 0 & K_y^{(k)} \end{bmatrix}$$

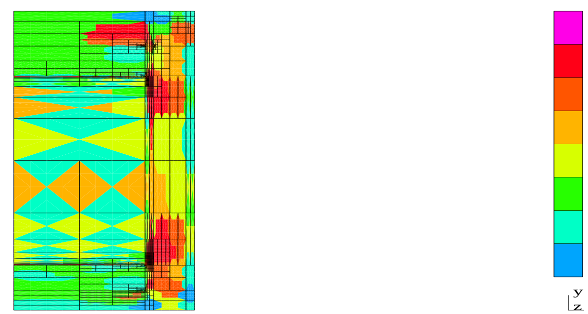
$$K_x^{(k)} = (25,\ 7,\ 5,\ 0.2,\ 0.05)$$
$$K_y^{(k)} = (25,\ 0.8,\ 0.0001,\ 0.2,\ 0.05)$$



Solution: unknown
Boundary Conditions:
$$K^{(i)}\nabla u \cdot n = g^{(i)} - \alpha^{(i)} u$$



Convergence history
(tolerance error = 0.1 %)

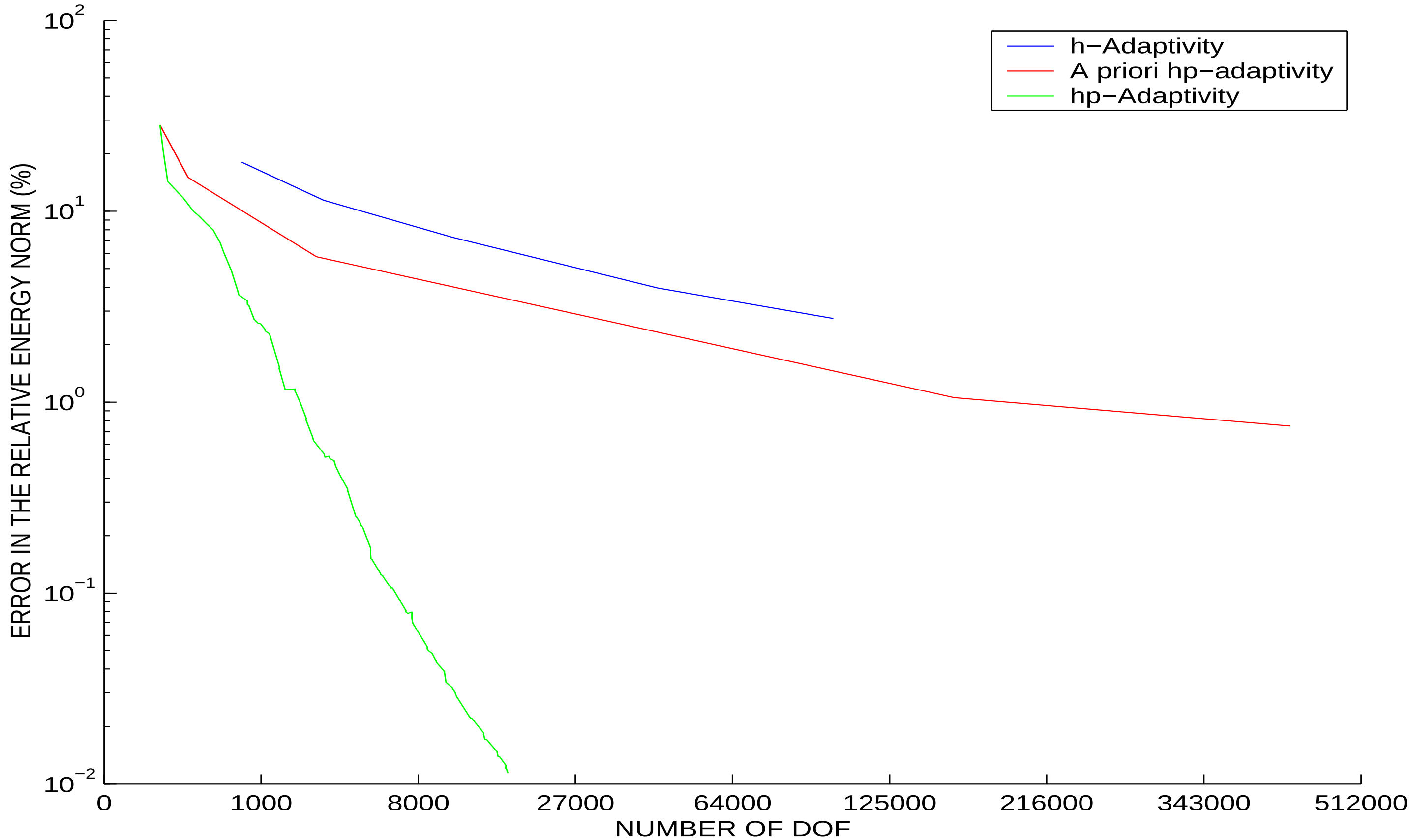


Final hp grid

4. *HP*-ADAPTIVITY

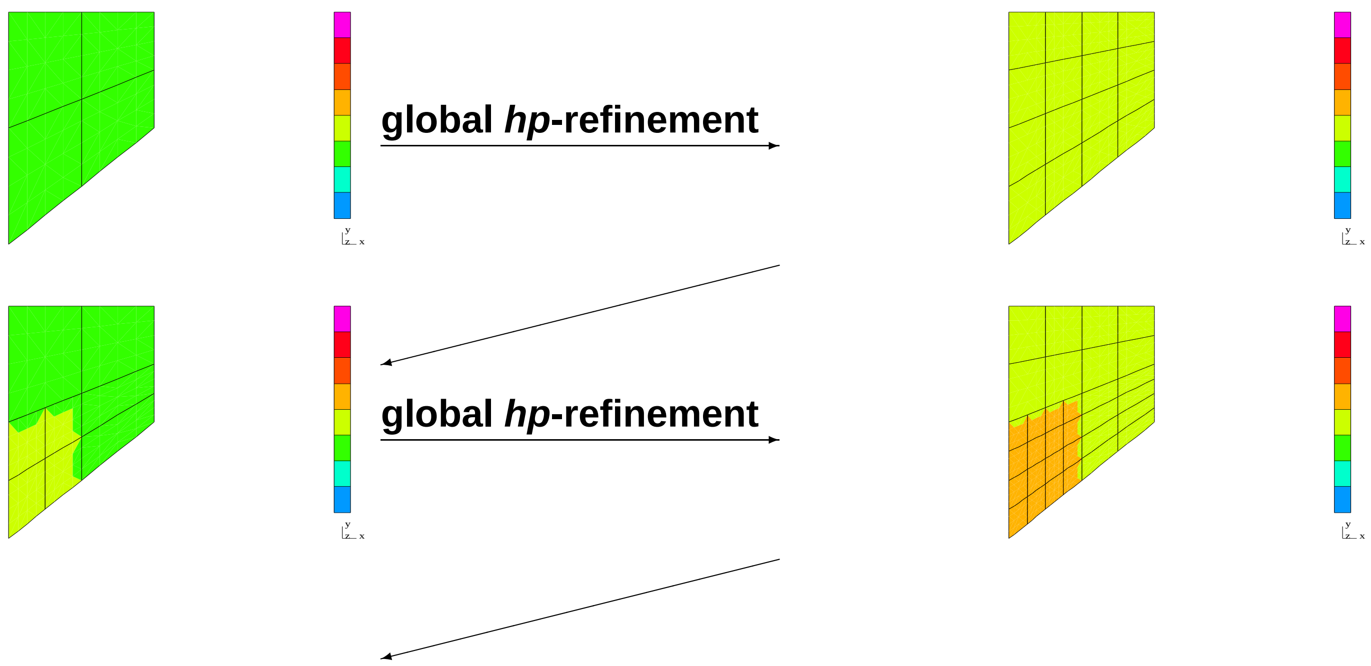
Convergence comparison

Orthotropic heat conduction example



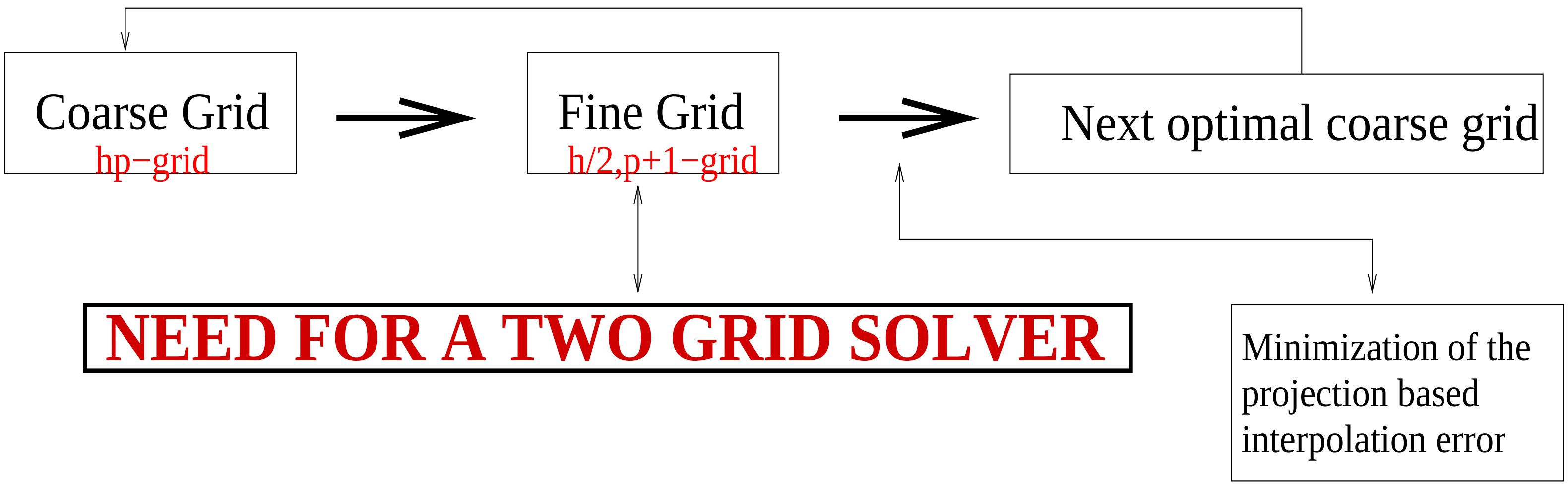
5. THE FULLY AUTOMATIC *HP*-ADAPTIVE STRATEGY

Fully automatic *hp*-adaptive strategy



5. THE FULLY AUTOMATIC *HP*-ADAPTIVE STRATEGY

Automatic *hp*-adaptivity delivers exponential convergence and enables solution of challenging EM problems



6. A TWO GRID SOLVER FOR SPD PROBLEMS

We seek x such that $Ax = b$. Consider the following iterative scheme:

$$r^{(n+1)} = [I - \alpha^{(n)} AS] r^{(n)}$$
$$x^{(n+1)} = [I - \alpha^{(n)} S] r^{(n)}$$

where S is a matrix, and $\alpha^{(n)}$ is a relaxation parameter. $\alpha^{(n)}$ *optimal* if:

$$\alpha^{(n)} = \arg \min \| x^{(n+1)} - x \|_A = \frac{(A^{-1} r^{(n)}, S r^{(n)})_A}{(S r^{(n)}, S r^{(n)})_A}$$

Then, we define our two grid solver as:

1 Iteration with $S = S_F = \sum A_i^{-1} \quad +$

1 Iteration with $S = S_C = P A_C^{-1} R$

6. A TWO GRID SOLVER FOR SPD PROBLEMS

Error reduction and stopping criteria

Let $e^{(n)} = x^{(n)} - x$ the error at step n, $\tilde{e}^{(n)} = [I - S_C A]e^{(n)} = [I - P_C]e^{(n)}$. Then:

$$\frac{\| e^{(n+1)} \|_A^2}{\| e^{(n)} \|_A^2} = 1 - \frac{| (\tilde{e}^{(n)}, S_F A \tilde{e}^{(n)})_A |^2}{\| \tilde{e}^{(n)} \|_A^2 \| S_F A \tilde{e}^{(n)} \|_A^2} = 1 - \frac{| (\tilde{e}^{(n)}, (P_C + S_F A)\tilde{e}^{(n)})_A |^2}{\| \tilde{e}^{(n)} \|_A^2 \| S_F A \tilde{e}^{(n)} \|_A^2}$$

Then:

$$\frac{\| e^{(n+1)} \|_A^2}{\| e^{(n)} \|_A^2} \leq \sup_e [1 - \frac{| (e, (P_C + S_F A)e)_A |^2}{\| e \|_A^2 \| S_F A e \|_A^2}] \leq C < 1$$ **(Error Reduction)**

For our stopping criteria, we want: Iterative Solver Error $\approx$ Discretization Error. That is:

$$\frac{\| e^{(n+1)} \|_A}{\| e^{(0)} \|_A} \leq 0.01$$ **(Stopping Criteria)**

A TWO GRID SOLVER FOR ELECTROMAGNETICS

We seek x such that $Ax = b$. Consider the following iterative scheme:

$$\boldsymbol{r}^{(n+1)} = [\boldsymbol{I} - \boldsymbol{\alpha}^{(n)}\boldsymbol{A}\boldsymbol{S}]\boldsymbol{r}^{(n)}$$
$$\boldsymbol{x}^{(n+1)} = [\boldsymbol{I} - \boldsymbol{\alpha}^{(n)}\boldsymbol{S}]\boldsymbol{r}^{(n)}$$

where S is a matrix, and $\alpha^{(n)}$ is a relaxation parameter. $\alpha^{(n)}$ *optimal* if:

$$\boldsymbol{\alpha}^{(n)} = \arg\ \min \parallel \boldsymbol{x}^{(n+1)} - \boldsymbol{x} \parallel_B = \frac{(\boldsymbol{A}^{-1}\boldsymbol{r}^{(n)}, \boldsymbol{S}\boldsymbol{r}^{(n)})_B}{(\boldsymbol{S}\boldsymbol{r}^{(n)}, \boldsymbol{S}\boldsymbol{r}^{(n)})_B}$$ **(NOT COMPUTABLE)**

Then, we define our two grid solver for Electromagnetics as:

1 Iteration with $S = S_F = \sum A_i^{-1} \quad +$

1 Iteration with $S = S_\nabla = \sum G_i^{-1} \quad +$

1 Iteration with $S = S_C = P A_C^{-1} R$

A TWO GRID SOLVER FOR ELECTROMAGNETICS

A two grid solver for discretization of Maxwell's equations using hp-FE

Consider the following two problems:

Problem I: $\nabla \times \nabla \times \mathbf{E} - k^2\mathbf{E} = \mathbf{J}$

Matrix form: $Au = v$

Two grid solver V-cycle:

$TG = (I - \alpha_1 S_F A)(I - \alpha_2 S_\nabla A)(I - S_C A_C)$

Problem II: $\nabla \times \nabla \times \mathbf{E} + \mathbf{E} = \mathbf{J}$

Matrix form: $\hat{A}u = v$

Two grid solver V-cycle:

$\widehat{TG} = (I - \alpha_1 \hat{S}_F \hat{A})(I - \alpha_2 \hat{S}_\nabla \hat{A})(I - \hat{S}_C \hat{A}_C)$

Theorem: If h is small enough, then:

$$\| TGe^{(n)} \| \leq \| \widehat{TG}e^{(n)} \| + Ch$$

Notice that C is independent of h and p.

A TWO GRID SOLVER FOR ELECTROMAGNETICS

A two grid solver for discretization of Maxwell's equations using hp-FE

Helmholtz decomposition:

$$H_D(\mathrm{curl};\Omega) = (Ker(\mathrm{curl})) \oplus (Ker(\mathrm{curl}))^{\perp}$$

We define the following subspaces (T =grid, K =element, v =vertex, e =edge):

$\Omega^v_{k,i} = \mathrm{int}(\bigcup\{\bar{K} \in T_k : v_{k,i} \in \partial K\})$; $\Omega^e_{k,i} = \mathrm{int}(\bigcup\{\bar{K} \in T_k : e_{k,i} \in \partial K\})$ *Domain decomposition*

$M^v_{k,i} = \{u \in M_k : \mathrm{supp}(u) \subset \Omega^v_{k,i}\}$; $M^e_{k,i} = \{u \in M_k : \mathrm{supp(u)} \subset \Omega^e_{k,i}\}$ *Nedelec's elements decomposition*

$W^v_{k,i} = \{u \in W_k : \mathrm{supp}(u) \subset \Omega^v_{k,i}\}$; $W^e_{k,i} = \{u \in W_k : \mathrm{supp}(u) \subset \Omega^e_{k,i}\} = \emptyset$ *Polynomial spaces decomposition*

Hiptmair proposed the following decomposition of M_k:

$$M_k = \sum_e M^e_{k,i} + \sum_v \nabla W^v_{k,i}$$

Arnold *et. al* proposed the following decomposition of M_k:

$$M_k = \sum_v M^v_{k,i}$$

8. PERFORMANCE OF THE TWO GRID SOLVER

Numerical Studies

2002

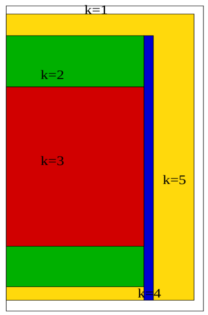
- Importance of the choice of shape functions.
- Importance of the relaxation parameter.
- Selection of patches for the block Jacobi smoother.
- Effect of averaging.
- Error estimation.
- Smoothing vs two grid solver.
- Guiding hp-adaptivity with a partially converged fine grid solution.

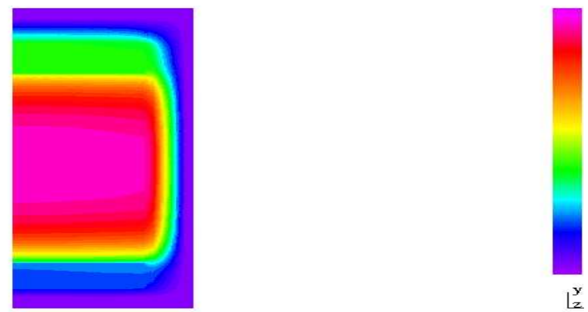
2003

- Guiding hp-adaptivity with a partially converged fine grid solution for EM problems.
- Efficiency of the two grid solver.
- Number of elements per wavelength required by the two grid solver to converge.
- Control of the dispersion error.
- Applications to real world problems.

8. PERFORMANCE OF THE TWO GRID SOLVER

Orthotropic heat conduction example





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Solution: unknown
Boundary Conditions:
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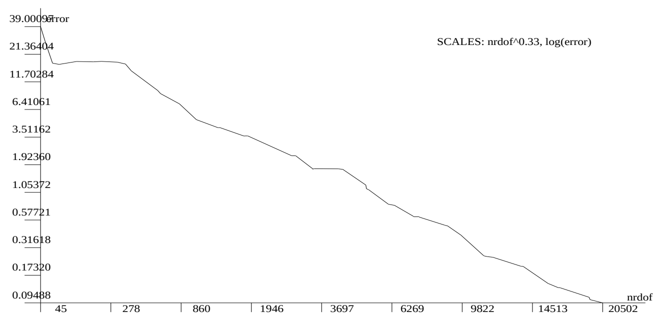
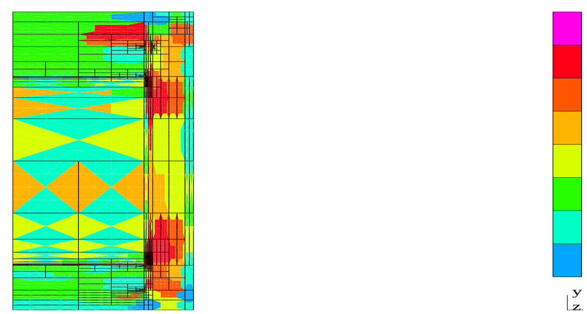




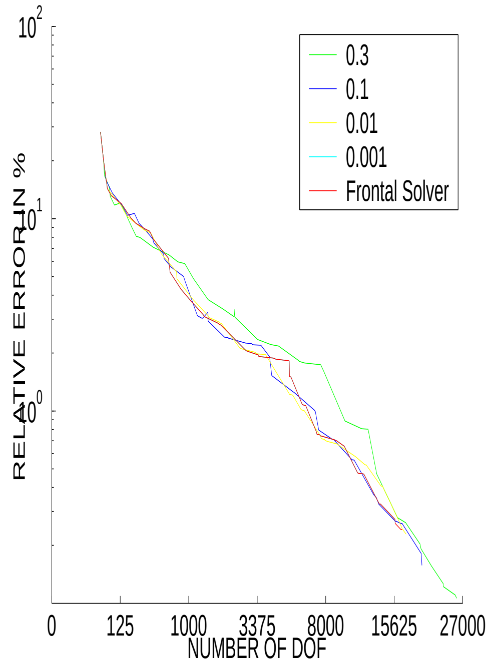
Convergence history
(tolerance error = 0.1 %)

Final hp grid

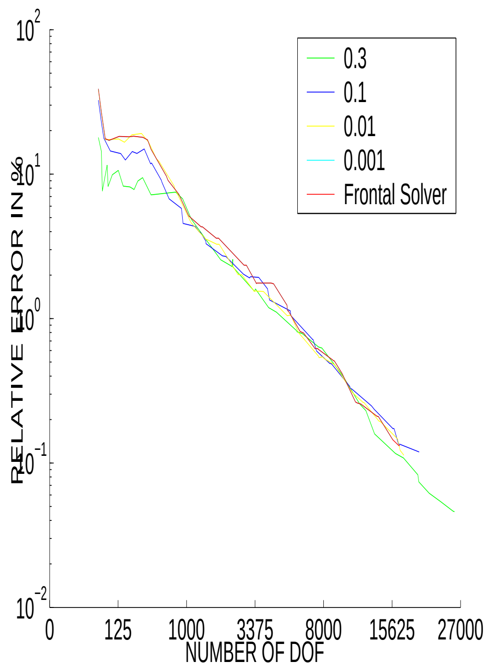
8. PERFORMANCE OF THE TWO GRID SOLVER

Guiding automatic hp-refinements

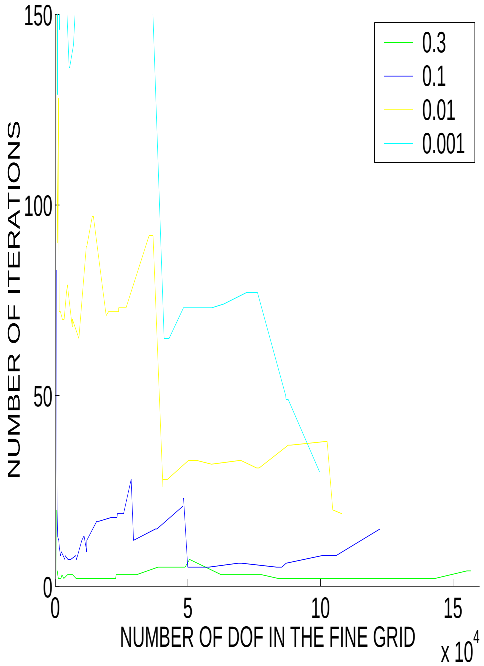
Orthotropic heat conduction. Guiding hp-refinements with a partially converged solution.



Energy error estimate



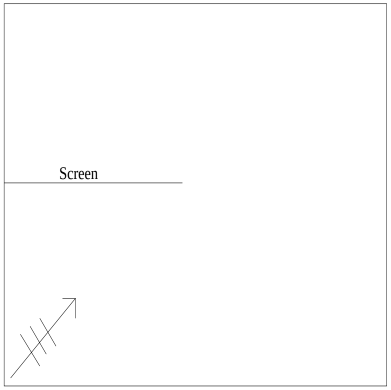
Discretization error estimate



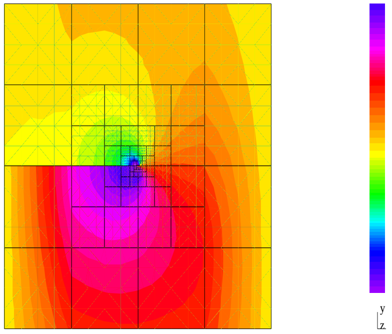
Number of iterations

8. PERFORMANCE OF THE TWO GRID SOLVER

Plane Wave incident into a screen (diffraction problem)

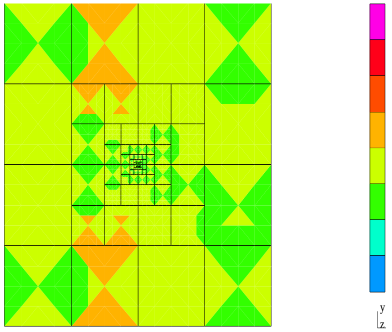


Geometry



Second component of electric field

Convergence history
(tolerance error = 0.1 %)

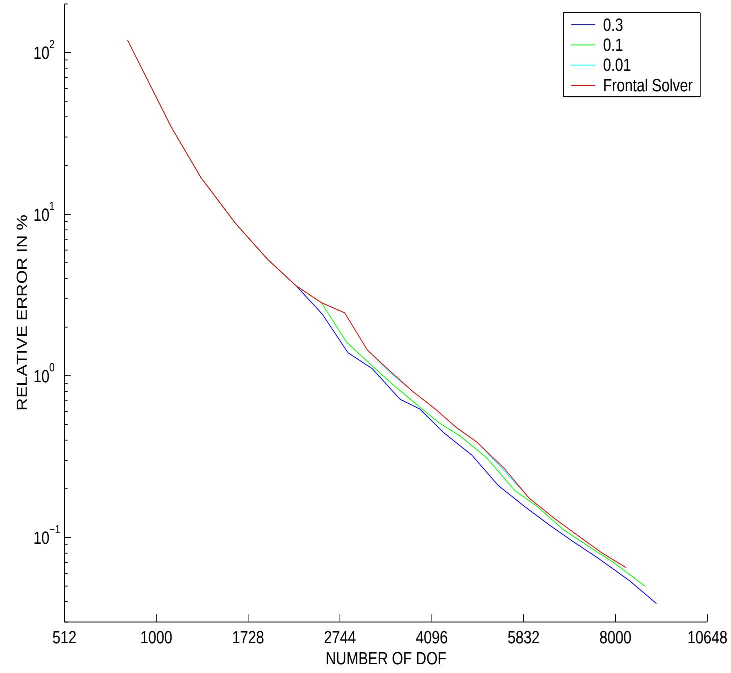


Final hp-grid

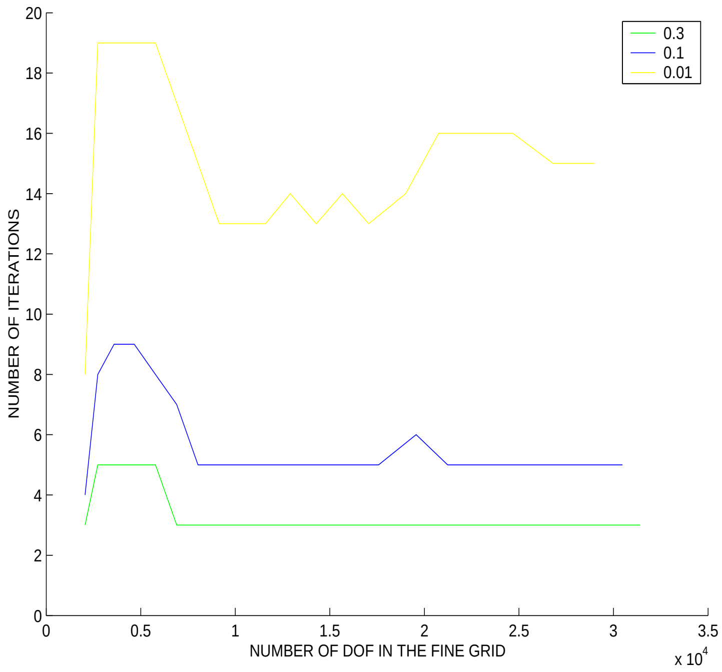
Numerical Results

Guiding automatic hp-refinements

Diffraction problem. Guiding hp-refinements with a partially converged solution.



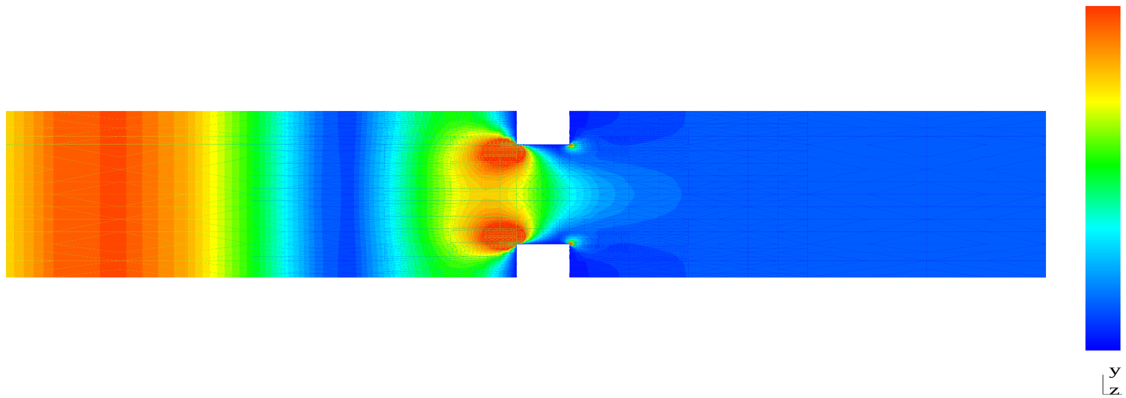
Discretization error estimate



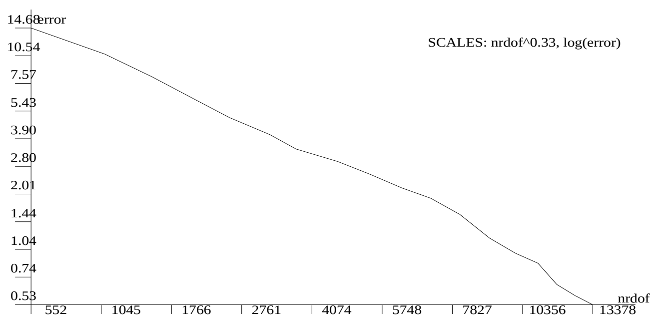
Number of iterations

8. PERFORMANCE OF THE TWO GRID SOLVER

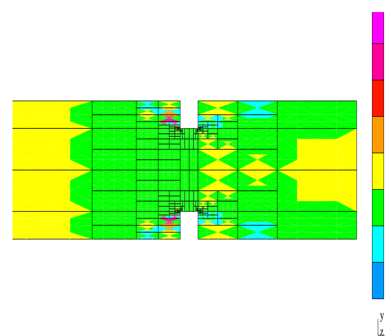
Waveguide example



Module of Second Component of Magnetic Field



Convergence history
(tolerance error = 0.5 %)

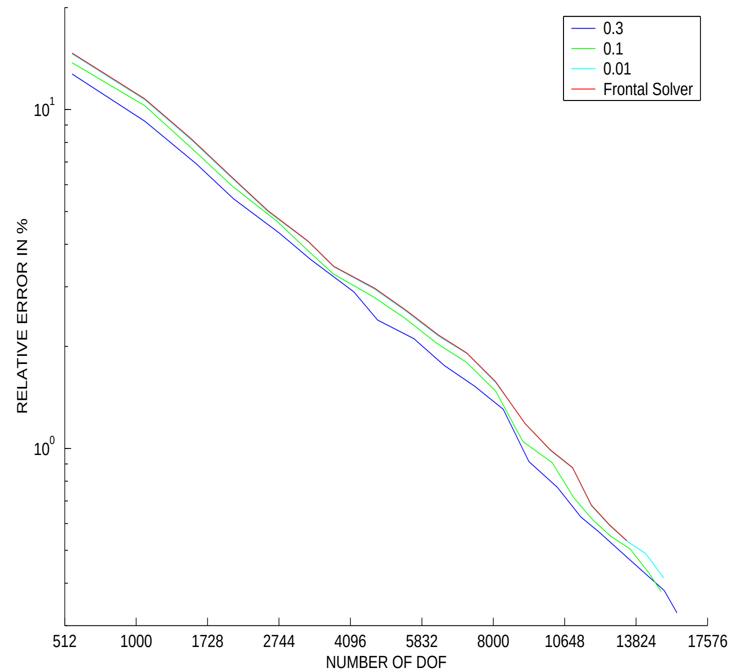


Final hp-grid

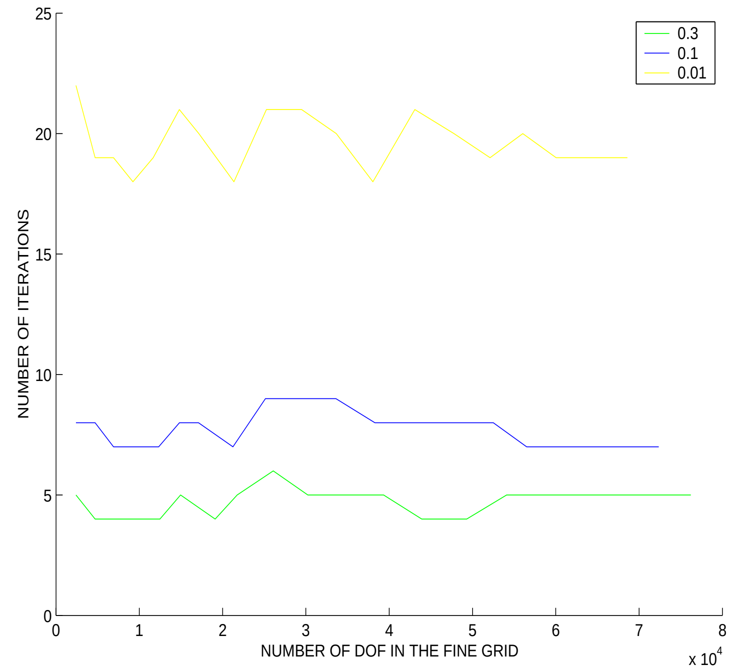
8. PERFORMANCE OF THE TWO GRID SOLVER

Guiding automatic hp-refinements

Waveguide example. Guiding hp-refinements with a partially converged solution.



Discretization error estimate



Number of iterations

8. PERFORMANCE OF THE TWO GRID SOLVER

Efficiency of the two grid solver

We studied scalability of the solver with respect h and p.

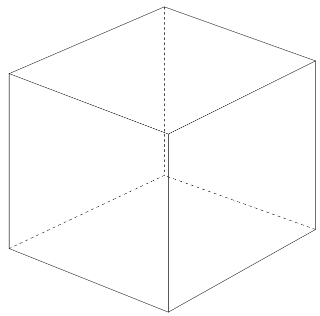
Speed = Coarse grid solve $+\mathcal{O}(p^9 N)$

We implemented an efficient solver.

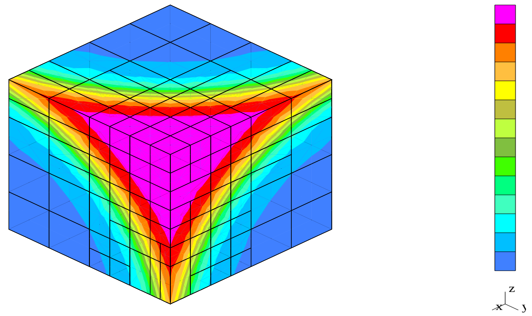
- **Fast integration rules.**
- **Fast matrix vector multiplication.**
- **Fast assembling.**
- **Fast patch inversion.**
- **Fast construction of prolongation/restriction operator.**

8. PERFORMANCE OF THE TWO GRID SOLVER

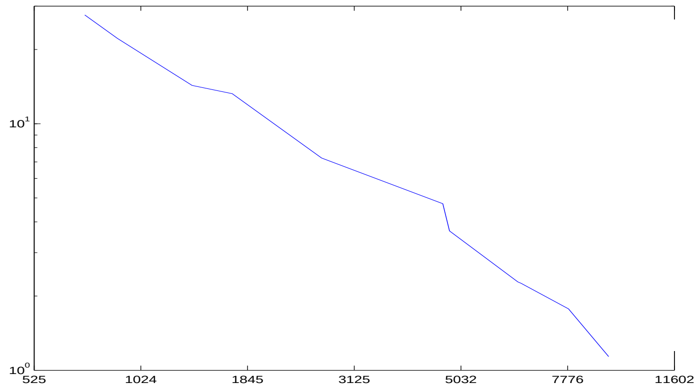
3D shock like solution example



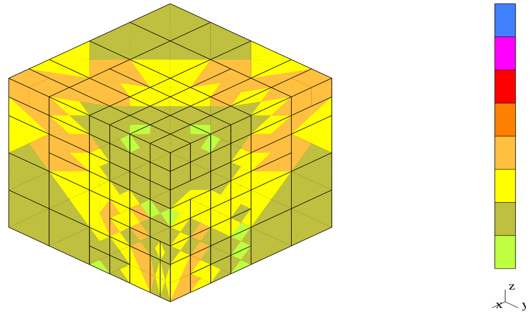
Equation: $-\Delta u = f$
Geometry: unit cube



Solution: $u = atan(20 * \sqrt{r} - \sqrt{3}))$
$r = (x - .25) * *2 + (y - .25) * *2 + (z - .25) * *2$
Dirichlet Boundary Conditions



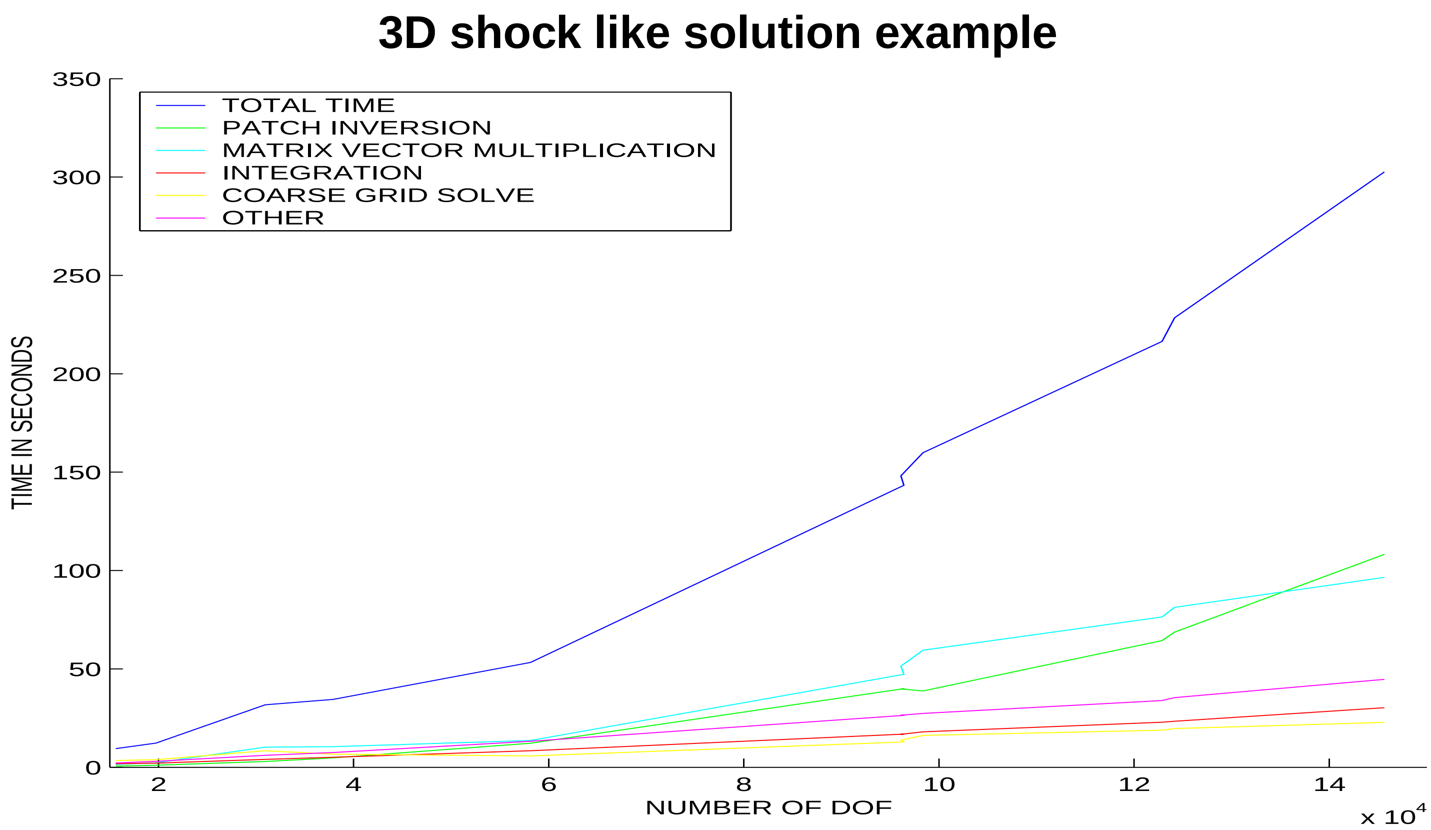
Convergence history
(tolerance error = 1%)



Final hp grid

8. PERFORMANCE OF THE TWO GRID SOLVER

Performance of the two grid solver

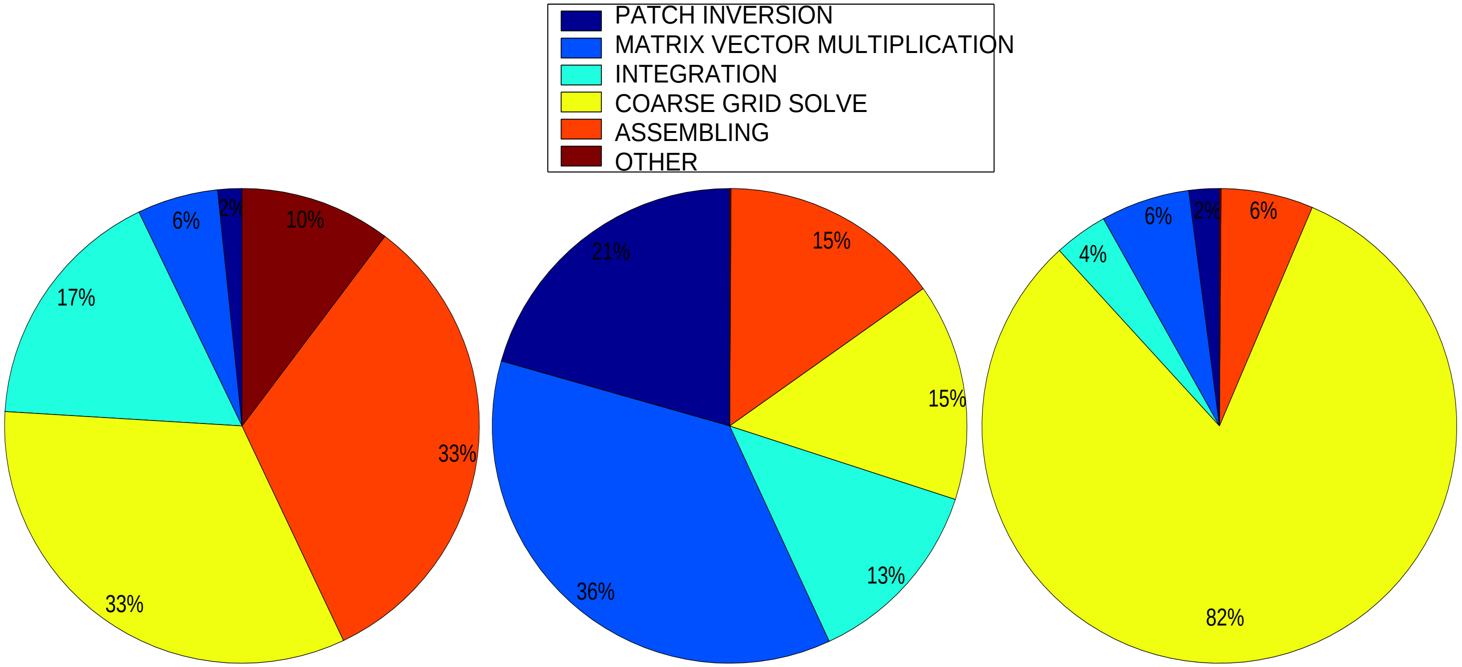


In core computations, AMD Athlon 1 Ghz processor.

8. PERFORMANCE OF THE TWO GRID SOLVER

Performance of the two grid solver

3D shock like solution problem



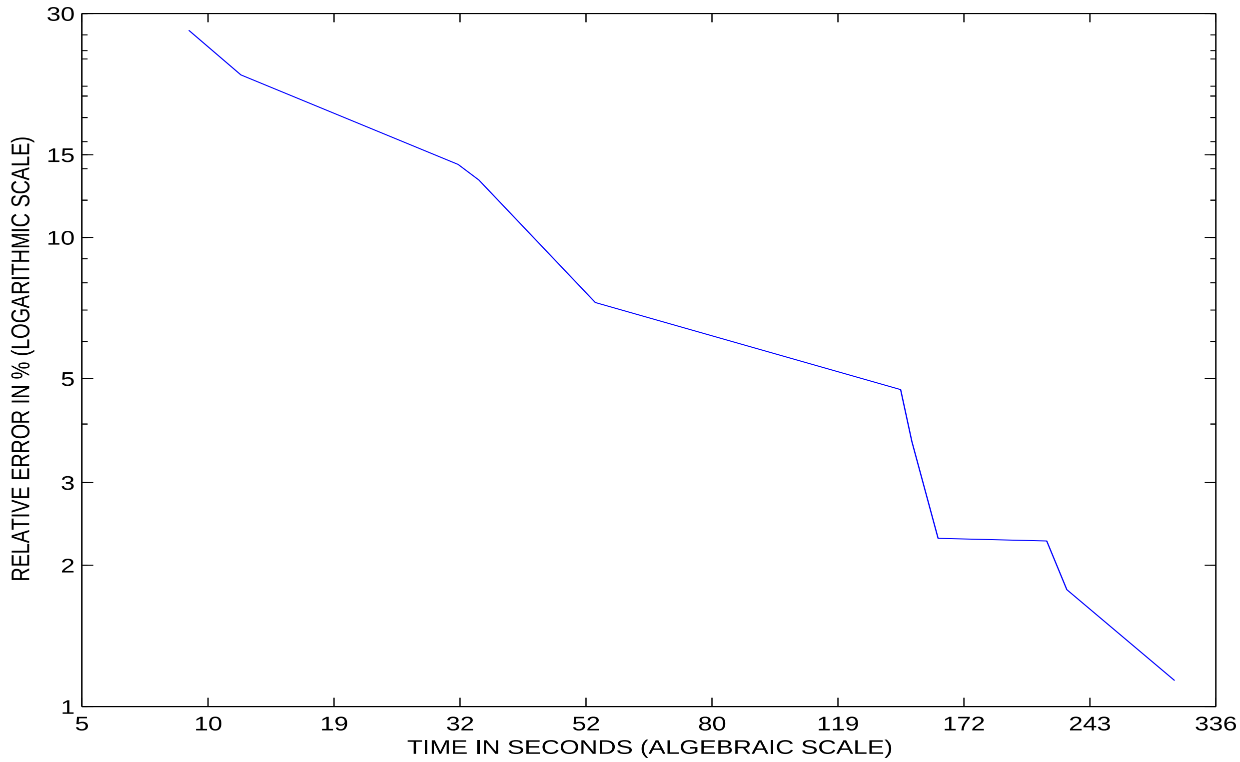
Nrdofs ≈ 2.15 Million	Nrdofs ≈ 0.27 Million	Nrdofs ≈ 2.15 Million
Total time ≈ 8 minutes	Total time ≈ 10 minutes	Total time ≈ 50 minutes
Memory* ≈ 1.0 Gb	Memory* ≈ 2.0 Gb	Memory* ≈ 3.5 Gb
p=2	p=8	p=4

*Memory = memory used by nonzero entries of stiffness matrix
In core computations, IBM Power4 1.3 Ghz processor.

8. PERFORMANCE OF THE TWO GRID SOLVER

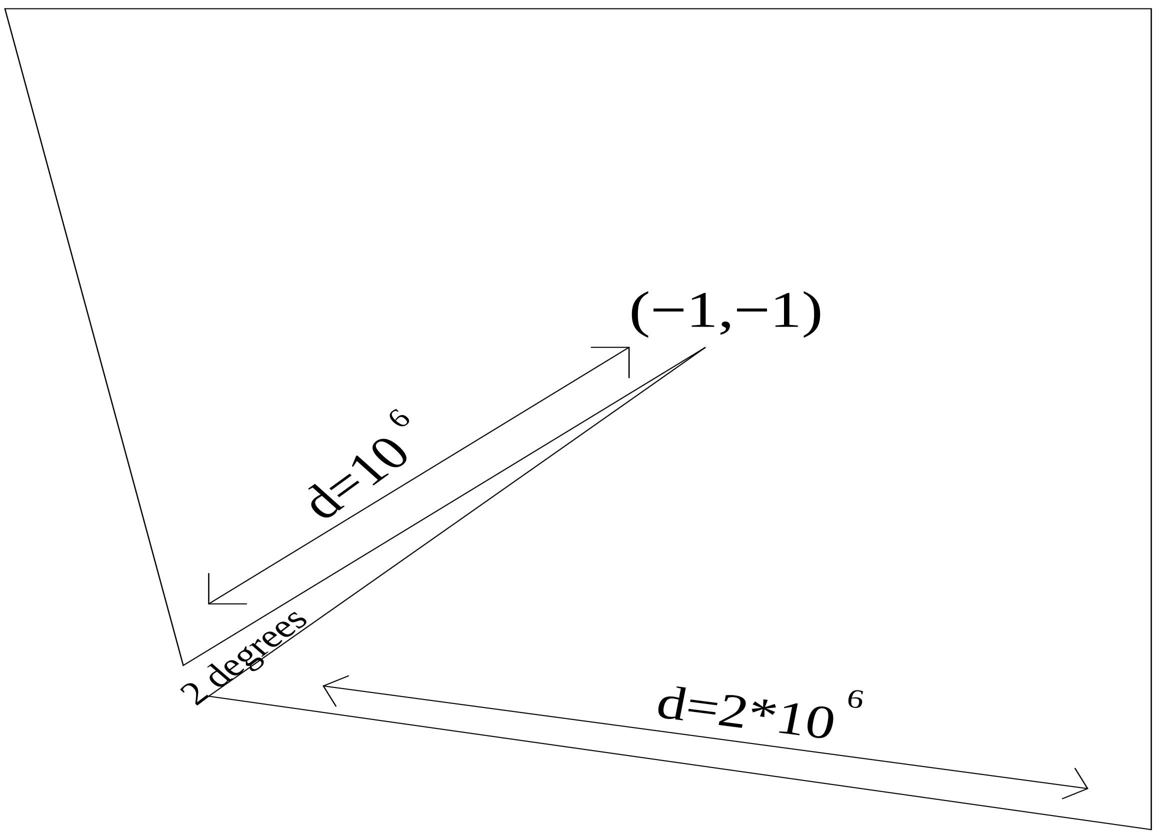
Convergence history

3D shock like solution example.
Scales: ERROR VS TIME.



9. ELECTROMAGNETIC APPLICATIONS

Edge diffraction example (Baker-Hughes): Electrostatics

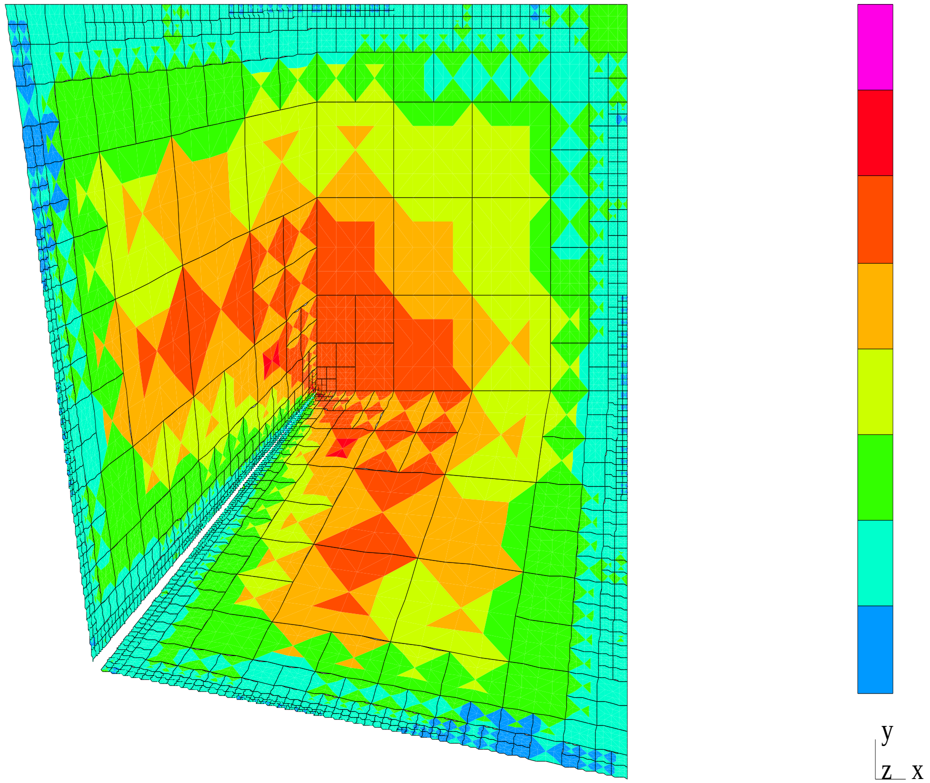


Dirichlet Boundary Conditions
u(boundary)=−ln r, r=sqrt (x*x+y*y)

9. ELECTROMAGNETIC APPLICATIONS

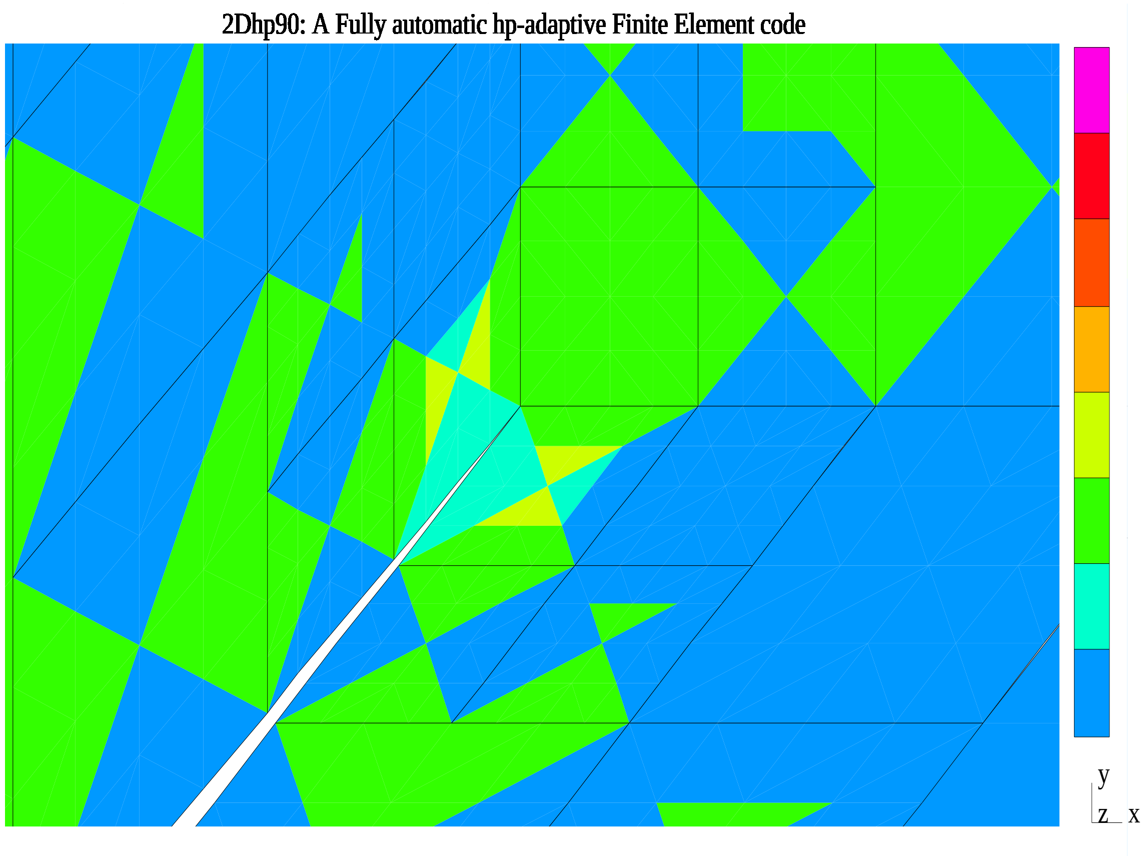
Edge diffraction example: final *hp*-grid, Zoom = 1

2Dhp90: A Fully automatic hp-adaptive Finite Element code



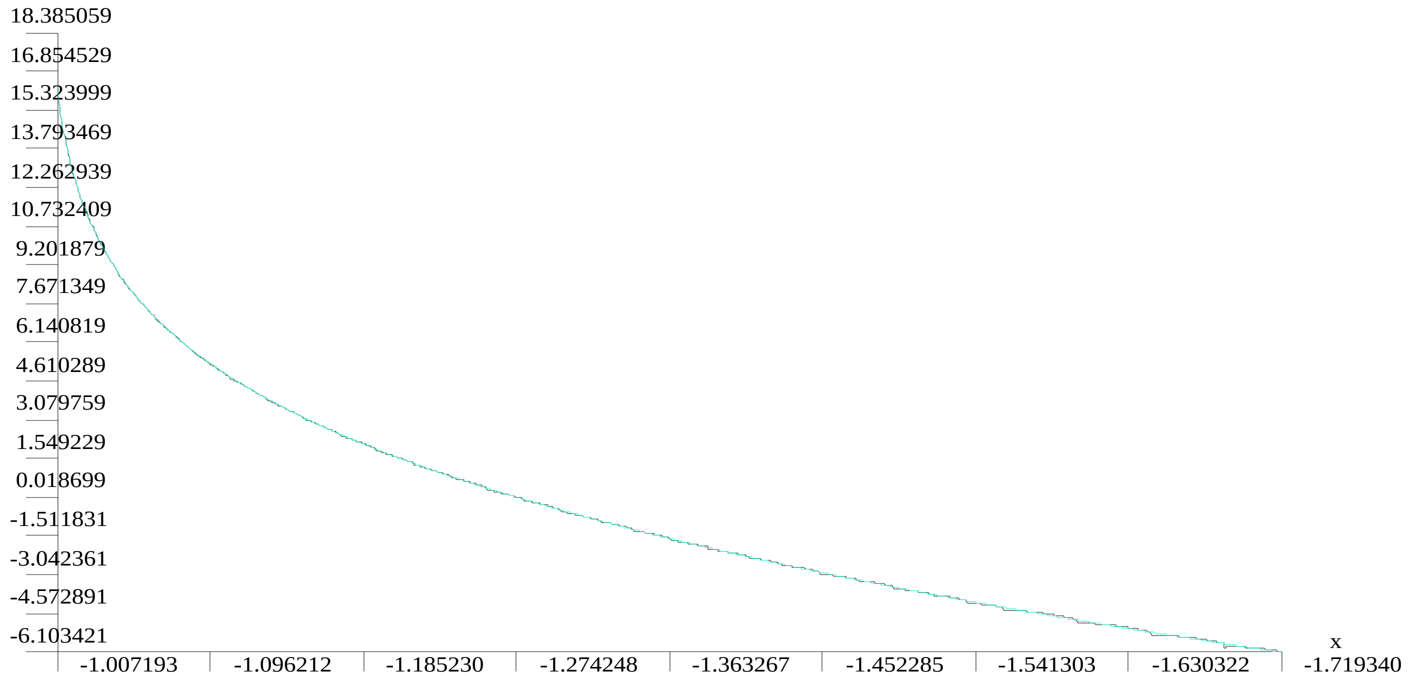
9. ELECTROMAGNETIC APPLICATIONS

Edge diffraction example: final *hp*-grid, Zoom = 10^{13}



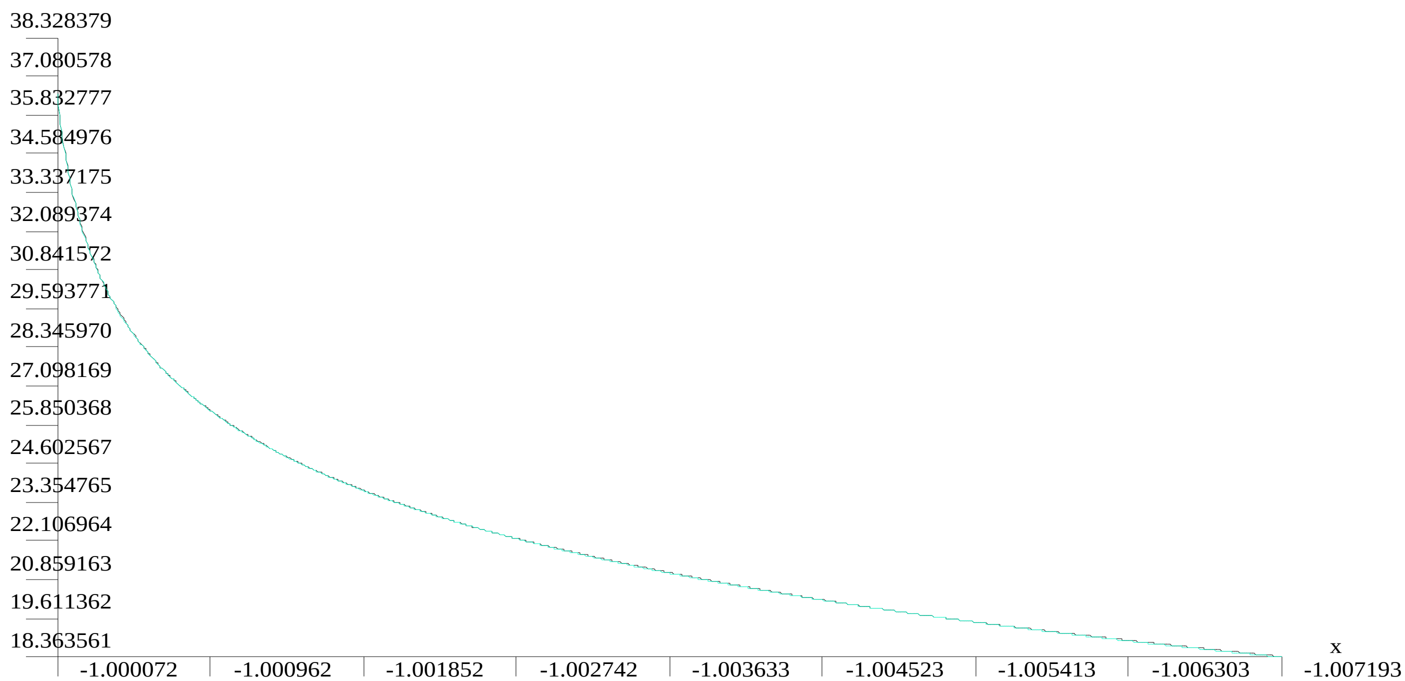
9. ELECTROMAGNETIC APPLICATIONS

Edge diffraction example: Comparison between exact and approximate solution at distances 0.01-1 from the singularity



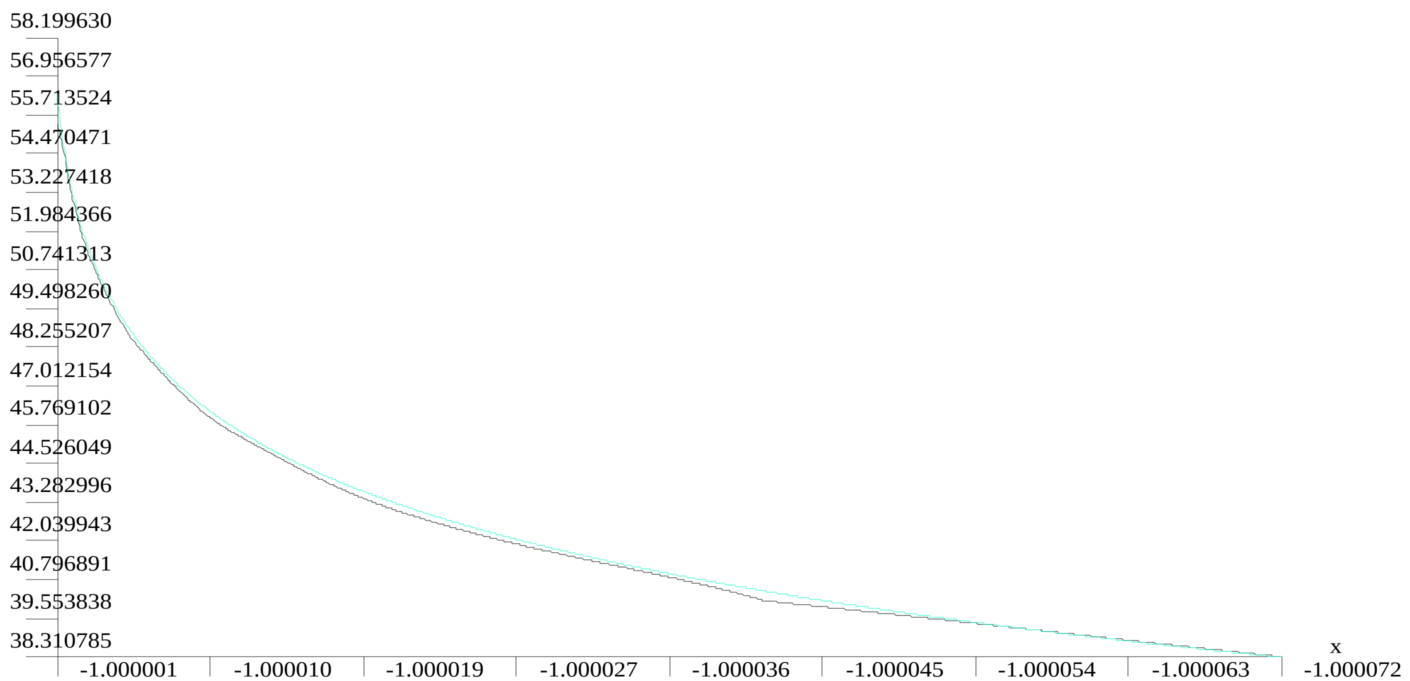
9. ELECTROMAGNETIC APPLICATIONS

Edge diffraction example: Comparison between exact and approximate solution at distances 0.0001-0.01 from the singularity



9. ELECTROMAGNETIC APPLICATIONS

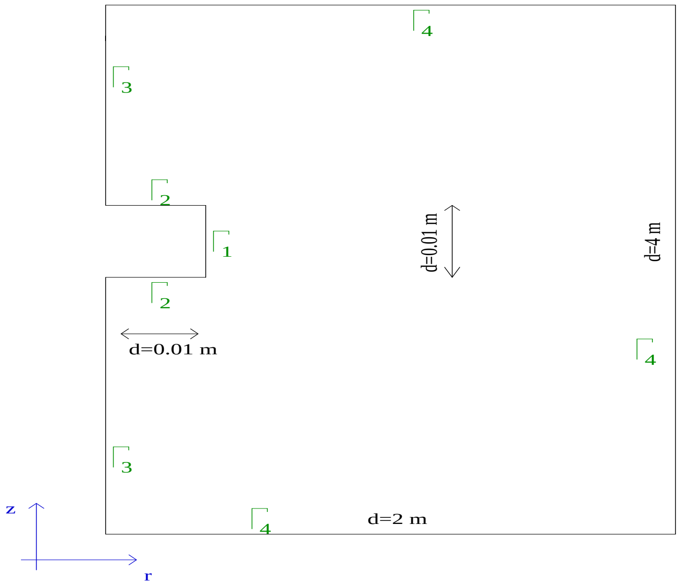
Edge diffraction example: Comparison between exact and approximate solution at distances 0.000001-0.0001 from the singularity



9. ELECTROMAGNETIC APPLICATIONS

Time Harmonic Maxwell's Equations

$$\nabla \times \mathbf{E} = -j\mu\omega\mathbf{H}$$
$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E} + \sigma\mathbf{E}$$



Reduced Wave Equation:

$$\boldsymbol{\nabla} \times \left(\frac{1}{\mu}\boldsymbol{\nabla} \times \boldsymbol{E}\right) - (\omega^2\epsilon - j\omega\sigma)\boldsymbol{E} = -j\omega J^{imp}$$

Boundary Conditions (BC):

Dirichlet BC at a PEC surface:

$$\mathbf{n} \times \boldsymbol{E} = 0 \;\; on \;\; \Gamma_2 \cup \Gamma_4$$

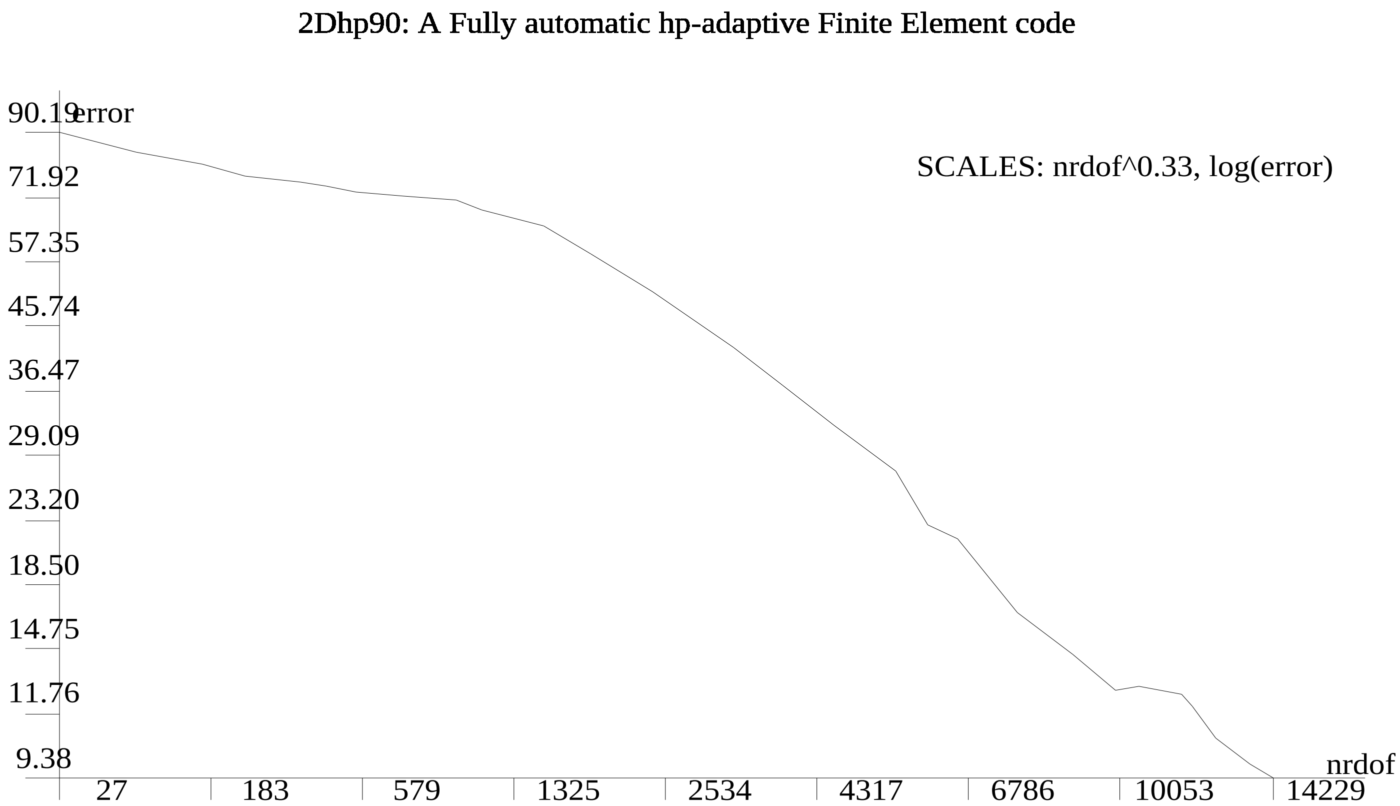
Neumann BC's:

$$\mathbf{n} \times \frac{1}{\mu}\nabla \times \boldsymbol{E} = -j\omega \;\; on \;\; \Gamma_1$$

$$\mathbf{n} \times \frac{1}{\mu}\nabla \times \boldsymbol{E} = 0 \;\; on \;\; \Gamma_3$$

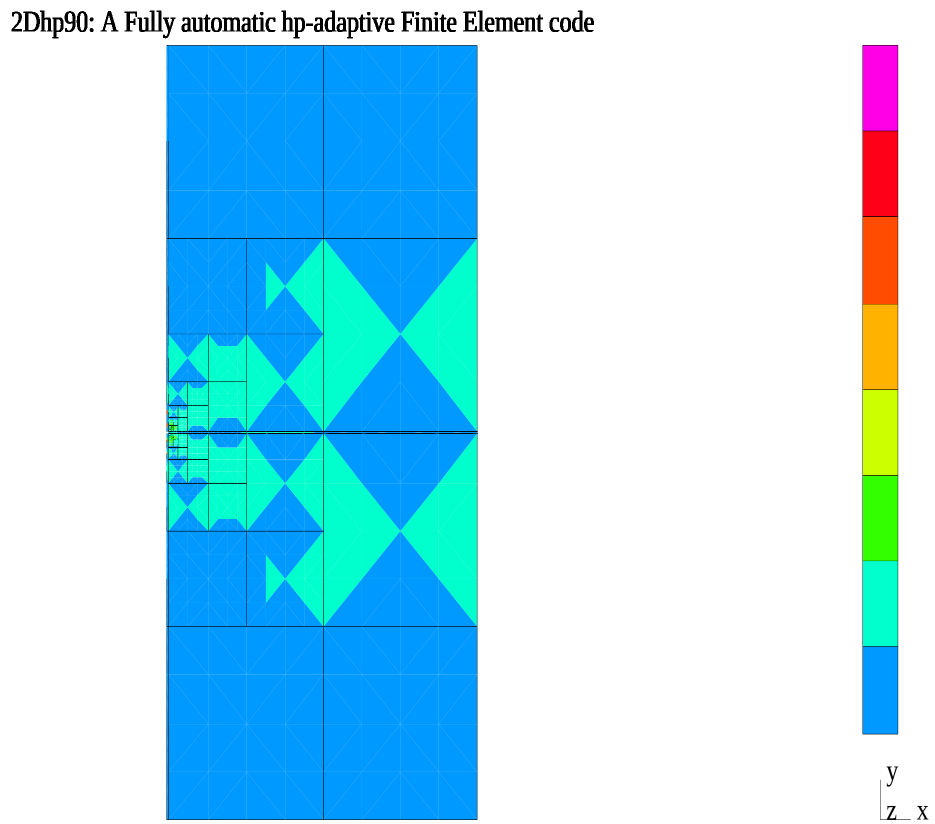
9. ELECTROMAGNETIC APPLICATIONS

Battery example: Convergence history



9. ELECTROMAGNETIC APPLICATIONS

Battery example: final *hp*-grid, Zoom = 1



9. ELECTROMAGNETIC APPLICATIONS

Why the optimal grid is so bad?

Optimization is based on minimization of the ENERGY NORM of the error, given by:

$$\| \, error \, \|^2 = \int | \, error \, |^2 + \int | \, \nabla \times error \, |^2$$

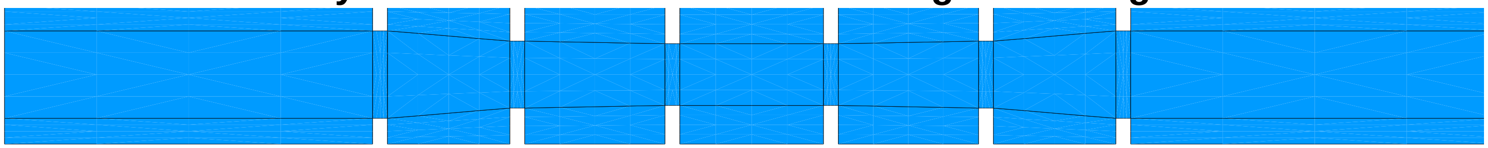
Interpretation of results:

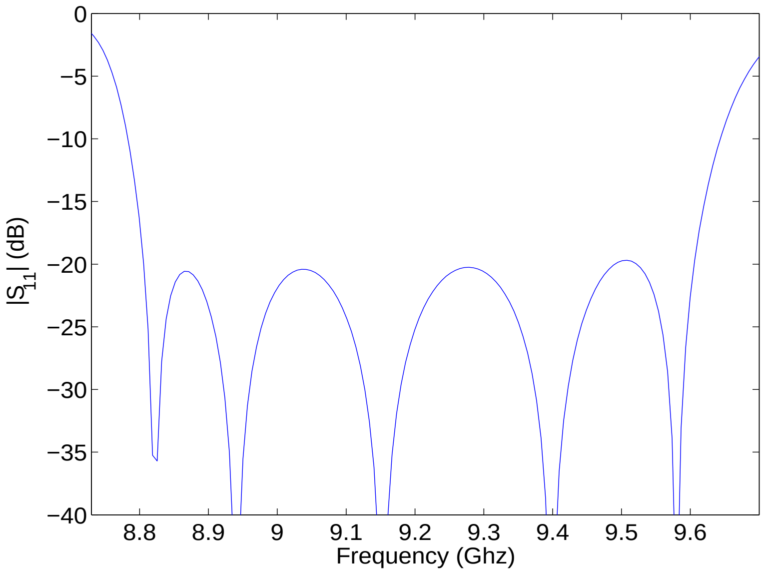
- **The grid is optimal for the selected refinement criteria,**
- **but our refinement criteria is inadequate for our pourposes.**

9. ELECTROMAGNETIC APPLICATIONS

Waveguide example with five iris

Geometry of a cross section of the rectangular waveguide





Return loss of the waveguide structure

H-plane five resonant iris filter.

Dominant mode (source): $TE_{10}-$mode.

Dimensions $\approx 20 \times 2 \times 1$ cm.

Operating Frequency $\approx 8.8 - 9.6$ Ghz

Cutoff frequency ≈ 6.56 Ghz

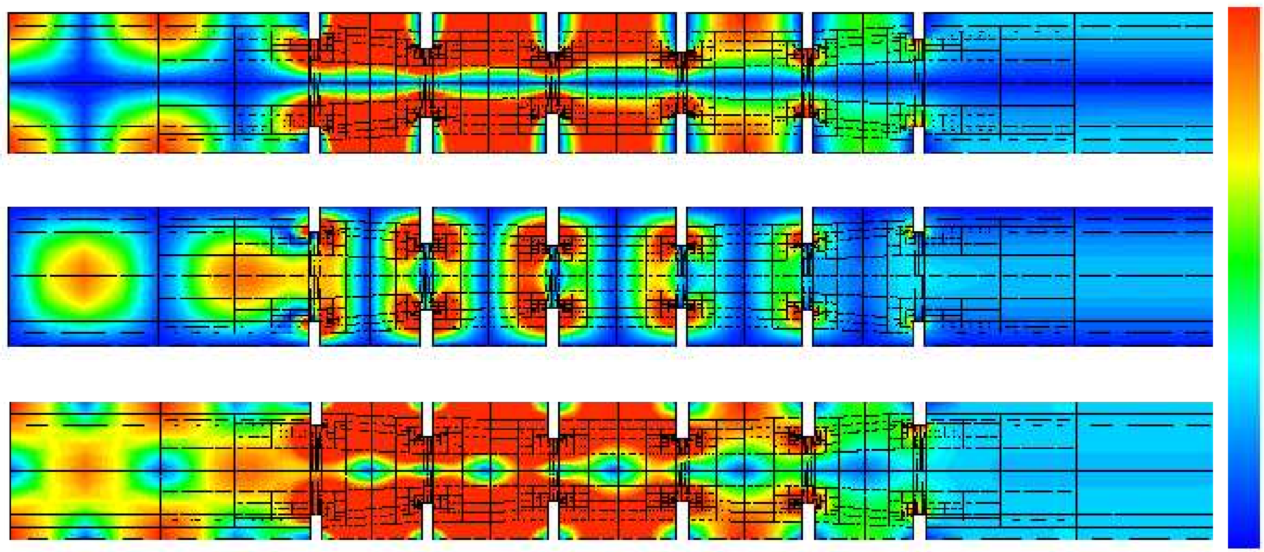
9. ELECTROMAGNETIC APPLICATIONS

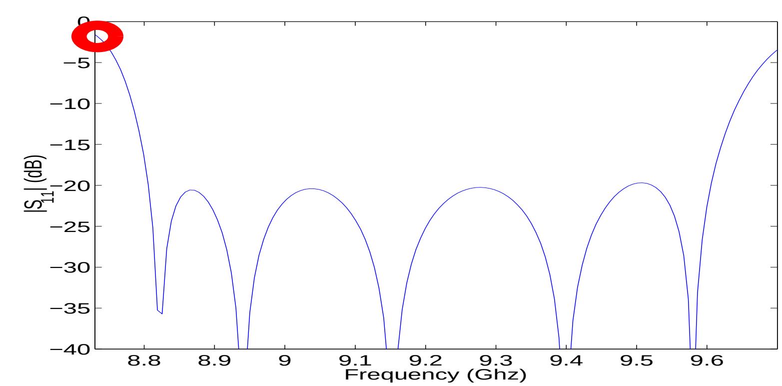
FEM solution for frequency $= 8.72$ Ghz

$|H_x|$

$|H_y|$

$\sqrt{|H_x|^2+|H_y|^2}$





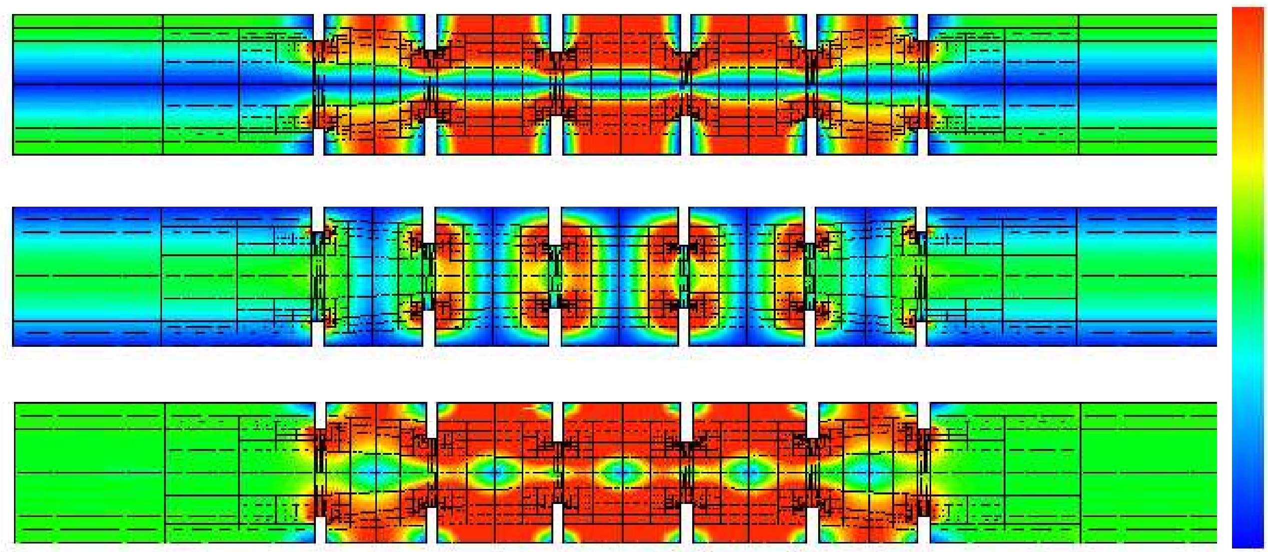
9. ELECTROMAGNETIC APPLICATIONS

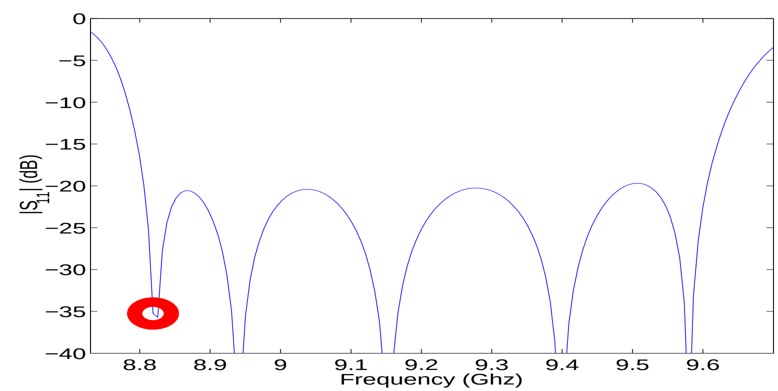
FEM solution for frequency $= 8.82$ Ghz

$|H_x|$

$|H_y|$

$\sqrt{|H_x|^2+|H_y|^2}$





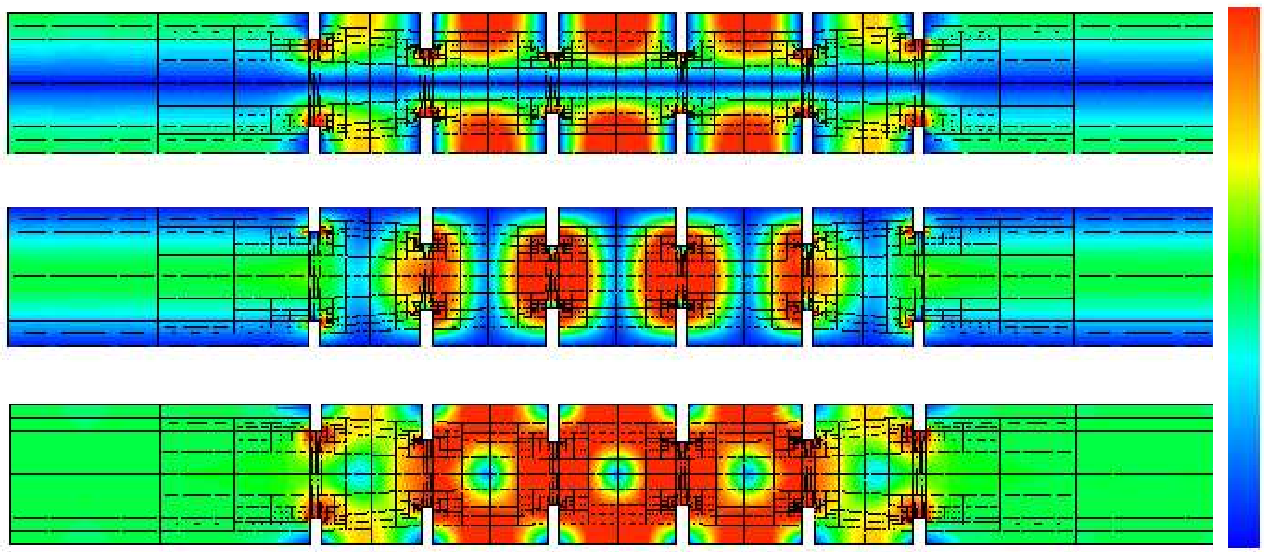
9. ELECTROMAGNETIC APPLICATIONS

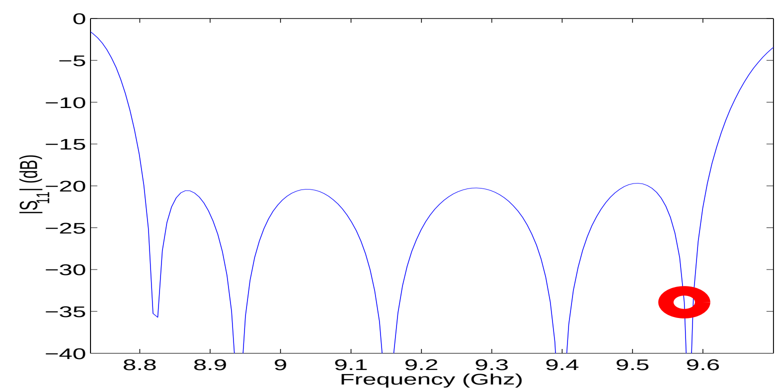
FEM solution for frequency $= 9.58$ Ghz

$|H_x|$

$|H_y|$

$\sqrt{|H_x|^2+|H_y|^2}$





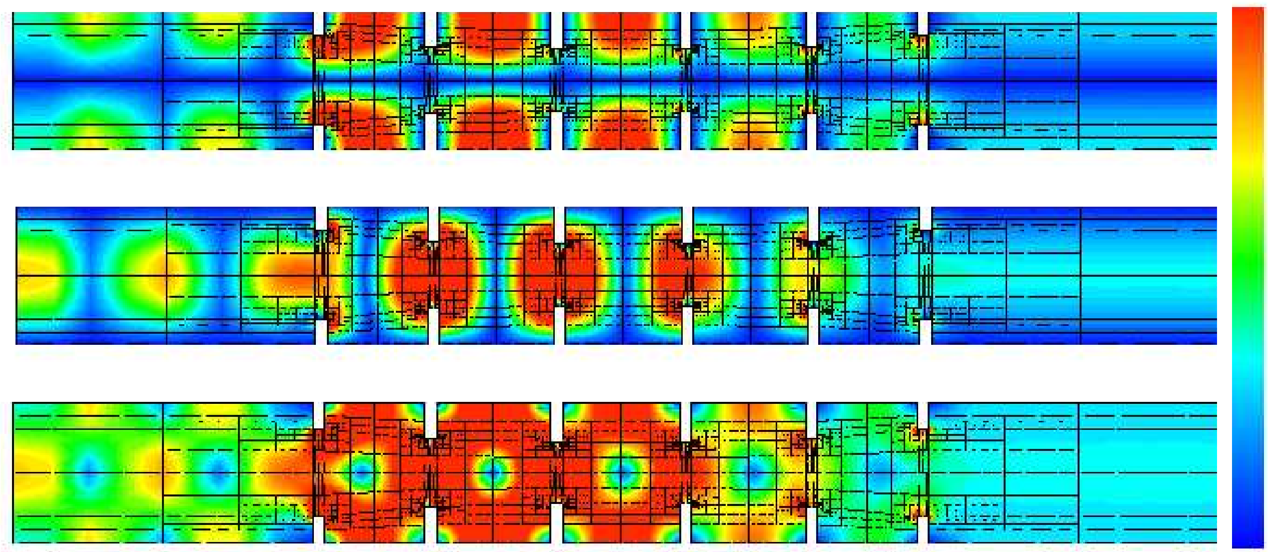
9. ELECTROMAGNETIC APPLICATIONS

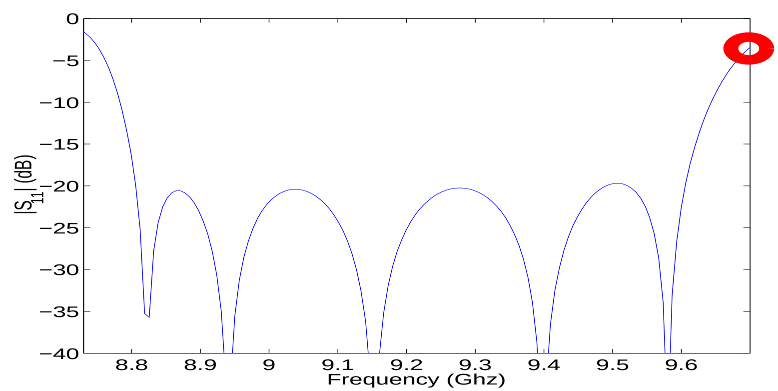
FEM solution for frequency $= 9.71$ Ghz

$|H_x|$

$|H_y|$

$\sqrt{|H_x|^2+|H_y|^2}$



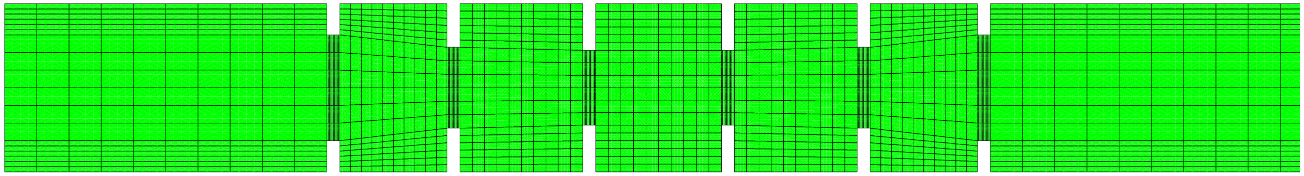


9. ELECTROMAGNETIC APPLICATIONS

Griding Techniques for the Waveguide Problem

Our refinement technology incorporates:

An hp-adaptive algorithm	A two grid solver
Low dispersion error	Convergence of iterative solver
Small h is not enough	Insensitive to p-enrichment ($1 \leq p \leq 4$)
Large p required	Coarse grid sufficiently fine
Waveguide example: $p \approx 3$	Waveguide example: $\lambda / h \approx 9$



Limitations of the hp-strategy for wave propagation problems:

We need large p and small h.

9. ELECTROMAGNETIC APPLICATIONS

Griding Techniques for the Waveguide Problem

Does convergence (or not) of the two grid solver depends upon h and/or p? How?

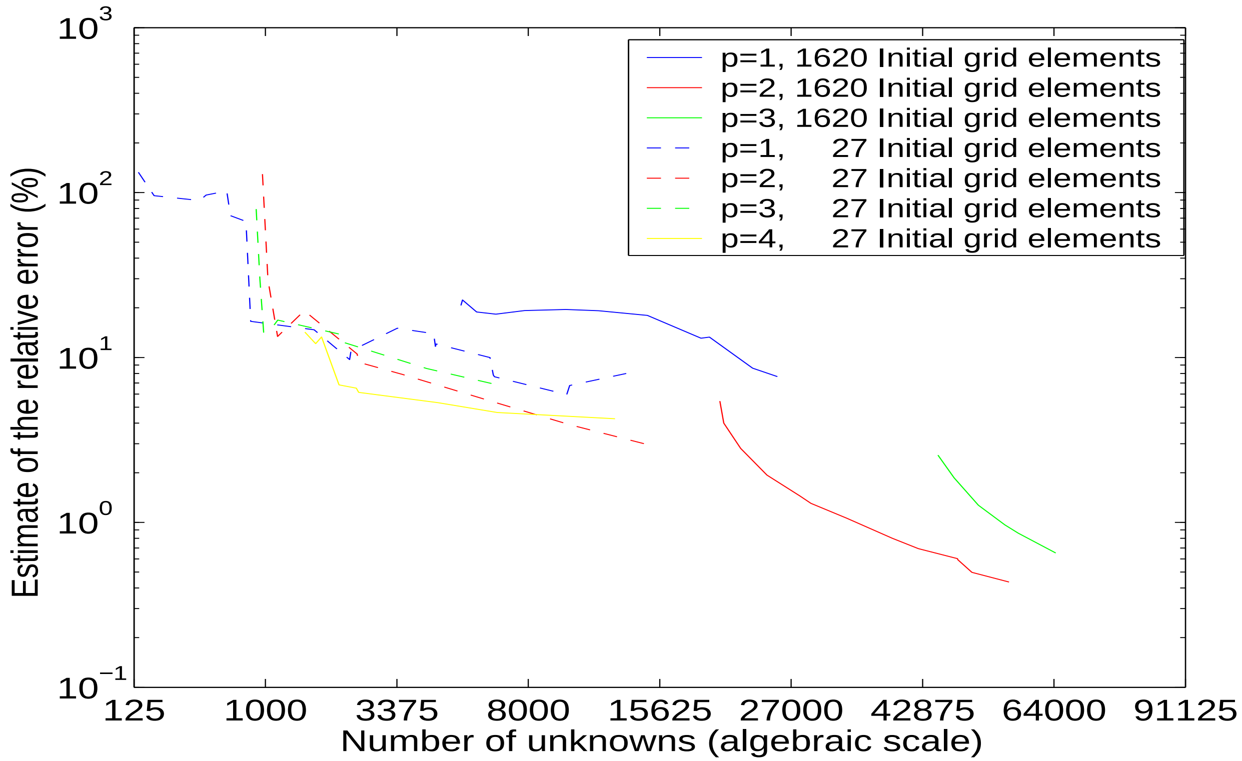
Convergence of two grid solver	$p = 1$	$p = 2$	$p = 3$	$p = 4$
Nr. of elements per $\lambda = 7, 13$	YES	YES	YES	YES
Nr. of elements per $\lambda = 7, 11$	NO	NO	NO	YES
Nr. of elements per $\lambda = 6, 13$	NO	NO	NO	NO

Convergence (or not) of the two grid solver is (almost) insensitive to p-enrichment.

9. ELECTROMAGNETIC APPLICATIONS

Griding Techniques for the Waveguide Problem

Convergence history for different initial grids

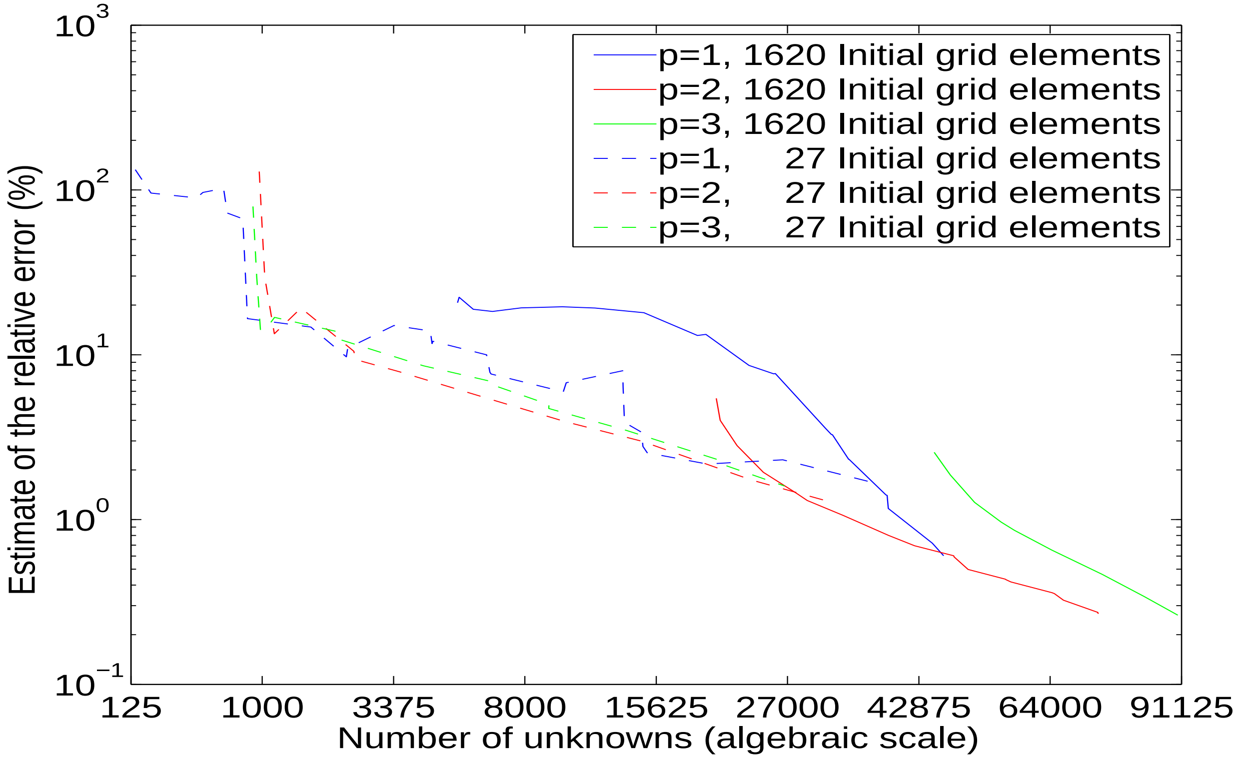


Conclusion : We need to control the dispersion error.

9. ELECTROMAGNETIC APPLICATIONS

Griding Techniques for the Waveguide Problem

Convergence history for different initial grids



Conclusion : Do we need to control the dispersion error?

10. CONCLUSIONS AND FUTURE WORK

- **Exponential convergence is achieved for real world problems by using a fully automatic hp-adaptive strategy.**
- **Multigrid for highly nonuniform hp-adaptive grids is an efficient iterative solver.**
- **It is possible to guide hp-adaptivity with partially converged solutions.**
- **There is a compromise between large p and small h on the design of the initial grid.**
- **This numerical method can be applied to a variety of real world EM problems.**

10. CONCLUSIONS AND FUTURE WORK

Completed tasks

- Designed and implemented a 2D and 3D version of the two grid solver for elliptic problems.
- Studied numerically the 2D and 3D versions of the two grid solver.
- Designed, studied and implemented a two grid solver for 2D Maxwell's equations.
- Studied and designed an error estimator for a two grid solver for Maxwell's equations.
- Studied performance of different smoothers (in context of the two grid solver) for Maxwell's equations.
- Designed, studied, and implemented a flexible CG/GMRES method that is suitable to accelerate the two grid solver for Maxwell's equations.
- Developed a convergence theory for all algorithms mentioned above.
- Applied the hp-adaptive strategy combined with the two grid solver in order to solve a number of problems related to waveguide filters design, and modeling of LWD electromagnetic measuring devices.

Future Tasks	Completion date
• Solve the 3D Fickera problem using hp-adaptivity and the two grid solver.	• NOV 2003
• Implement and study a two grid solver for 3D Maxwell's equations.	• DEC 2003
• Utilize this technology to solve a 3D model problem related to Radar Cross Section (RCS) analysis.	• JAN 2004
• Write and defend dissertation.	• MAR 2004